Hedging forward positions: Basis risk versus liquidity costs

Stefan Ankirchner, Peter Kratz, Thomas Kruse

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Companies operating a gas power plant have an immanent
  ▶ short forward position of natural gas (NG),
  ▶ long forward position of power.

To reduce price risk they
  ▶ buy natural gas on forward markets,
  ▶ sell power on forward markets.

Suppose that a German energy company wants to buy today the NG it needs in 2015.

Problem: German gas forward market is illiquid.
- Bid-ask-spread ↓ as time to delivery approaches
- Dutch and German gas prices are **highly** correlated

**Germany**
- Ask: 1 €/MWh
- Bid: 0.1 €/MWh

**Netherlands**
- Ask: 0.1 €/MWh
- Bid: 0.1 €/MWh
2 Ways of Hedging

- **Hedge 1:**
  - Buy natural gas in G

- **Hedge 2:**
  - Buy natural gas in NL.
  - Shortly before delivery: sell in NL and buy in G.

**Pros & Cons:**

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<th>Hedge 1</th>
<th>Hedge 2</th>
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<tr>
<td>Pro</td>
<td>No risk</td>
<td>Low liqu. costs</td>
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<tr>
<td>Con</td>
<td>High liqu. costs</td>
<td>Basis risk</td>
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Basis risk versus liquidity costs
Trade-off: High liquidity costs versus basis risk

Question: What is the optimal position in German and Dutch NG at any time before 2015?

Aim: Explicit formulas for stylized models
The model

- initial short position: $x_0 < 0$
- $X_t =$ primary asset position (e.g. German NG)
- $Y_t =$ proxy position (e.g. Dutch NG)
Model cont’d: execution costs

- \( K_t \) = bid-ask-spread of primary asset at time \( t \)
- \( L \) = bid-ask-spread of proxy

**Expected execution costs**

\[
\text{Costs}(X, Y) = E \left[ \int_{[0, T]} \frac{K_s}{2} |dX_s| + \int_{[0, T]} \frac{L}{2} |dY_s| \right]
\]
Control problem with constraints:

\[ \text{Costs}(X, Y) + \lambda \text{Risk}(X, Y) \rightarrow \min! \]

How to solve that?

**Analytic solution approach**

The value function \( V(t, x, y) \) satisfies the variational equality

\[
\min \left\{ -V_x(t, x, y) - K_t, -V_y(t, x, y) - L, -V_t(t, x, y) - g(f(x, y)) \right\} = 0.
\]

**Probabilistic solution approach**

Connection between singular control and optimal stopping
Lemma
Let \( X \) be a given primary position path and assume that \( L = 0 \). Then the optimal cross hedge is given by

\[
Y^*_t = -hX_t
\]

\[
h = \rho \frac{\sigma_1}{\sigma_2} = \text{minimum variance hedge ratio}
\]
Optimal position paths are (piecewise) monotone
Our method for getting explicit solutions

Assumption A: The optimal cross hedge $Y(X)$ associated to any $X$ is non-increasing after 0, i.e. of the form

$$Y(X)_t = y \wedge -hX_t.$$ 

Iterative Method:

1. For a given $y \geq 0$ determine the optimal primary position $X = X(y)$. To this end reformulate the problem as a family of stopping problems.

2. Determine optimal initial cross hedge position $y^*$. 

3. The optimal positions are given by

$$X_t^* = X_t(y^*) \text{ and } Y_t^* = y^* \wedge -hX_t^*.$$ 

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Basis risk versus liquidity costs
Examples

Explicit solutions for the following examples:

- Liquidity does not improve soon
- Liquidity does improve soon
- Liquidity kicks in at a random time
Example 1: Concave deterministic costs

"Liquidity does not improve soon" (Mathematically: liquidity costs are deterministic, decreasing and concave)
Proposition

Suppose that $K$ is decreasing and concave on $[0, T]$. Then the optimal position strategy is of the form

$$X_t^* = x^* 1_{[0,T]}(t) \text{ and } Y_t^* = y^* 1_{[0,T]}(t),$$

with $x^* \leq 0$ and $y^* \geq 0$.

The optimal positions $x^*$ and $y^*$ can be calculated explicitly (tedious!).
Example cont’d: Decision tree

\[ L < \bar{L} \]

- yes
  - keep primary open
  - cross hedge with \( A \) forwards
- no
  - no
  - keep primary open
  - cross hedge with \( A \) forwards

\[ \Delta K \geq \lambda \sigma_1 T \]

\[ \bar{L} = \frac{\sigma_2}{2\sigma_1} \left( \Delta K \rho - \sqrt{(1 - \rho^2)(\lambda^2 \sigma_1^2 T^2 - \Delta K^2)}^+ \right) \]

\[ A = -\frac{\sigma_1}{\sigma_2} \max \left( 0, \rho - 2\bar{L} \frac{\sqrt{1-\rho^2}}{\sqrt{\lambda^2 \sigma_2^2 T^2 - 4\bar{L}^2}}^+ \right) x_0 \]
Example 2: Active trading kicks in at a random time

- $K$ jumps at a random time $\tilde{\tau}$ from a high level $K_+$ to a low level $K_-$. 
- $\tilde{\tau}$ is the first jump time of an inhomogeneous Poisson process with non-decreasing jump intensity.

→ Close positions at time $\tilde{\tau}$: $X_s = Y_s = 0$ for all $s \geq \tilde{\tau}$. 

![Diagram showing the liquidation costs over time with a step at $\tilde{\tau}$]
Example 2 cont’d: Optimal strategies are static

Proposition

Suppose that $K$ jumps from $K_+$ to $K_-$ at time $\tilde{\tau}$. Then the optimal position strategy is of the form

$$X_t^* = x^* \mathbf{1}_{[0, \tilde{\tau})}(t) \quad \text{and} \quad Y_t^* = y^* \mathbf{1}_{[0, \tilde{\tau})}(t),$$

with $x^* \leq 0$ and $y^* \geq 0$.

The optimal positions $x^*$ and $y^*$ can be calculated explicitly.
There is a trade-off between liquidity costs and basis risk when hedging on forward markets.

We present an iterative, probabilistic method for deriving optimal hedging positions.

We have explicit decision trees for stylized examples.

If liquidity does not improve soon, then it is optimal to trade only at 0 or $T$.

If liquidity does improve soon, then it is optimal to close the position when the marginal cost saving = marginal add. risk.
Thank you!