Singular BSDEs & Emissions Derivative Pricing

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Price Formation: Equilibrium Analysis

R.C, M. Fehr, J. Hinz, A. Prochet

- Putting a Price on CO_2 by **internalizing** its Social Cost
- Regulatory Economic Instruments
  - Carbon **TAX**
  - Permits Allocation & Trading (**Cap-and-Trade**)
- Calibrate the Different Schemes for
  - **MEANINGFUL** & **FAIR** comparisons
- **Dynamic Stochastic General Equilibrium**
- **Inelastic** Demand (to start with)
  - Electricity Production for the purpose of **illustration**
- Novelty (if any): **Random** Factors
  - **Demand** for goods $\{D_t^k\}_{t\geq 0}$
  - **Costs** of Production $\{C_t^{i,j,k}\}_{t\geq 0}$
    - Spot Price of Coal
    - Spot Price of Natural Gas

Reduced Form Models & Option Pricing

(**Uhrig-Homburg-Wagner, R.C - Hinz**)

- Emissions (Cap-and-Trade) Markets **MAY** or **MAY NOT** exist in the US (and Canada, Australia, Japan, ...)
- In any case, a **liquid** option market **ALREADY** exists in Europe
  - Underlying $\{A_t\}_t$ non-negative martingale with **binary terminal value** (single phase model)
  - Think of A_t as of a binary option
  - Underlying of binary option should be *Emissions*
- Need for **Formulae** (closed or computable)
  - Prices and Hedges difficult to compute (only numerically)
  - Jumps due to announcements (**Cetin** et al.)
- **Reduced Form Models**

Option quotes on Jan. 3, 2008

Option Maturity	Option Type	Volume	Strike	Allowance Price	Implied Vol	Settlement Price
Dec-08	Call	150,000	24.00	23.54	50.50%	4.19
Dec-08	Call	500,000	26.00	23.54	50.50%	3.50
Dec-08	Call	25,000	27.00	23.54	50.50%	3.20
Dec-08	Call	300,000	35.00	23.54	50.50%	1.56
Dec-08	Call	1,000,000	40.00	23.54	50.50%	1.00
Dec-08	Put	200,000	15.00	23.54	50.50%	0.83

Option quotes on Jan. 4, 2008

Option Maturity	Option Type	Volume	Strike	Allowance Price	Implied Vol	Settlement Price
Dec-08	Cal	200,000	22.00	23.55	51.25%	5.06
Dec-08	Call	150,000	26.00	23.55	51.25%	3.57
Dec-08	Call	450,000	27.00	23.55	51.25%	3.27
Dec-08	Call	100,000	28.00	23.55	51.25%	2.99
Dec-08	Call	125,000	29.00	23.55	51.25%	2.74
Dec-08	Call	525,000	30.00	23.55	51.25%	2.51
Dec-08	Call	250,000	40.00	23.55	51.25%	1.04
Dec-08	Call	700,000	50.00	23.55	51.25%	0.45
Dec-08	Put	1,000,000	14.00	23.55	51.25%	0.64
Dec-08	Put	200,000	15.00	23.55	51.25%	0.86
Dec-08	Put	200,000	15.00	23.55	51.25%	0.86
Dec-08	Put	400,000	16.00	23.55	51.25%	1.13
Dec-08	Put	100,000	17.00	23.55	51.25%	1.43
Dec-08	Put	1,000,000	18.00	23.55	51.25%	1.78
Dec-08	Put	500,000	20.00	23.55	51.25%	2.60
Dec-08	Put	200,000	21.00	23.55	51.25%	3.07
Dec-08	Put	200,000	22.00	23.55	51.25%	3.57

Reduced Form Models and Calibration

Allowance price should be of the form

$$A_t = \lambda \mathbb{E}\{\mathbf{1}_N \, | \mathcal{F}_t\}$$

for a non-compliance set $N \in \mathcal{F}_t$. Choose

$$N = \{\Gamma_T \geq 1\}$$

for a random variable Γ_T representing the normalized emissions at compliance time. So

$$A_t = \lambda \mathbb{E}\{\mathbf{1}_{\{\Gamma_T \geq 1\}} \, | \mathcal{F}_t\}, \qquad t \in [0, T]$$

We choose Γ_T in a parametric family

$$\Gamma_T = \Gamma_0 \exp\Big[\int_0^T \sigma_s dW_s - \frac{1}{2}\int_0^T \sigma_s^2 \, ds\Big]$$

for some square integrable deterministic function

$$(0, T) \ni t \hookrightarrow \sigma_t$$

Dynamic Price Model for $a_t = \frac{1}{\lambda} A_t$

- a_t is given by

$$a_t = \Phi\left(\frac{\Phi^{-1}(a_0)\sqrt{\int_0^T \sigma_s^2 ds} + \int_0^t \sigma_s dW_s}{\sqrt{\int_t^T \sigma_s^2 ds}}\right) \qquad t \in [0, T)$$

  where Φ is standard normal c.d.f.

- a_t solves the SDE

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{z_t}dW_t$$

  where the positive-valued function $(0, T) \ni t \hookrightarrow z_t$ is given by

$$z_t = \frac{\sigma_t^2}{\int_t^T \sigma_u^2 du}, \qquad t \in (0, T)$$

Risk Neutral Densities

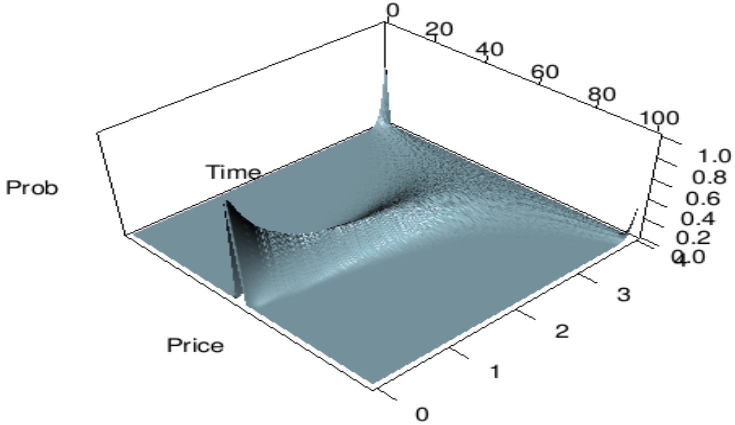


Figure: Histograms for each day of a 4 yr compliance period of 10^5 simulated risk neutral allowance price paths.

Aside: Binary Martingales as Underliers

Allowance prices are given by $A_t = \lambda a_t$ where $\{a_t\}_{0 \le t \le T}$ satisfies

- $\{a_t\}_t$ is a martingale
- $0 \le a_t \le 1$
- $\mathbb{P}\{\lim_{t \to T} a_t = 1\} = 1 - \mathbb{P}\{\lim_{t \to T} a_t = 0\} = p$ for some $p \in (0, 1)$

The model

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{z_t}dW_t$$

suggests looking for martingales $\{Y_t\}_{0 \le t < \infty}$ satisfying

- $0 \le Y_t \le 1$
- $\mathbb{P}\{\lim_{t \to \infty} Y_t = 1\} = 1 - \mathbb{P}\{\lim_{t \to \infty} T_t = 0\} = p$ for some $p \in (0, 1)$

and do a **time change** to get back to the (compliance) interval $[0, T)$

Feller's Theory of 1-D Diffusions

Gives conditions for the SDE

$$da_t = \Theta(a_t)dW_t$$

for $x \hookrightarrow \Theta(x)$ satisfying

- $\Theta(x) > 0$ for $0 < x < 1$
- $\Theta(0) = \Theta(1) = 0$

to

- Converge to the boundaries 0 and 1
- NOT explode (i.e. NOT reach the boundaries in finite time)

Interestingly enough the solution of

$$dY_t = \Phi'(\Phi^{-1}(Y_t))dW_t$$

IS ONE OF THEM !

Explicit Examples

The SDE

$$dX_t = \sqrt{2}dW_t + X_t dt$$

has the solution

$$X_t = e^t \left(x_0 + \int_0^t e^{-s} dW_s\right)$$

and

$$\lim_{t\to\infty} X_t = +\infty \qquad \text{on the set } \{\int_0^\infty e^{-s} dW_s > -x_0\}$$

$$\lim_{t\to\infty} X_t = -\infty \qquad \text{on the set } \{\int_0^\infty e^{-s} dW_s < -x_0\}$$

Moreover Φ is **harmonic** so if we choose

$$Y_t = \Phi(X_t)$$

we have a martingale with the desired properties.

Another (explicit) example can be constructed from Ph. Carmona, Petit and Yor on Dufresne formula.

Calibration

Publicly available option data being unreliable, calibration

Has to Be Historical !!!!

- Choose **Constant** Market Price of Risk
- **Two-parameter** Family for Time-change

$$\{z_t(\alpha,\beta) = \beta(T-t)^{-\alpha}\}_{t\in[0,T]}, \qquad \beta > 0, \alpha \geq 1.$$

Volatility function $\{\sigma_t(\alpha,\beta)\}_{t\in(0,T)}$ given by

$$\begin{aligned}\sigma_t(\alpha,\beta)^2 &= z_t(\alpha,\beta)e^{-\int_0^t z_u(\alpha,\beta)du} \\ &= \begin{cases} \beta(T-t)^{-\alpha} e^{\beta\frac{T^{-\alpha+1}-(T-t)^{-\alpha+1}}{-\alpha+1}} & \text{for } \beta > 0, \alpha > 1 \\ \beta(T-t)^{\beta-1}T^{-\beta} & \text{for } \beta > 0, \alpha = 1 \end{cases}\end{aligned}$$

Maximum Likelihood

Call Option Price in One Period Model

for $\alpha = 1$, $\beta > 0$, the price of an European call with strike price $K \geq 0$ written on a one-period allowance futures price at time $\tau \in [0, T]$ is given at time $t \in [0, \tau]$ by

$$\begin{aligned} C_t &= e^{-\int_t^\tau r_s ds} \mathbb{E}\{(A_\tau - K)^+ \mid \mathcal{F}_t\} \\ &= \int (\lambda \Phi(x) - K)^+ N(\mu_{t,\tau}, \nu_{t,\tau})(dx) \end{aligned}$$

where

$$\begin{aligned} \mu_{t,\tau} &= \Phi^{-1}(A_t/\lambda) \sqrt{\left(\frac{T-t}{T-\tau}\right)^\beta} \\ \nu_{t,\tau} &= \left(\frac{T-t}{T-\tau}\right)^\beta - 1. \end{aligned}$$

Easily extended to several periods

Price Dependence on T and Sensitivity to β

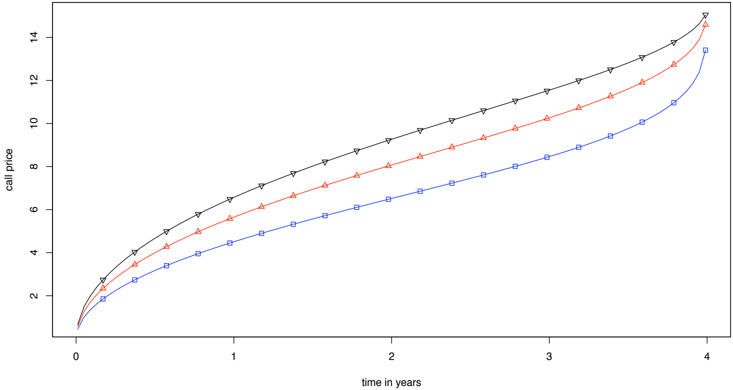
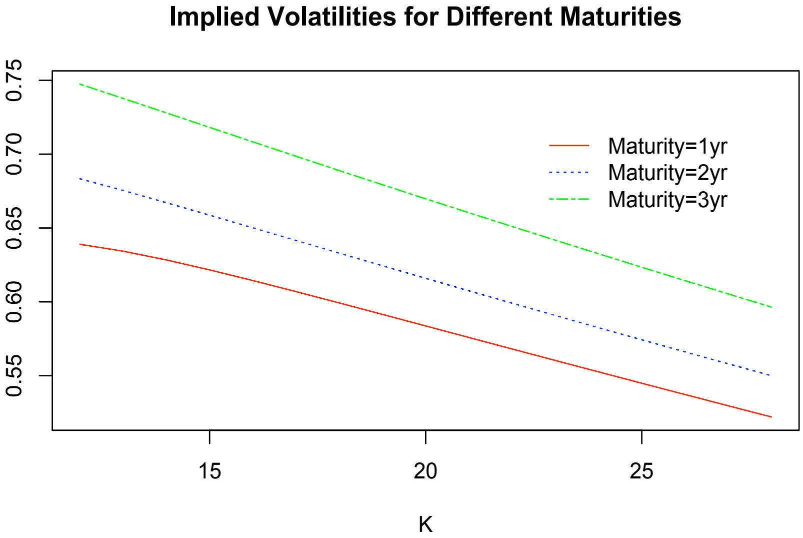
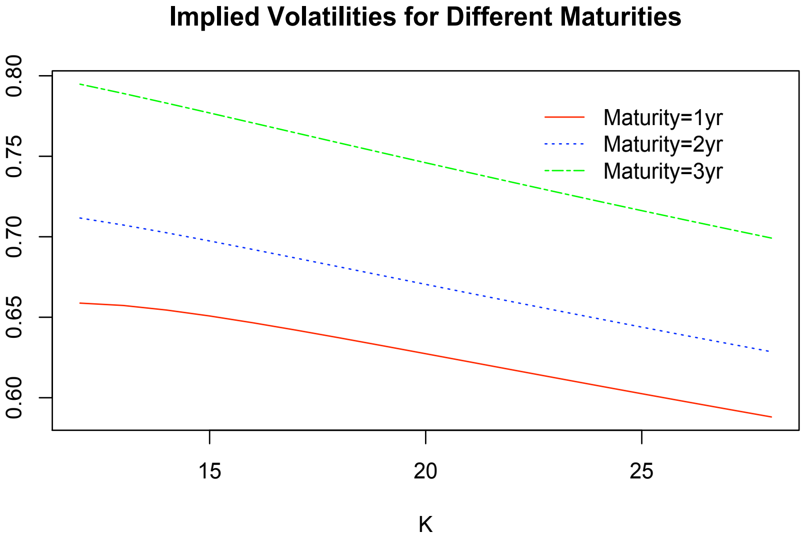


Figure: Dependence $\tau \mapsto C_0(\tau)$ of Call prices on maturity τ. Graphs $\square$, $\triangle$, and $\triangledown$ correspond to $\beta = 0.5$, $\beta = 0.8$, $\beta = 1.1$.


Implied Volatilities for Different Maturities
0.75
0.70
0.65
0.60
0.55
15
20
25
K
Maturity=1yr
Maturity=2yr
Maturity=3yr

Implied Volatilities $\beta = 0.6$, $\lambda = 100$



Option quotes on April 9, 2010

With a Smile Now!

Option Maturity	Option Type	Volume	Strike	Allowance Price	Implied Vol	Settlement Price
Dec-10	Call	750,000	14.00	13.70	29.69	1.20
Dec-10	Call	150,000	15.00	13.70	29.89	0.85
Dec-10	Call	250,000	16.00	13.70	30.64	0.61
Dec-10	Call	250,000	18.00	13.70	32.52	0.34
Dec-10	Call	1,000,000	20.00	13.70	33.07	0.17
Dec-10	Put	1,000,000	10.00	13.70	37.42	0.29
Dec-10	Put	500,000	12.00	13.70	32.12	0.67
Dec-10	Put	500,000	13.00	13.70	30.37	1.01

- Relax **demand inelasticity**
- Include preferences to relax **risk neutrality** (**RC-Delarue-Espinosa-Touzi**)
- "Representative Agent" form already considered in **Seifert-Uhrig-Homburg-Wagner**, **RC-Fehr-Hinz**

Mathematical Set-Up (continuous time)

- $(\Omega, \mathcal{F}, \mathbb{P})$ **historical** probability structure
- W D-dimensional Wiener process on $(\Omega, \mathcal{F}, \mathbb{P})$
- $T > 0$ finite horizon (end of the **single** compliance period)
- $\mathbb{F} = \{\mathcal{F}_t;\ 0 \leq t \leq T\}$ filtration of W

Goal of equilibrium analysis is to derive pollution permit price $\{A_t;\ 0 \leq t \leq T\}$ allowing firms to **maximize their expected utilities simultaneously**

Emissions Dynamics

Assume allowance price $A = \{A_t;\, 0 \le t \le T\}$ exists.

- A is a $\mathbb{F}$-martingale under $\mathbb{Q}$
- $dA_t = Z_t\, dB_t$ for some adapted process Z s.t. $Z_t \neq 0$ a.s. and B D-dim Wiener process for spot martingale measure $\mathbb{Q}$
- $A_T = \lambda \mathbf{1}_{[\Lambda,\infty)}(E_T)$ where
  - λ is the penalty
  - $E_t = \sum_{i \in \mathcal{I}} E^i_t$ is the aggregate of the E^i_t representing the **cumulative emission** up to time t of firm i
  - Λ is the cap imposed by the regulator

Assume the following dynamics **under** $\mathbb{P}$

$$dE^i_t = (b^i_t - \xi^i_t)dt + \sigma^i_t dW_t, \qquad E^i_0 = 0.$$

- $\{E^i_t(\xi^i_t \equiv 0)\}_{0 \le t \le T}$ cumulative emissions of firm i in BAU
- $\{\xi^i_t)\}_{0 \le t \le T}$ abatement rate of firm i
- Assumptions on emission rates b^i_t and volatilities σ^i_t to be articulated later

Individual Firm Optimization Problems

Abatement costs for firm i given by cost function $c_t^i : \mathbb{R} \to \mathbb{R}$

- c^i is C^1 and strictly convex
- c^i satisfies Inada-like conditions for each $t \in [0, T]$

$$(c^i)'(-\infty) = -\infty \qquad \text{and} \qquad (c^i)'(+\infty) = +\infty.$$

- $c^i(0) = \min c_t^i$ ($\xi^i \equiv 0$ corresponds to BAU)

Typical example for c^i

$$\lambda |x|^{1+\alpha},$$

for some $\lambda > 0$ and $\alpha > 0$.

Each firm chooses its **abatement strategy** ξ^i and its **investment** θ^i in allowances. Its **wealth** is given by

$$X_t^i = X_t^{i,\xi,\theta} = x^i + \int_0^T \theta_t^i dA_t - \int_0^T c^i(\xi_t^i)dt - E_T^i A_T.$$

Solving the Individual Firm Optimization Problems

Preferences of firm i given by a C^1, increasing, strictly concave **utility function** $U^i : \mathbb{R} \to \mathbb{R}$ satisfying Inada conditions:

$$(U^i)'(-\infty) = +\infty \qquad \text{and} \qquad (U^i)'(+\infty) = 0.$$

The optimization problem for firm i is:

$$V(x^i) := \sup_{(\xi^i,\theta^i)\in\mathcal{A}^i} \mathbb{E}^{\mathbb{P}}\{U^i(X_T^{i,\xi^i\theta^i})\}$$

If no non-standard restriction on $\mathcal{A}^i$ set of admissible strategies for firm i

Proposition

If an equilibrium allowance price $\{A_t\}_{0\le t\le T}$ exists, then the optimal abatement strategy $\hat{\xi}^i$ is given by

$$\hat{\xi}^i_t = [(c^i)']^{-1}(A_t).$$

NB: The optimal abatement strategy $\hat{\xi}^i$ is independent of the utility function U^i!

Finding the Equilibrium Allowance Price

- Recall

$$dE^i_t = \left[\tilde{b}^i_t - [(c^i)']^{-1}(A_t)\right] dt + \sigma^i_t dB_t, \qquad E^i_0 = 0, \text{ for each } i$$

- Assume
  - $\forall i, \tilde{b}^i_t = \tilde{b}^i(t)E^i_t$ or $\forall i, \tilde{b}^i_t = \tilde{b}^i(t)$
  - $\forall i, \sigma^i_t = \sigma^i(t)$.
- Set

$$b := \sum_{i\in\mathcal{I}} \tilde{b}^i, \ \sigma := \sum_{i\in\mathcal{I}} \sigma^i, \text{ and } f := \sum_{i\in\mathcal{I}} [(c^i)']^{-1}.$$

Therefore we have the following FBSDE

$$dE_t = \{b(t, E_t) - f_t(A_t)\}dt + \sigma(t)dB_t, \qquad E_0 = 0 \tag{1}$$

$$dA_t = Z_t dB_t, \qquad A_T = \lambda \mathbf{1}_{[\Lambda,+\infty)}(E_T), \tag{2}$$

with $b(t, E_t) = b(t)E^\beta_t$ with $\beta \in \{0, 1\}$ and f increasing.

Theoretical Existence and Uniqueness

Theorem

If $\sigma(t) \geq \underline{\sigma} > 0$ then for any $\lambda > 0$ and $\Lambda \in \mathbb{R}$, FBSDE (1)-(2) admits a unique solution $(E, A, Z) \in M^2$. Moreover, A_t is nondecreasing w.r.t λ and nonincreasing w.r.t Λ.

Proof

- Approximate the singular terminal condition $\lambda \mathbf{1}_{[\Lambda,+\infty)}(E_T)$ by increasing and decreasing sequences $\{\varphi_n(E_T)\}_n$ and $\{\psi_n(E_T)\}_n$ of smooth monotone functions of E_T
- Use
  - comparison results for BSDEs
  - the fact that E_T has a density (implying $\mathbb{P}\{E_T = \Lambda\} = 0$)

  to control the limits

PDE Characterization

Assume GBM for BAU emissions (**Chesney-Taschini**, **Seifert-Uhrig-Homburg-Wagner**, **Grüll-Kiesel**) i.e. $b(t,e) = be$ and $\sigma(t,e) = \sigma e$

$$\begin{cases} E_t = E_0 + \int_0^t (bE_s - f(Y_s))ds + \int_0^t \sigma E_s d\tilde{W}_s \\ A_t = \lambda\, \mathbf{1}_{[\Lambda,\infty)}(E_T) - \int_t^T Z_t d\tilde{W}_t. \end{cases} \quad (3)$$

Allowance price A_t constructed as $A_t = v(t, E_t)$ for a function v which **MUST** solve

$$\begin{cases} \partial_t v(t,e) + (be - f(v(t,e)))\partial_e v(t,e) + \frac{1}{2}\sigma^2 e^2 \partial^2_{ee} v(t,e) = 0, \\ v(T,.) = \mathbf{1}_{[\Lambda,\infty)} \end{cases} \quad (4)$$

The price at time t of a **call option** with maturity τ and strike K on an allowance forward contract maturing at time $T > \tau$ is given by

$$V(t, E_t) = \mathbb{E}_t\{(Y_\tau - K)^+\} = \mathbb{E}_t\{(v(\tau, E_\tau) - K)^+\}.$$

V solves:

$$\begin{cases} \partial_t V(t,e) + (be - f(v(t,e)))\partial_e V(t,e) + \frac{1}{2}\sigma^2 e^2 \partial^2_{ee} V(t,e) = 0, \\ V(\tau,.) = (v(\tau,.) - K)^+ \end{cases} \quad (5)$$

Black-Scholes Case: $f \equiv 0$.

$$\begin{aligned} v^0(t,e) &= \lambda \mathbb{P}\left[E^0_T \geq \Lambda | E^0_t = e\right] = \lambda \Phi\left(\frac{\ln(e/\Lambda e^{-b(T-t)})}{\sigma\sqrt{T-t}} - \frac{\sigma\sqrt{T-t}}{2}\right) \\ V^0(t,e) &= \mathbb{E}\left[(v^0(\tau, E^0_\tau) - K)^+ | E^0_t = e\right], \end{aligned}$$

where E^0 is the geometric Brownian motion:

$$dE^0_t = E^0_t[bdt + \sigma d\tilde{W}_t].$$

used as proxy estimation of the cumulative emissions in business as usual.

R.C. - Delarue - Espinosa - Touzi For $\epsilon \geq 0$ small, let v^ϵ and V^ϵ be the prices of the allowances and the option for $f = \epsilon f_0$. We denote by .

$$v^\epsilon(T,.) = \lambda \mathbf{1}_{[\Lambda,\infty)} \quad \text{and} \quad -\partial_t v^\epsilon - (be - \epsilon f_0(v^\epsilon))\partial_e v^\epsilon - \frac{1}{2}\sigma^2 e^2 \partial^2_{ee} v^\epsilon = 0,$$

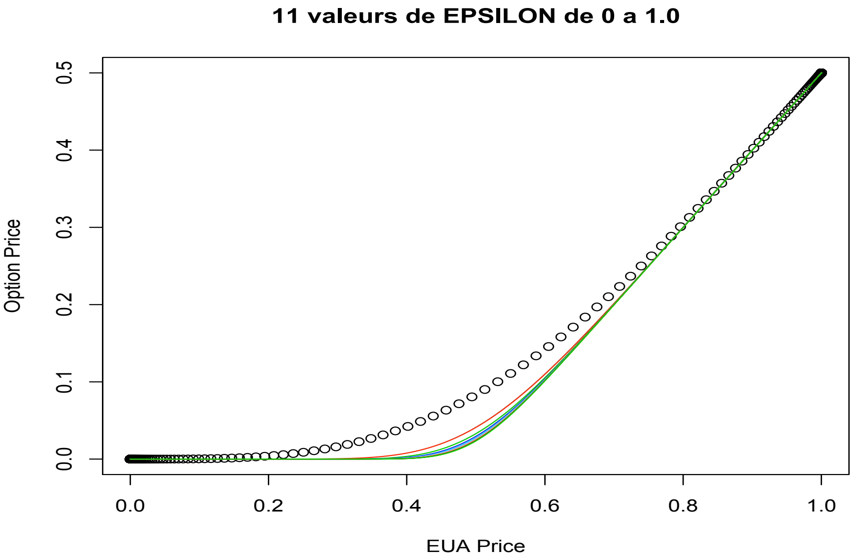
$$V^\epsilon(T,.) = (v^\epsilon(T,.) - K)^+ \quad \text{and} \quad -\partial_t V^\epsilon - (be - \epsilon f_0(v^\epsilon))\partial_e V^\epsilon - \frac{1}{2}\sigma^2 e^2 \partial^2_{ee} V^\epsilon = 0,$$

Proposition

As $\epsilon \to 0$, we have

$$V^\epsilon(t,s) = V^0(t,s)$$
$$+\epsilon\, \mathbb{E}_{t,e}\left[\mathbf{1}_{[\Lambda,\infty)}(v^0)(\tau, E^0_\tau) \int_t^T f_0(v^0)(s, E^0_s)\partial_e v^0(s \vee \tau, E^0_{s\vee\tau}) \frac{E^0_{s\vee\tau}}{E^0_s} ds \right]$$
$$+ \circ(\epsilon),$$

Option Prices



A Slightly Different Model

Single good (e.g. **electricity**) regulated economy, with price dynamics given **exogenously**!

$$\frac{dP_t}{P_t} = \mu(t, P_t)dt + \sigma(t, P_t)dW_t$$

Firm i

- Controls its *instataneous rate of production* q_t^i
- **Production** over $[0, t]$

$$Q_t^i := \int_0^t q_t^i dt.$$

- **Costs of production** given by $c_t^i : \mathbb{R}_+ \mapsto \mathbb{R}$ C^1 strictly convex satisfying Inada-like conditions

$$(c_t^i)'(0) = 0, \qquad (c_t^i)'(+\infty) = +\infty$$

- **Cumulative emissions** $E_t^i := e^i Q_t^i$
- **P&L** (wealth)

$$X_t^i = X_t^{i,q^i,\theta^i} = x^i + \int_0^T \theta_t^i dA_t - \int_0^T [P_t q_t^i - c_t^i(q_t^i)]dt - e^i Q_T^i A_T.$$

Individual Firm Optimization Problem

Proposition

If such an equilibrium exits, the optimal production strategy $\hat{q}^i$ is given by:

$$\hat{q}^i_t = [(c^i)']^{-1}(P_t - e^i A_t).$$

NB: As before the optimal production schedule $\hat{q}^i$ **DOES NOT DEPEND** upon the utility function!

Existence of Allowance Equilibrium Prices

- Set $E_t := \sum_{i \in \mathcal{I}} E_t^i$ for the total aggregate emissions up to time t
- Define $f(p, y) := \sum_{i \in \mathcal{I}} \varepsilon^i [(c^i)']^{-1}(p - \varepsilon^i y)$

Then the corresponding FBSDE under $\mathbb{Q}$ reads

$$\begin{cases} dP_t &= \sigma(t, P_t) dB_t, \qquad P_0 = p \\ dE_t &= f(P_t, A_t) dt, \qquad E_0 = 0 \\ dA_t &= Z_t dB_t, \qquad A_T = \lambda \mathbf{1}_{[\Lambda, +\infty)}(E_T). \end{cases}$$

NB: The volatility of the forward equation is **degenerate!**

Still, **Natural Conjecture**: For $\lambda > 0$ and $\Lambda \in \mathbb{R}$, the above FBSDE has a unique solution (P, E, A, Z).

An Enlightening Example (R.C. - Delarue)

$$\begin{cases} dP_t = dW_t, & P_0 = p \\ dE_t = (P_t - A_t)\,dt, & E_0 = e \\ dA_t = Z_t dW_t, \quad 0 \le t \le T, & A_T = \mathbf{1}_{[\Lambda,\infty)}(E_T) \end{cases} \tag{6}$$

Theorem

- There exists a unique progressively measurable triple $(P_t, E_t, A_t)_{0 \le t \le T}$ satisfying (6) and

$$\mathbf{1}_{(\Lambda,\infty)}(E_T) \le A_T \le \mathbf{1}_{[\Lambda,\infty)}(E_T).$$

- The marginal distribution of E_t
  - is absolutely continuous for $0 \le t < T$
  - has a Dirac mass at Λ when $t = T$, $\mathbb{P}\{E_T = \Lambda\} > 0$.

The terminal condition $A_T = \mathbf{1}_{[\Lambda,\infty)}(E_T)$ may not be satisfied!

Comments on the Existence of a Point Mass fot E_T

- Ruled out (by assumption) in early equilibrium model studies

Assumption

the $\mathcal{F}_{T-1}$-conditional distribution of $\sum_{i\in I}\Delta^i$ possesses almost surely no point mass, or equivalently, for all $\mathcal{F}_{T-1}$-measurable random variables Z

$$\mathbb{P}\left\{\sum_{i\in I}\Delta^i = Z\right\} = 0 \qquad (7)$$

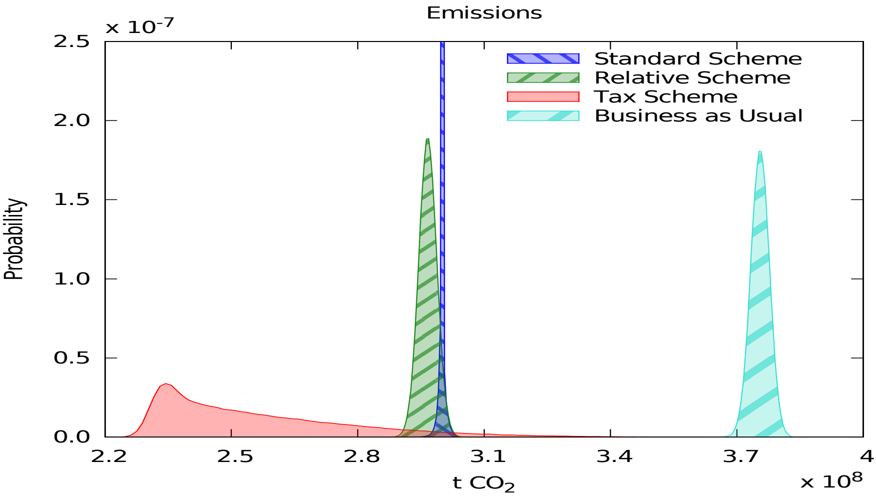
- Thought to be *innocent* !
- We should have known better!
  - Numerical Evidence from Case Studies Shows High Emission Concentration below Λ

Numerical Illustration: **Japan** Case Study

Japanese Electricity Market:

- **TOKYO** recently unveiled a **Carbon Scheme**
- Eastern & Western Regions (1GW Interconnection)
- Electricity Production: Nuclear, **Coal, Natural Gas**, Oil
  - Coal is **expensive**
  - Visible Impact of Regulation (**fuel switch**)
- **Regulation** Gory Details
  - **Cap** (Emission Target) 300 Mega-ton CO_2 = 20% w.r.t. 2012 BAU
  - Calibration for Fair Comparisons: **Meet Cap 95% of time**
    - Penalty 100 USD
    - Tax Level 40 USD
  - Numerical Solution of a **Stochastic Control** Problem (**HJB**) in 4-D

Yearly Emissions Equilibrium Distributions



Yearly emissions from electricity production for the Standard Scheme, the Relative Scheme, a Tax Scheme and BAU.

Lectures based on

1. R.C., M. Fehr and J. Hinz: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM J. Control and Optimization* (2009)
2. R.C., M. Fehr, J. Hinz and A. Porchet: Mathematical Equilibrium and Market Design for Emissions Markets Trading Schemes. *SIAM Review* (2010)
3. R.C., and J. Hinz: Risk-Neutral Modeling of Emission Allowance Prices and Option Valuation (working paper)
4. R.C., F. Delarue, G.E. Espinosa and N. Touzi: Singular BSDEs and Emission Derivative Valuation (working paper)