A structural risk-neutral model for pricing and hedging power derivatives Energy Finance 2010 Conference - Duisburg University - Essen

René Aïd, Luciano Campi, Nicolas Langrené Paris-Dauphine University - Paris Diderot University EDF R&D - FiME Research Centre



## Agenda



- Electricity prices modeling
- Related works

### 2 Spot model

- Design
- Estimation
- O Pricing & hedging
  - Futures
  - Options



Electricity prices modeling Related works

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## Looking for a power spot price model

### Applications

- pricing of derivatives on the spot
- asset valuation (strip of hourly fuel spread options)
- hedging
- energy market risk management

- realistic
- o robust
- tractable
- consistent

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## Modeling strategies

### Modeling futures prices

pros modeling the real available instruments cons introduction of many parameters to reconstruct hourly futures prices

### Modeling spot prices

Exogeneous

O Equilibrium

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### Related works

### Electricity prices exogeneous dynamics

Deng (00), Benth et al. (03, 07, 09), Burger et al. (04), Kolodnyi (04), Cartea & Figueroa (05), Geman & Roncoroni (06)

### Equilibrium model

Pirrong & Jermakyan (00) Barlow (02) Kanamura & Ohashi (07) Cartea & Villaplana (08) Coulon & Howison (09) Lyle & Elliot (09) Spot Futures Options

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Pirrong & Jermakyan (00)			

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Pirrong & Jermakyan (00)	×	×	
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### This talk

### Objectives

pricing and hedging power derivatives... ... using an improved version of A., Campi Nguyen & Touzi (09) Structural Risk-Neutral model Spot Futures Options .. Campi, Nguyen & Touzi (09) × × improved SRN model × × ×

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Design Estimatior

# Initial SRN Model

### Variables

fuels, $1 \le i \le n$

### Electricity price (€/MWh)

# $\widehat{P}_t = \sum_{i=1}^n h_i S_t^i \mathbf{1}_{ig\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^kig\}}$

Design Estimation

# Initial SRN Model

### Variables

n	fuels, $1 \le i \le n$
	demand (MW)
	heat rates $(h_i S^i_t$ en $\in$ /MWh, $\nearrow$ en $i)$

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Design Estimation

# Initial SRN Model

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n	fuels, $1 \leq i \leq n$
$D_t$	demand (MW)

- capacities (en MW)
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Design Estimation

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Design Estimation

# Initial SRN model

### Pros

• Consistency between electricity prices and fuel prices

• Consistency between electricity prices and demand

Design Estimation

# Initial SRN model

### Pros

• Consistency between electricity prices and fuel prices

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#### Cons

Marginal fuel cost is not the spot price.

Design Estimation

## Initial SRN model

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Marginal fuel cost is not the spot price.

Design Estimation

## Initial SRN model

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# Marginal fuel cost is not the spot price Non-convex technical constraints (may lead to negative prices) Strategic behaviour (Hortacsu & Puller, RAND 1 of Economics 2008)

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Design Estimation

## Initial SRN model

### Pros

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- Marginal fuel cost is not the spot price
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  - Fixed cost recovery problem for peak-load generation plants

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# Initial SRN model

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Design Estimation

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# Initial SRN Model - illustration

### Spot price (in €/MWh)



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## Initial SRN Model - illustration

#### Spot price (in €/MWh)



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## Initial SRN Model - illustration

#### Spot price (in €/MWh)



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### Improved SRN model

• Marginal fuel cost  $\widehat{P}_t := \sum_{i=1}^n h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^{i} C_t^k\right\}}$ 

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- Available capacity  $\overline{C}_t := \sum_{k=1}^n C_t^k$

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Design Estimation

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- Available capacity  $\overline{C}_t := \sum_{k=1}^n C_t^k$
- Price spikes occur when the electric system is under stress, i.e.  $\overline{C}_t D_t$  is small

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Design Estimation

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- Corresponds to peak-load fixed cost problem recovery...

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Design Estimation

## Improved SRN model

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- Corresponds to peak-load fixed cost problem recovery...

$$y_t := \frac{P_t}{\widehat{P}_t}$$
 as a (nonlinear) function of  $x_t := \overline{C}_t - D_t$ 

Design Estimation

## Improved SRN model - Estimation





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### Figure: PowerNext - 19th hours Nov, 13th 06 to April 30th 10

Design Estimation

### Improved SRN model - Estimation





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### Figure: PowerNext - 19th hours Nov, 13th 06 to April 30th 10

Design Estimation

## Improved SRN model - Estimation





- Decreasing relation
- Difficult estimation



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Figure: PowerNext - 19th hours Nov, 13th 06 to April 30th 10

Design Estimation

## Improved SRN model - Estimation





- Decreasing relation
- Difficult estimation



Figure: PowerNext - 19th hours Nov, 13th 06 to April 30th 10

Design Estimation

## Improved SRN model - Estimation



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Design Estimation

## Improved SRN model - Estimation



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Design Estimation

# Improved SRN model - Estimation



Design Estimation

## Improved SRN model - Estimation



Design Estimation

## Improved SRN model - Estimation



Design Estimation

### Improved SRN model - Estimation



Design Estimation

# Improved SRN model - Estimation

Estimated relation : 
$$y_t = \frac{\gamma}{x_t^{\nu}}$$

### Improved SRN model

$$P_t = g\left(\sum_{k=1}^n C_t^k - D_t\right) \times \left(\sum_{i=1}^n h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^i C_t^k\right\}}\right)$$

with scarcity function

$$g(x) := \min\left(\frac{\gamma}{x^{\nu}}, M\right) \mathbf{1}_{\{x > 0\}} + M \mathbf{1}_{\{x \le 0\}}$$

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Design Estimation

# Improved SRN model - Back-testing

#### Spot price (in €/MWh)



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Design Estimation

## Improved SRN model - Back-testing

#### Spot price (in €/MWh)


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### Improved SRN model - Backtesting

#### Spot price (in €/MWh)



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Futures Options

# Pricing & hedging

### Pricing

- incomplete market
- need for a hedging criterion
- Super-replication, utility indifference or mean-variance
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Futures Options

# Pricing & hedging

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Futures Options

### Futures

# Futures prices $F_{t}^{e}\left(T ight)=\mathbb{E}_{t}^{\widehat{\mathbb{Q}}}\left[e^{-r\left(T-t ight)}P_{T} ight]$

$$F_{t}^{e}(T) = \sum_{i=1}^{n} h_{i}G_{i}^{T}(t, C_{t}, D_{t}) F_{t}^{i}(T)$$

with :

$$G_i^{T}(t,C_t,D_t) = \mathbb{E}_t \left[ g \left( \sum_{k=1}^n C_T^k - D_T \right) \mathbf{1}_{\left\{ \sum_{k=1}^{i-1} C_T^k \le D_T \le \sum_{k=1}^{i} C_T^k \right\}} \right]$$

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### Futures prices - hedging

### Demand & capacities

$$dD_{t} = a(t, D_{t}) dt + b(t, D_{t}) dW_{t}^{D}$$
$$dC_{t}^{i} = \alpha_{i} (t, C_{t}^{i}) dt + \beta_{i} (t, C_{t}^{i}) dW_{t}^{C,i}$$

### Futures price dynamics

$$dF_t^e(T) = \sum_{i=1}^n h_i \left[ G_i^T(t, C_t, D_t) dF_t^i(T) + F_t^i(T) dG_i^T(t, C_t, D_t) \right]$$

$$dG_i^{T}(t, C_t, D_t) = \sum_{k=1}^{n} \frac{\partial G_i^{T}}{\partial c_k}(t, C_t, D_t) \beta_k(t, C_t^k) dW_t^{C,k} + \frac{\partial G_i^{T}}{\partial z}(t, C_t, D_t) b(t, D_t) dW_t^D$$

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Futures prices - hedging

• To go further, need to choose dynamics for demand and capacities

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Futures prices - hedging

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- $G_i^T$  explicite as function of extended incomplete Goodwin-Staton integral :

$$\widetilde{\mathcal{G}}(x,y;\nu) = \int_{x}^{\infty} \frac{1}{(y+z)^{\nu}} e^{-z^{2}} dz$$

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 ... for which efficient numerical algorithms are provided in A., Campi & Langrené (10).

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Futures Options

# Futures prices - hedging : spot simulations

#### Spot price (in €/MWh)



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# Futures prices - hedging : spot simulations

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### Futures prices - hedging

### Numerical test

- Hedging an electricity futures with a delivery period of 1 hour
- with a daily rebalanced basket of futures contracts on fuels

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# Futures prices - hedging



#### Sample paths (in €)

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### Futures prices - hedging



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### Futures prices - hedging



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# Futures prices - hedging



### Remarks

- Positive values are losses
- Far from maturity : perfect hedge; electricity futures is equivalent to a basket of fuels

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Futures Options

# Futures prices - hedging



#### Remarks

- Positive values are losses
- Far from maturity : perfect hedge; electricity futures is equivalent to a basket of fuels
- Close to maturity : inefficient hedge

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Futures Options

# Spread options (do not panic)

#### Spread option with a 2 fuel model

The price  $\pi_0$  at time t = 0 of a call spread option with pay-off  $H = (P_T - h_1 S_T^1 - K)^+$  is given by :

$$\pi_{0} = \int_{\mathbb{R}^{2}} f_{C_{T}^{1} - D_{T}}(z) f_{C_{T}^{2}}(c) \left\{ \phi_{1}(c, z) \mathbf{1}_{\{z \geq 0\}} + \phi_{2}(c, z) \mathbf{1}_{\{z \leq 0\}} \right\} dcdz,$$

$$\phi_1 = (g-1)BS_0(\sigma_1, K)\mathbf{1}_{\{g>1\}}$$

$$\phi_{2} = g \int_{0}^{\infty} \hat{f}_{Y_{T}^{1}}(y) BS_{0}\left(\sigma_{2}, \frac{K + (1 - g)y}{g}\right) \left(\mathbf{1}_{\{g \leq 1\}} + \mathbf{1}_{\{g > 1\}}\mathbf{1}_{\{y < \frac{K}{g - 1}\}}\right) dy$$

$$+\left(gY_0^2\mathcal{N}\left(\frac{\left(r-\frac{\sigma_1^2}{2}\right)T-\ln\left(\frac{\kappa}{(g-1)Y_0^1}\right)}{\sigma_1\sqrt{T}}\right)+\left(g-1\right)BS_0\left(\sigma_1,\frac{\kappa}{g-1}\right)\right)\mathbf{1}_{\{g>1\}}$$

with g := g(c + z).

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# Spread options

### • semi-explicit formula : numerical integration

- partial hedging with futures on fuels and electricity
- applied on European dark spread call option with a period of delivery of 1 hour

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# Spread options



#### Marginal oil probability (%)



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# Spread options



#### Marginal oil probability (%)



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seasonality pattern

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# Spread options



#### Marginal oil probability (%)



- seasonality pattern
- information on planned outages

# Conclusion

### Conclusions

- SRN electricity spot price model with a scarcity function
- allows futures and derivatives pricing and hedging
- nevertheless, only fuels dependancies can he hedged...
- ... and present work only treated hourly futures

#### Perspectives

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- comparison with calibration procedure
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