A structural risk-neutral model for pricing and hedging power derivatives

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René Aïd, Luciano Campi, Nicolas Langrené
Paris-Dauphine University - Paris Diderot University
EDF R&D - FiME Research Centre

Agenda

Looking for a power spot price model

Applications

- pricing of derivatives on the spot
- asset valuation (strip of hourly fuel spread options)
- hedging
- energy market risk management

Model requirements

- realistic
- robust
- tractable
- consistent

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Modeling futures prices

pros modeling the real available instruments

cons introduction of many parameters to reconstruct hourly futures prices

Modeling spot prices

Exogeneous

Equilibrium

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Electricity prices exogeneous dynamics

Deng (00), Benth et al. (03, 07, 09), Burger et al. (04), Kolodnyi (04), Cartea & Figueroa (05), Geman & Roncoroni (06)

Equilibrium model

	Spot	Futures	Options
Pirrong & Jermakyan (00)	×	×	
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pricing and hedging power derivatives...

... using an improved version of A., Campi Nguyen & Touzi (09) Structural Risk-Neutral model

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Initial SRN Model

Variables

n	fuels, $1 \leq i \leq n$
D_t	demand (MW)
C_t^i	capacities (en MW)
S_t^i	fuel prices
h_i	*heat rates* ($h_i S_t^i$ en €/MWh, $\nearrow$ en i)

Electricity price (€/MWh)

$$\widehat{P}_t = \sum_{i=1}^{n} h_i S_t^i \mathbb{1}_{\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\}}$$

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Pros

- Consistency between electricity prices and fuel prices
- Consistency between electricity prices and demand

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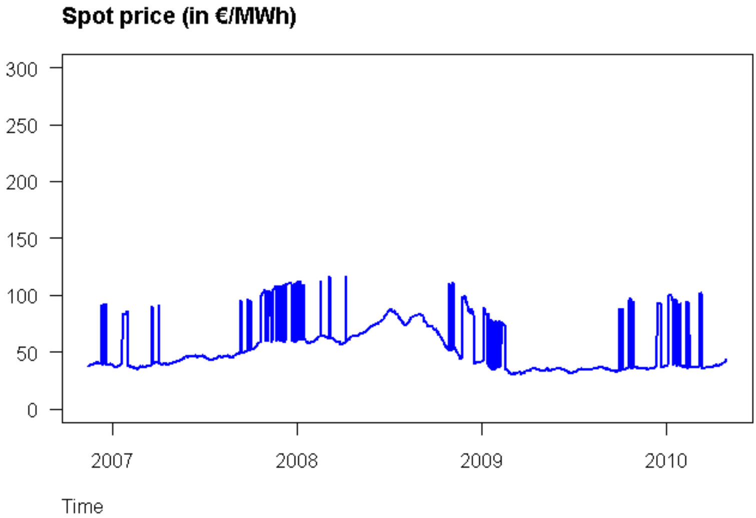
Initial SRN Model - illustration



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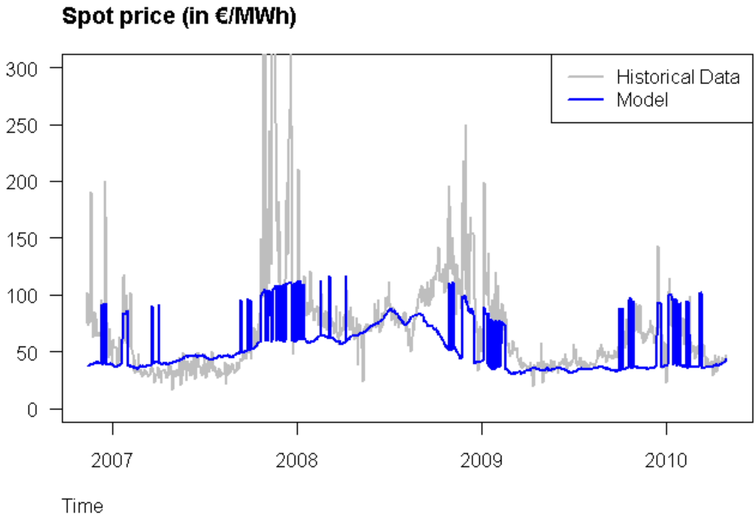

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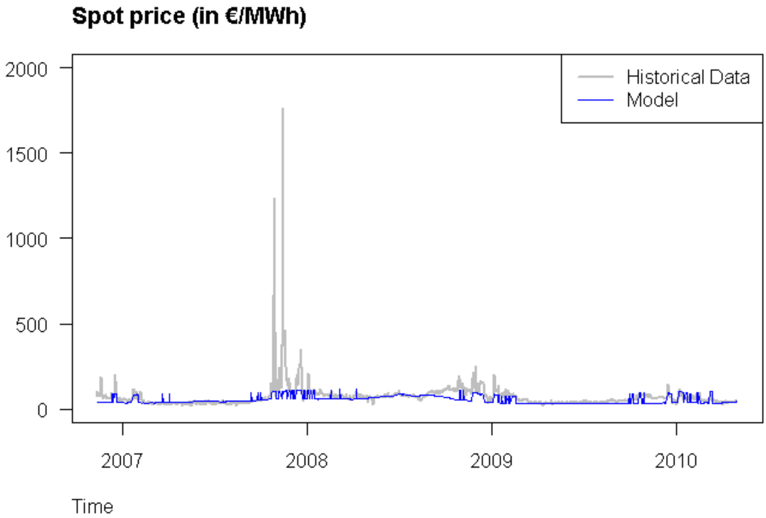

Improved SRN model

- Marginal fuel cost $\widehat{P}_t := \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\}}$

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$y_t := \frac{P_t}{\widehat{P}_t}$ **as a (nonlinear) function of** $x_t := \overline{C}_t - D_t$

Improved SRN model - Estimation

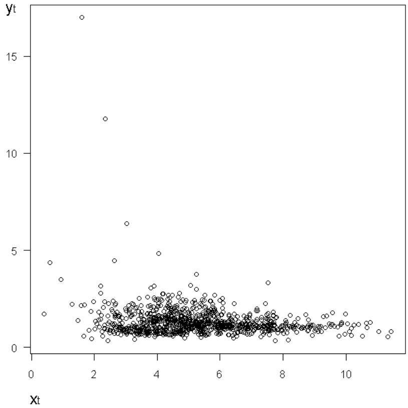


Figure: PowerNext - 19th hours
Nov, 13th 06 to April 30th 10

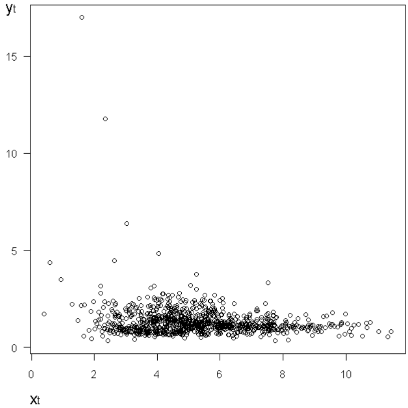
Observation

- Decreasing relation
- Difficult estimation

Idea

- Quantile

Improved SRN model - Estimation



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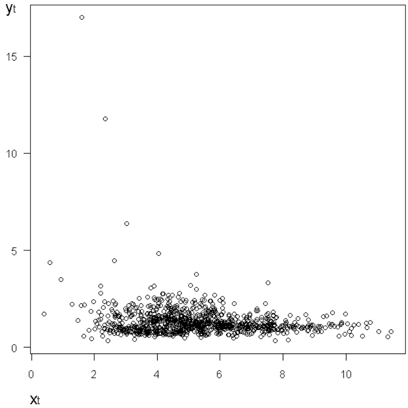

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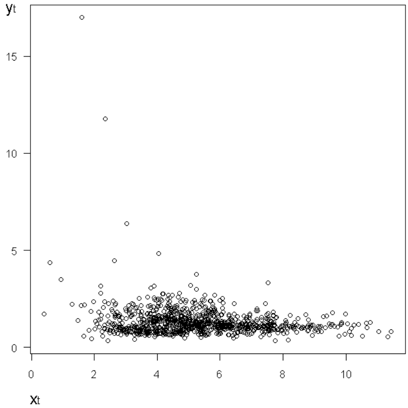

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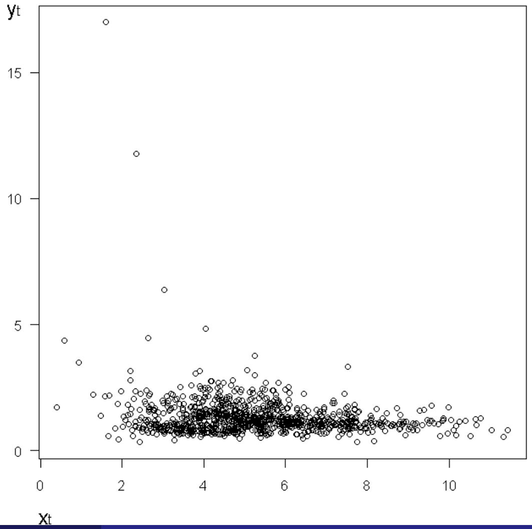

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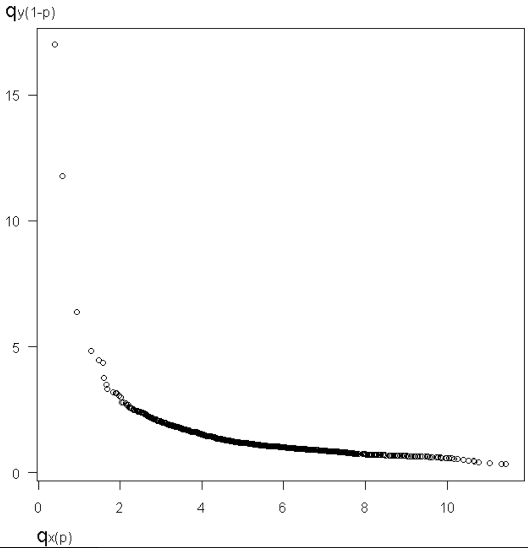

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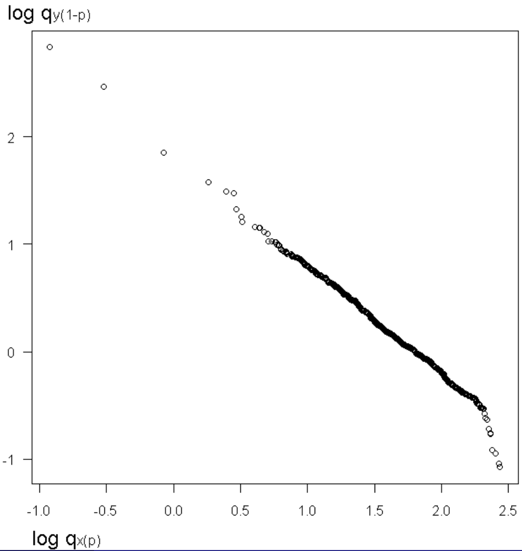

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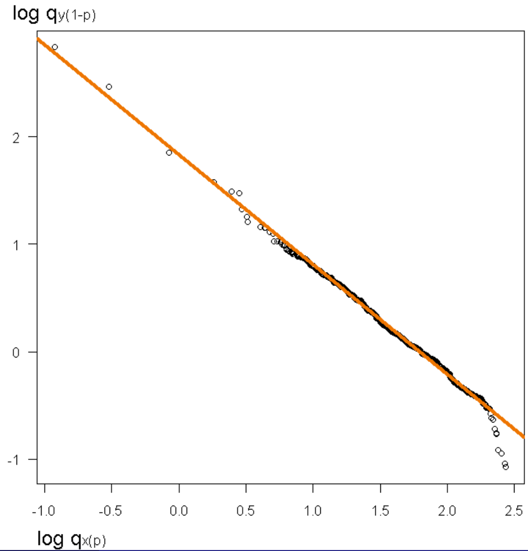

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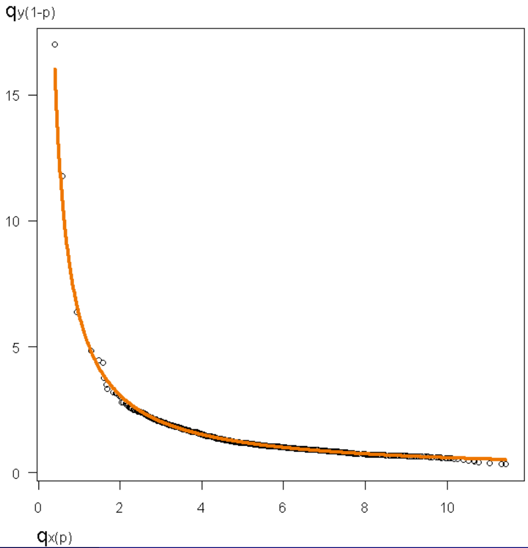

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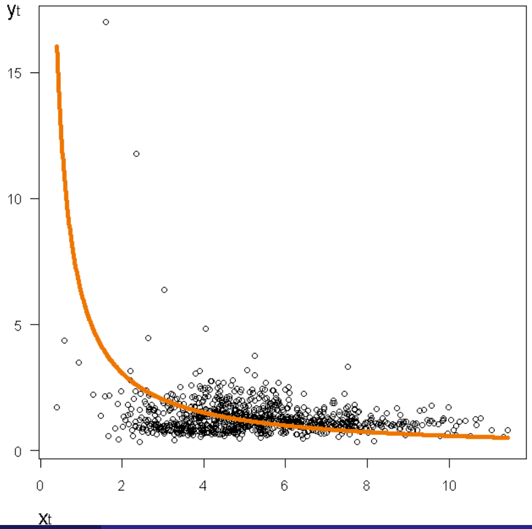


Improved SRN model - Estimation

Estimated relation : $y_t = \frac{\gamma}{x_t^\nu}$

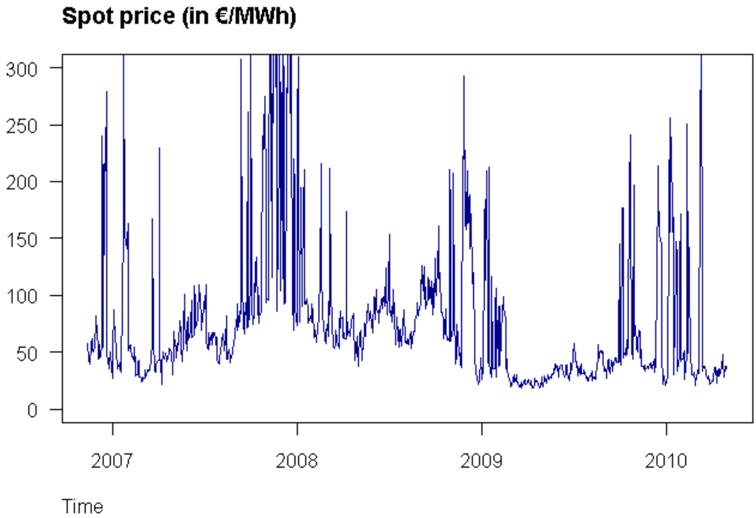
Improved SRN model

$$P_t = g\left(\sum_{k=1}^{n} C_t^k - D_t\right) \times \left(\sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\right\}}\right)$$

with scarcity function

$$g(x) := \min\left(\frac{\gamma}{x^\nu}, M\right) \mathbf{1}_{\{x>0\}} + M \mathbf{1}_{\{x \leqslant 0\}}$$

Improved SRN model - Back-testing



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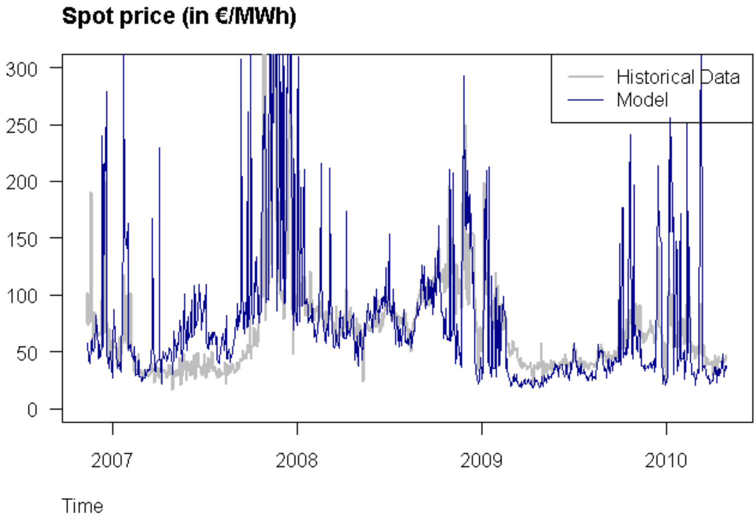
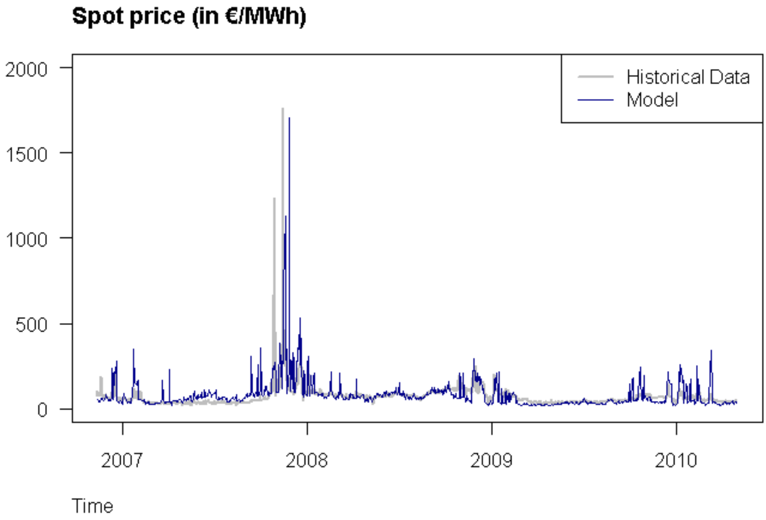


Improved SRN model - Backtesting



Pricing & hedging

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- incomplete market
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Futures

Futures prices $F^e_t(T) = \mathbb{E}^{\widehat{\mathbb{Q}}}_t\left[e^{-r(T-t)}P_T\right]$

$$F^e_t(T) = \sum_{i=1}^{n} h_i G_i^T(t, C_t, D_t)\, F^i_t(T)$$

with :

$$G_i^T(t,C_t,D_t) = \mathbb{E}_t\left[g\left(\sum_{k=1}^{n} C_T^k - D_T\right)\mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_T^k \leq D_T \leq \sum_{k=1}^{i} C_T^k\right\}}\right]$$

Futures prices - hedging

Demand & capacities

$$dD_t = a(t, D_t)\, dt + b(t, D_t)\, dW_t^D$$
$$dC_t^i = \alpha_i\left(t, C_t^i\right) dt + \beta_i\left(t, C_t^i\right) dW_t^{C,i}$$

Futures price dynamics

$$dF_t^e(T) = \sum_{i=1}^n h_i \left[G_i^T(t, C_t, D_t) dF_t^i(T) + F_t^i(T) dG_i^T(t, C_t, D_t)\right]$$

$$dG_i^T(t, C_t, D_t) = \sum_{k=1}^n \frac{\partial G_i^T}{\partial c_k}(t, C_t, D_t)\beta_k(t, C_t^k) dW_t^{C,k} + \frac{\partial G_i^T}{\partial z}(t, C_t, D_t) b(t, D_t) dW_t^D$$

Futures prices - hedging

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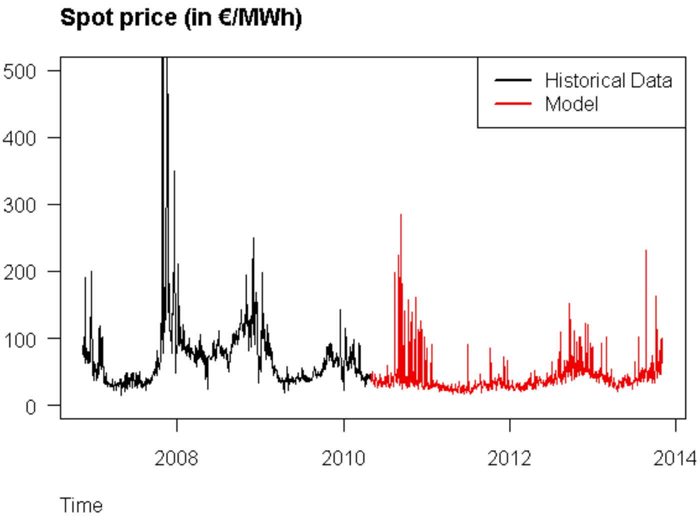
- ... for which efficient numerical algorithms are provided in A., Campi & Langrené (10).

Futures prices - hedging : spot simulations



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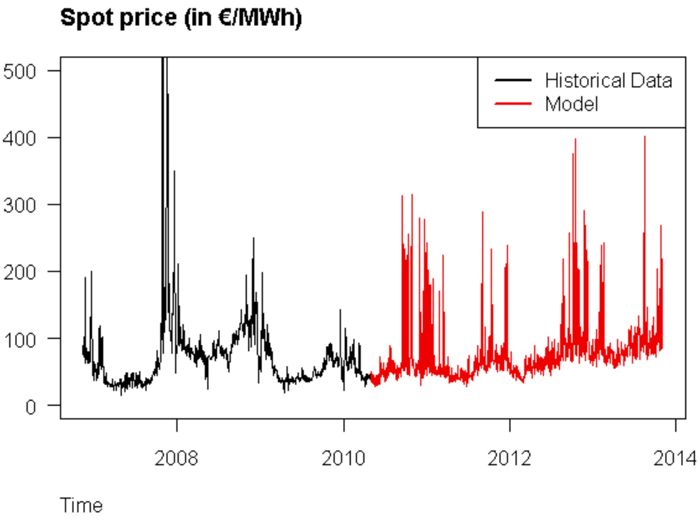

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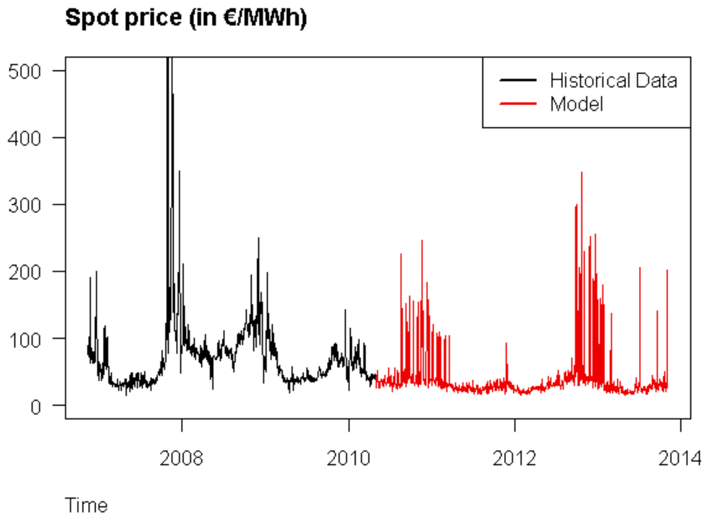

Futures prices - hedging

Numerical test

- Hedging an electricity futures with a delivery period of 1 hour
- with a daily rebalanced basket of futures contracts on fuels

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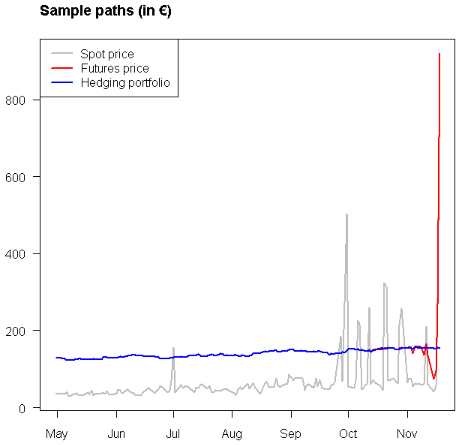

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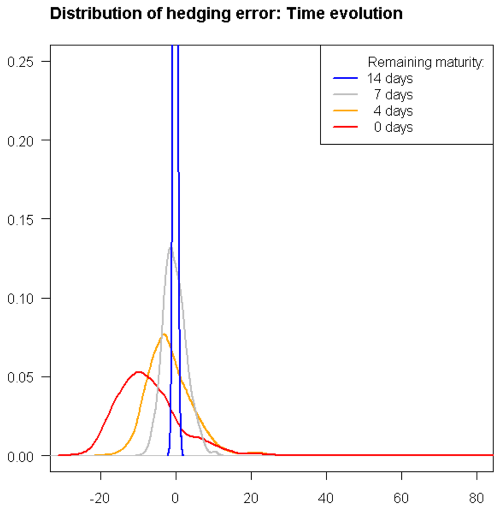

Remarks

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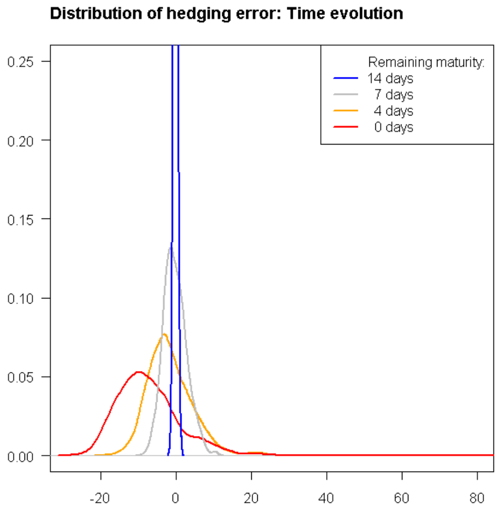

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- Positive values are losses

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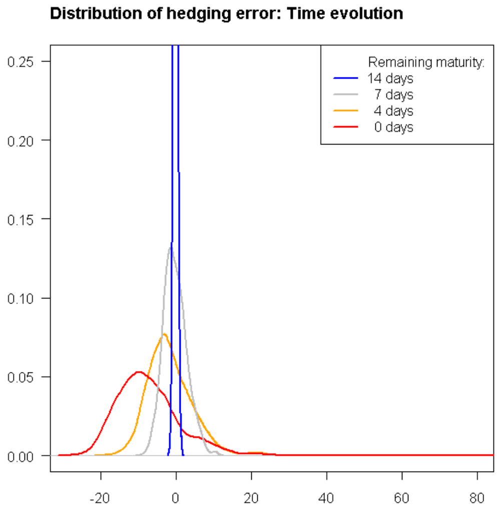

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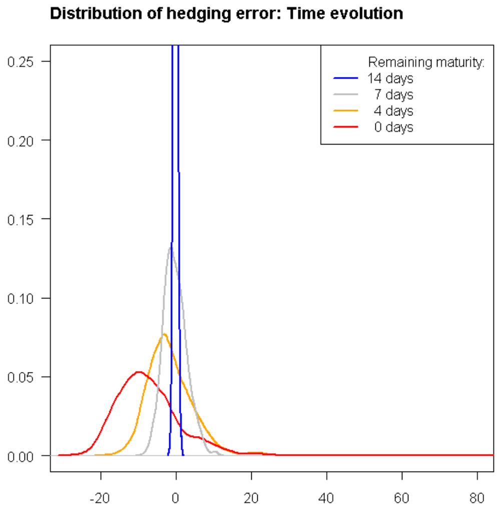

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Spread options (do not panic)

Spread option with a 2 fuel model

The price π_0 at time $t = 0$ of a call spread option with pay-off $H = (P_T - h_1 S_T^1 - K)^+$ is given by :

$$\pi_0 = \int_{\mathbb{R}^2} f_{C_T^1 - D_T}(z) f_{C_T^2}(c) \left\{ \phi_1(c,z) \mathbf{1}_{\{z>0\}} + \phi_2(c,z) \mathbf{1}_{\{z\le 0\}} \right\} dcdz,$$

$$\phi_1 = (g-1) BS_0(\sigma_1, K) \mathbf{1}_{\{g>1\}}$$

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$$+ \left(g Y_0^2 \mathcal{N}\left(\frac{\left(r - \frac{\sigma_1^2}{2}\right) T - \ln\left(\frac{K}{(g-1)Y_0^1}\right)}{\sigma_1 \sqrt{T}} \right) + (g-1)\, BS_0\left(\sigma_1, \frac{K}{g-1}\right) \right) \mathbf{1}_{\{g>1\}}$$

with $g := g(c+z)$.

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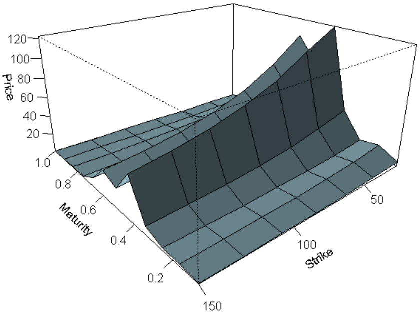

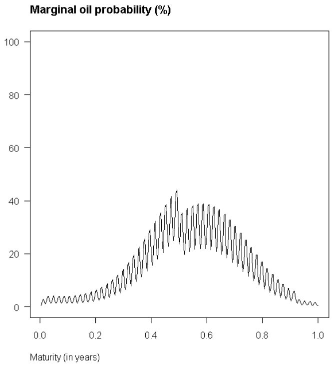

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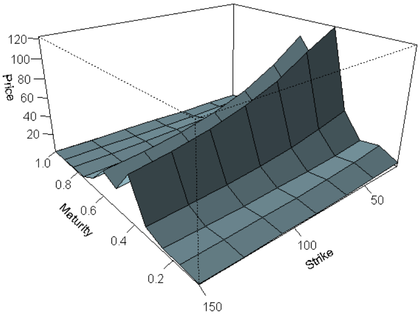

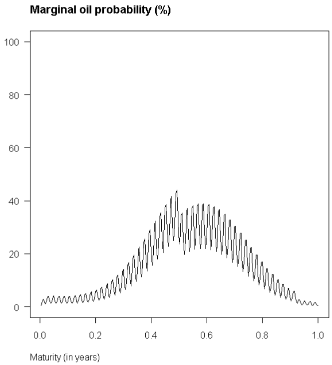

- seasonality pattern

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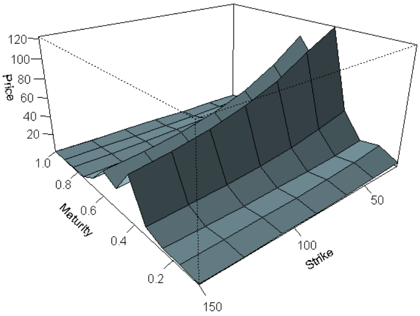

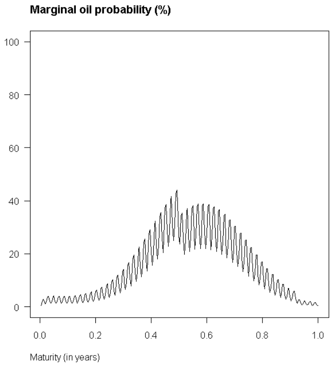

- seasonality pattern
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References

- A., Campi, Nguyen Huu & Touzi, *Int. J. Theoretical & Applied Finance*, 2010
- Barlow, *Math. Finance*, 2002
- Benth & Koekebakker, *J. of Energy Economics*, 2007
- Benth & Vos, *Tech. Rep., Math. Dept. Oslo*, 2009
- Benth, Ekeland, Hauge & Nielsen, *Applied Math. Finance*, 2003
- Burger, Klar, Müller & Schlindlmayr, *Quantitative Finance*, 2004

References

- Cartea & Figueroa, *Applied Math. Finance*, 2005
- Cartea & Villaplana, *J. of Banking & Finance*, 2008
- Coulon & Howison, *J. of Energy Markets*, 2009
- Deng, *Tech. Rept.,California Energy Institute*, 2000
- Geman & Roncoroni, *J. of Business*, 2006
- Kanamura & Ohashi, *Energy Economics*, 2007
- Kolodnyi, *J. of Engineering Mathematics*, 2004

References

- Lyle & Elliott, *Energy Economics*, 2009
- Pham, *Math. Meth. of Operations Research*, 2000
- Pirrong & Jermakyan, *J. of Banking & Finance*, 2008 [1]
- Schweizer, *Handbook Math. Finance, Cambridge Univ. Press*, 2001

1. Olin Business School Tech. Rep. 2000