Energy Risk, Framework Risk, and FloVaR

Two Case-Studies

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Energy Risk and FloVaR

Notion of Risk

- **Question**: What is risk?
- **Risk** = Exposure to uncertainty.
- **Examples**:
    1. *Uncertainty with no exposure*:
    Temperature in Santiago is uncertain, yet a London resident has no exposure to it;
    2. *Positive exposure*:
    The gain of a gambler who knows that his lottery ticket has been selected is a source of uncertainty to which he is favorably exposed.

Risk Matrix

Accounting	Credit	Force majeure
Compliance	Liquidity	Geographical
Raw data	**Market**	Legal
Proxy	Political	Procedural
Curve building	Settlement	Reputation
Mapping	Volumetric	Operational
Modelling	Taxation	Regulatory
Knowledge	Index	Sovereign
Concentration	Legal	Technological

(source: Leppard: Energy Risk Management. Risk Publications)

Target of Analysis: Dynamics of Risk

- Market dynamics determine the **time evolution of risk**.
- Variance, VaR, and Exp.shortfall offer a **static** picture of risk exposure.
- **Time** acts on risk through channels:
  1. Contract *tenor* (=payment times);
  2. Increase of *uncertainty* about the future;
  3. *Periodicity features* (e.g., seasonal vol.).

P&L Distribution

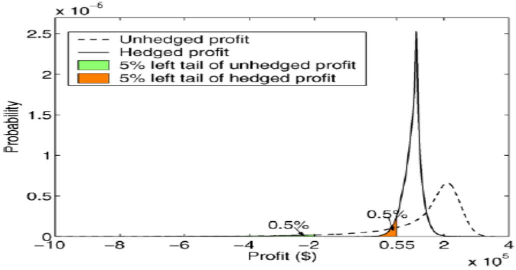
- The **Mark-To-Market Value** $V(t)$ of a position (or portfolio) is the assessment resulting from pricing each component at the standing market quote (or fair value) at time t.
- Let us consider a **time lag** Δ in the future.
  ($e.g.$, $1/250$ = trading day).
- The MTM variation over $[t, t+\Delta]$:

  $$\text{P\&L}(t, \Delta) = V(t + \Delta t) - V(t)$$

  is a random variable representing the position **Profit&Loss** (P&L).
- **Risk measure** = descriptive statistics of $\text{P\&L}(t, \Delta)$ distribution.

Value-at-Risk

- It is a synthetic measure of the riskiness of a complex portfolio.
- Responds to the **issue**: "We are confident at a level of $\alpha\%$ that the portfolio will not experience a *loss* exceeding *VaR* Euros during the next Δ days".
- **Input**: Confidence level α + Time horizon Δ. (*e.g.*, $\alpha = 99\%, \Delta = 10\text{d}$).
- **Output** $=$ *VaR* $= (1-\alpha)$-quantile of the P&L distribution:



Application: FloVaR Methodology

- **FloVaR** → Estimation of the time evolution of the distribution of the mark-to-market value of a portfolio gathering energy and commodity positions.
- **Input**
  - Portfolio composition (=position),
  - Quoted forward curves + Estimated risk factors,
  - Future cash flow dates $t_1, ..., t_n$ = last maturity.
- **Output** → **FloVaR**
  - The MTM value distributions at dates $t_1, ..., t_n$ are n functions assigning estimated probability of occurrence to a number of MTM values, one function per date in the future.

Example: Contract and Method

- **Load contract**:

$$\begin{aligned} V(t_k, S_k) &= \mathbf{E}_k^* \left(\sum_{i=k+1}^{n} e^{-r(t_i - t_0)} [K - S_i] \times L_i \right) \\ &= \sum_{i=k+1}^{n} e^{-r(t_i - t_0)} [K - \underbrace{\mathbf{E}_k^*(S_i))}_{=F(t_k, t_i) = f(S_k)}] L_i. \end{aligned}$$

- **Parameters**: $T = 2$ years, tenor $=$ monthly, n.simul. $= 20,000$, $r = 4\%$ *p.a.*, $\alpha = 95\%$.

Experiment: Setting

- **Model for** $S(t)$: 1-factor Schwartz (1997) model:

$$d\ln(S(t)) = \kappa[\mu - \ln(S(t)) - \frac{1}{2}\sigma_S^2]dt + \sigma_S dW_S(t).$$

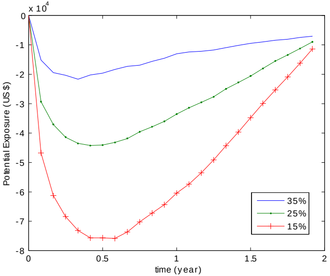
- **Futures price**:

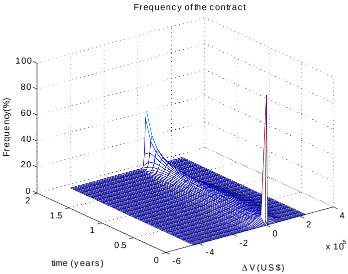
$$\lg F = \frac{\lg S(t)}{e^{\kappa(T-t)}} + (1 - e^{-\kappa(T-t)})(\mu - \frac{\sigma_S^2}{2} - \lambda) + \frac{\sigma_S^2(1 - e^{-\kappa(T-t)})}{4\kappa}.$$

- **Model for** $L(t) = L_0 + L_1\cos(2\pi t)$.
- **Parameters**: Oil [2/1/1985,17/2/1995]

Price					Load		
κ	μ	σ_S	λ	$S(0)$	Constant	L_0 =1000	-
0.301	3.093	0.334	-0.242	20	Variable	L_0 =1000	L_1 =800

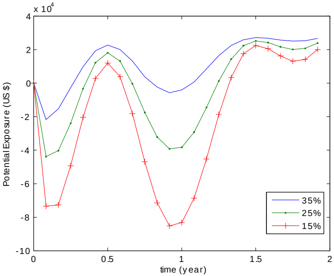
Experiment: Results under Constant Load

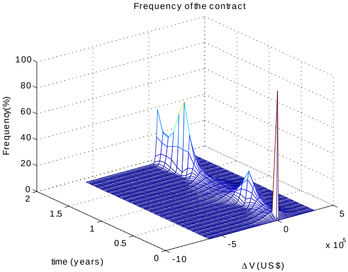




- **FloVaR shape** of an energy swap is driven by:
  1. *Uncertainty* about future spot/fwd prices: this figure increases with time horizon T as $\sqrt{T}$;
  2. *Load/Volume* to deliver: this figure linearly decreases with time horizon.

Experiment: Results under Variable Load





- **FloVaR shape** of an energy swap is additionally driven by:
  3. *Seasonal* effects (periodicity features).

Remark: portfolio contains a bullet bond maturing at position's expiration.

Framework Risk

Target of Analysis: Mapping Risk

- **Model risk** may refer to distinct aspects:
  - Structural representativeness;
  - Parameters uncertainty;
  - Framework appropriateness = **mapping risk**.
- **Mapping risk** can be assessed by comparing alternative formulations (=distinct primitives) of a common underlying model.
- *Example*: A commodity price model can be equivalently cast under a spot-convenience yield framework or a forward curve setting (Roncoroni-Id Brik (2010)).

Commodity Price Model Setting I

- **Spot-convenience yield formulation** under $\mathbb{P}$:

$$\begin{aligned} dS(t)/S(t) &= (\mu - \delta(t))dt + \sigma_1 dW_1(t) \\ d\delta(t) &= \kappa[\hat{\alpha} - \delta(t)]dt + \sigma_2 dW_2(t) \\ \rho &= \frac{d}{dt}\langle W_1, W_2\rangle_t , \end{aligned}$$

with $\hat{a} = a - \lambda\sigma_2/\kappa$, $\lambda =$ market price of convenience risk.

- **Estimation method**: Kalman filter.

Commodity Price Model Setting II

- **Forward price formulation** under $\mathbb{P}$:

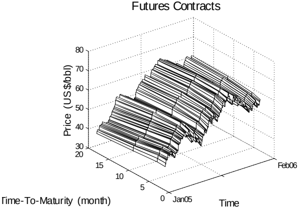
$$\frac{dF_T}{F_T} = \left[\mu - r + \lambda \frac{e^{-\kappa(T-t)} - 1}{\kappa}\right] dt + \sigma_1 dW_1 - \sigma_2 \left[\frac{1 - e^{-\kappa(T-t)}}{\kappa}\right] dW_2,$$

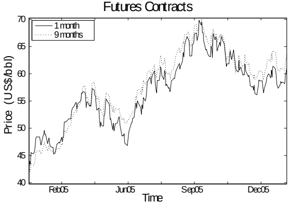
$$F_T(0) = S(0) e^{\left(r - \hat{\alpha} + \frac{\sigma_2^2}{2\kappa} - \rho\frac{\sigma_1\sigma_2}{\kappa}\right)T + \frac{\sigma_2^2}{4}\frac{1-e^{-2\kappa T}}{\kappa^3} + \left(\alpha\kappa + \rho\sigma_1\sigma_2 - \frac{\sigma_2^2}{\kappa}\right)\frac{1-e^{-\kappa T}}{\kappa}}$$

- **Estimation method**: GMM (or Exact Likelihood).

Data and Estimation Results

- **Market**: NYMEX WTI crude oil futures.
- **Periods**: 2005 [Jan.1 - Dec.30, 2005].

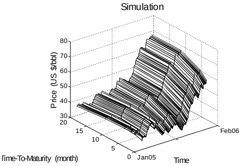




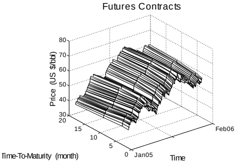
- **Results**:

Setting	σ_1	σ_2	ρ	κ	μ	λ	α	S(0)
Fwd	0.3310	0.3387	0.7717	2.0905	0.6450	0.0523	0.0934	42.0129
Sc	0.3074	0.1936	0.7803	0.9102	0.5599	0.0785	0.1884	

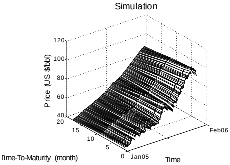
Test 1: Trajectorial Properties



Forward



Historical



Spot - Convenience Yield

Test 2: Re-estimation Stability: Forward Model

- **Description**:
  1. Sample 50 paths (1 path = 4 ttm's (1, 3, 9, 18m)$\times$150d);
  2. Estimation on simulated paths;
  3. Descriptive statistics of discrepancy re-estimated/initial par.

Fwd-2005	σ_1	σ_2	ρ	κ	λ	μ	α	S(0)
Mean	-0.0037	-0.0142	0.0215	0.0467	0.0482	0.0855	-0.0001	-0.7887
Std.Dev.	0.0163	0.0343	0.0476	0.3371	0.1520	0.2508	0.0122	5.9490
Skewness	-0.2790	-1.6778	0.4512	-4.3216	0.2820	-0.6367	5.8256	-6.8571
Kurtosis	3.0128	7.2904	4.7529	26.8326	2.7313	2.3636	39.0179	48.0202

Test 2: Re-estimation Stability: SC Model

	σ_1	σ_2	ρ	κ	λ	μ	α
Mean	-0.0214	-0.0777	0.0789	-0.4364	0.0189	0.0117	0.0208
Std.Dev.	0.0321	0.0290	0.0281	0.2361	0.0143	0.0209	0.0338
Skewness	0.6484	-1.5947	-0.5614	-1.9651	1.8062	-1.0884	2.2702
Kurtosis	6.2502	6.2514	4.4712	7.9757	8.3604	8.4803	9.8973

Test 3: Stability to Perturbations: Forward Model

- **Description**: Sample 50 paths perturbed at 10 rand pts by $\mathcal{N}(0,1)$.

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FD Model	σ_1	σ_2	ρ	κ	λ	μ	α	S(0)
Mean	0.0037	0.0281	-0.0024	0.0493	-0.0034	0.0069	0.0051	0.0085
SE	0.0083	0.1190	0.0460	0.6813	0.0092	0.0208	0.0244	0.1476
Skewness	0.1167	0.2581	1.6474	1.1414	-2.9557	2.9021	2.9021	1.3868
Kurtosis	2.8583	2.8332	9.3718	7.8096	11.0552	11.6183	12.2291	16.2726

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SC Model	σ_1	σ_2	ρ	κ	λ	μ	α
Mean	0.0320	-0.1065	-0.0761	-0.3460	0.0155	0.0537	0.0181
Std.Dev.	0.0331	0.0480	0.0689	0.1952	0.0137	0.0890	0.0138
Skewness	0.1520	-0.4963	-1.1401	1.1888	0.9438	2.6433	2.0937
Kurtosis	3.6762	2.4367	3.6116	5.6231	5.4545	12.2756	9.3147

Test 4: Convergence with Increasing Information

- **Descriptions**: Estimation across thicker&thicker term structures.

Tenor (Fwd model)	σ_1	σ_2	ρ	κ	λ	μ	α	S(0)
1,3,9,18	0.3310	0.3387	0.7717	2.0908	0.0523	0.6450	0.0934	42.0129
1,3,6,9,12,18	0.3305	0.3369	0.7640	2.0663	0.0522	0.6419	0.0954	41.9480
1,2,3,6,9,12,15,18	0.3296	0.3350	0.7620	2.0730	0.0525	0.6465	0.0988	42.0119
All	0.3273	0.3193	0.7569	2.0012	0.0531	0.6586	0.1009	41.9354

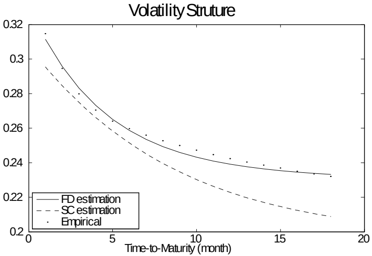
Tenor (SC model)	σ_1	σ_2	ρ	κ	λ	μ	α
1,3,9,18	0.3074	0.1936	0.7803	0.9102	0.0785	0.5599	0.1884
1,3,6,9,12,18	0.3610	0.1942	0.7856	0.9587	0.0595	0.5673	0.1488
1,2,3,6,9,12,15,18	0.3877	0.1882	0.7960	1.0050	0.0542	0.5673	0.1592
All	0.3653	0.1837	0.7869	1.0485	0.0568	0.5620	0.1884

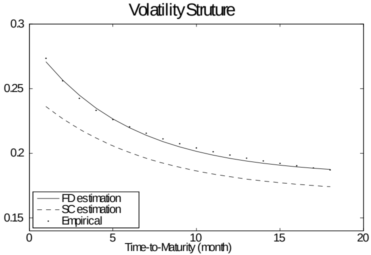
Tests 5-6: Computational Time and Volatility Structure

- **Computations time** over increasing tenors:

Tenor	1,3,9,18	1,3,6,9,12,18	1,2,3,6,9,12,15,18	all time to maturity
SC	50 sec,	100 sec,	120 sec,	380 sec,
Fwd	1 sec,	2-3 sec,	2-4 sec,	5-6 sec,

- **Recovery of volatility structure**:





Conclusions

- **Kalman filter on spot-convenience yield model estimation:**
  - Several parameters to estimate,
  - Weak statistical stability and convergence,
  - The optimizing function displays several or even no local maxima,
  - Time intensive computation.
- **GMM/Exact Likelihood on forward model estimation:**
  - Rather statistically stable ...
  - ... and quick to compute.

The Author

Andrea Roncoroni is Professor of Finance at ESSEC Business School (Paris - Singapore) and VP Lecturer at Bocconi University (Milan). He holds PhD's in Applied Mathematics and in Finance. His research interests cover Energy Finance, Financial Econometrics and Commodity-linked Derivative Structuring. He has consulted for private companies (e.g., Gaz de France, Edison Trading, EGL, Dong Energy) and lectured for public institutions (e.g., International Energy Agency, Central Bank of France, Italian Stock Exchange). He regularly published on academic journals (e.g., J. Business, J. Banking and Finance, J. Economic Dynamics) and financial book series (Implementing Models in Quantitative Finance: Methods and Cases, Springer, 2008, and the Handbook of Multi-Commodity Markets and Products, Wiley, forthcoming in 2011, with G.Fusai).

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