Hedging forward positions: Basis risk versus liquidity costs

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Intro: Risk management of a gas power plant

Companies operating a gas power plant have an immanent

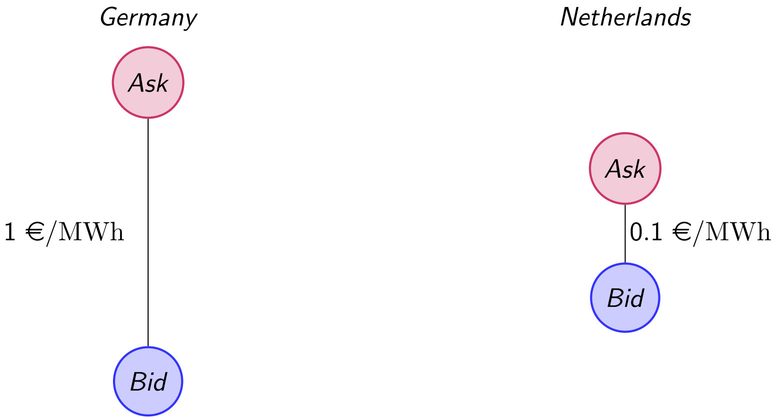
- short forward position of natural gas (NG),
- long forward position of power.

To reduce **price risk** they

- buy natural gas on forward markets,
- sell power on forward markets.

Suppose that a German energy company wants to buy today the NG it needs in 2015.

Problem: German gas **forward market** is **illiquid**.



- Bid-ask-spread ↓ as time to delivery approaches
- Dutch and German gas prices are **highly** correlated

2 Ways of Hedging

- Hedge 1:

  Buy natural gas in G

- Hedge 2:

  Buy natural gas in NL.
  Shortly before delivery: sell in NL and buy in G.

Pros & Cons:

	Hedge 1	Hedge 2
Pro	No risk	Low liqu. costs
Con	High liqu. costs	Basis risk

Optimal trade-off via stochastic control

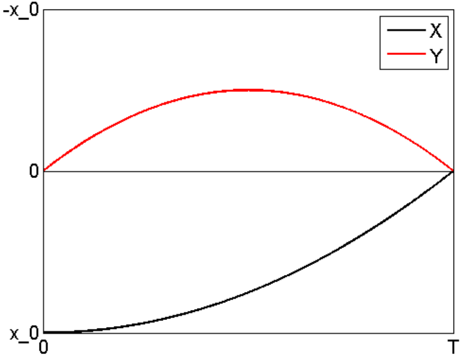
Trade-off: High liquidity costs versus basis risk

Question: What is the optimal position in German and Dutch NG at *any* time before 2015?

Aim: Explicit formulas for stylized models

The model

- initial short position : $x_0 < 0$
- X_t = primary asset position (e.g. German NG)
- Y_t = proxy position (e.g. Dutch NG)



Model cont'd: execution costs

- $K_t =$ bid-ask-spread of primary asset at time t
- $L =$ bid-ask-spread of proxy

Expected execution costs

$$\mathbf{Costs}(X, Y) = E\left[\int_{[0,T]} \frac{K_s}{2}|dX_s| + \int_{[0,T]} \frac{L}{2}|dY_s|\right]$$

Target function and minimum variance hedge

Control problem with constraints:

$$\mathrm{Costs}(X,Y) + \lambda\ \mathrm{Risk}(X,Y) \longrightarrow \text{min!}$$

How to solve that?

Analytic solution approach

The value function $V(t,x,y)$ satisfies the variational equality

$$\min\{-V_x(t,x,y) - K_t, -V_y(t,x,y) - L, -V_t(t,x,y) - g(f(x,y)\} = 0.$$

Probabilistic solution approach

Connection between singular control and optimal stopping

Minimum variance hedge

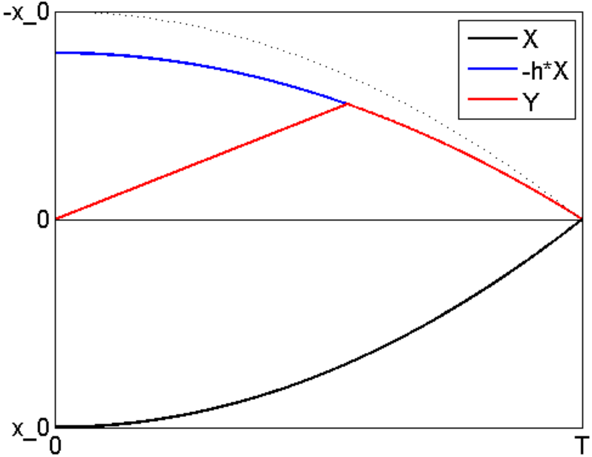
Lemma

Let X be a given primary position path and assume that $L = 0$. Then the optimal cross hedge is given by

$$Y_t^* = -hX_t$$

$h = \rho\frac{\sigma_1}{\sigma_2}$ = minimum variance hedge ratio

Optimal position paths are (piecewise) monotone



Our method for getting explicit solutions

Assumption A: The optimal cross hedge $Y(X)$ associated to any X is non-increasing after 0, i.e. of the form

$$Y(X)_t = y \wedge -hX_t.$$

Iterative Method:

1. For a given $y \geq 0$ determine the optimal primary position $X = X(y)$. To this end reformulate the problem as a family of stopping problems.
2. Determine optimal initial cross hedge position y^*.
3. The optimal positions are given by

$$X_t^* = X_t(y^*) \text{ and } Y_t^* = y^* \wedge -hX_t^*.$$

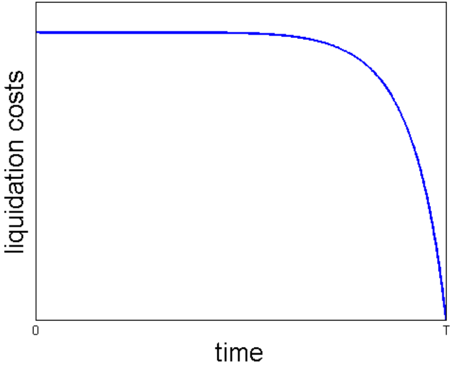
Examples

Explicit solutions for the following examples:

- Liquidity does **not** improve soon
- Liquidity does improve soon
- Liquidity kicks in at a **random time**

Example 1: Concave deterministic costs

- "Liquidity does not improve soon" (Mathematically: liquidity costs are deterministic, decreasing and concave)



Example cont'd: Optimal strategies are static

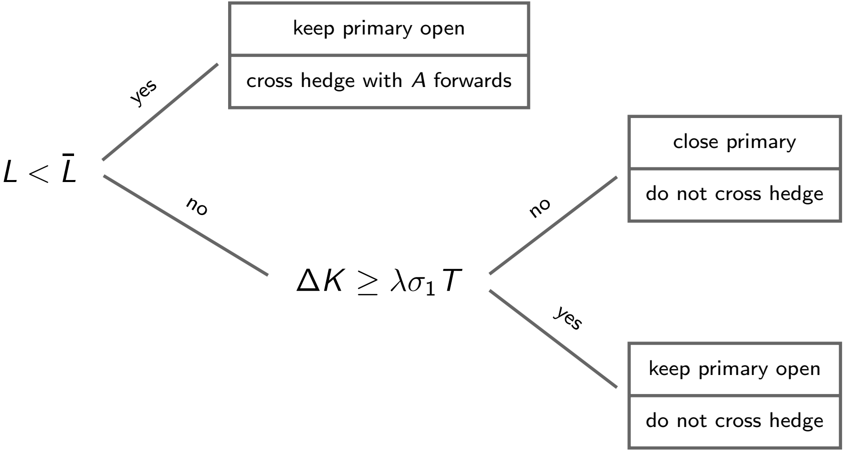
Proposition

Suppose that K is decreasing and concave on $[0, T]$. Then the optimal position strategy is of the form

$$X_t^* = x^* 1_{[0,T)}(t) \text{ and } Y_t^* = y^* 1_{[0,T)}(t),$$

with $x^* \leq 0$ and $y^* \geq 0$.
The optimal positions x^* and y^* can be calculated explicitly (tedious!).

Example cont'd: Decision tree

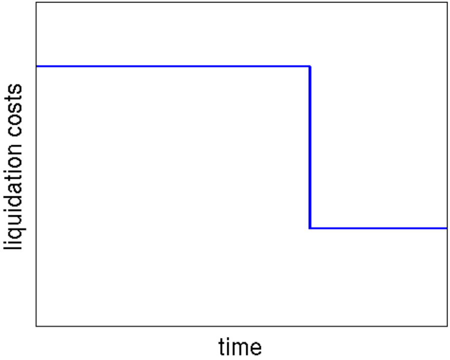


- $\bar{L} = \frac{\sigma_2}{2\sigma_1}\left(\Delta K \rho - \sqrt{(1-\rho^2)(\lambda^2\sigma_1^2 T^2 - \Delta K^2)^+}\right)$
- $A = -\frac{\sigma_1}{\sigma_2} \max\left(0, \rho - 2L\frac{\sqrt{1-\rho^2}}{\sqrt{(\lambda^2\sigma_2^2 T^2 - 4L^2)^+}}\right) x_0$

Example 2: Active trading kicks in at a random time

- K jumps at a random time $\tilde{\tau}$ from a high level K_+ to a low level K_-.
- $\tilde{\tau}$ is the first jump time of an inhomogeneous Poisson process with non-decreasing jump intensity.

$\rightarrow$ Close positions at time $\tilde{\tau}$: $X_s = Y_s = 0$ for all $s \geq \tilde{\tau}$.



Example 2 cont'd: Optimal strategies are static

Proposition

Suppose that K jumps from K_+ to K_- at time $\tilde{\tau}$. Then the optimal position strategy is of the form

$$X_t^* = x^* 1_{[0,\tilde{\tau})}(t) \text{ and } Y_t^* = y^* 1_{[0,\tilde{\tau})}(t),$$

with $x^* \leq 0$ and $y^* \geq 0$.
The optimal positions x^* and y^* can be calculated explicitly.

Conclusion

- There is a trade-off between liquidity costs and basis risk when hedging on forward markets.
- We present an iterative, probabilistic method for deriving optimal hedging positions.
- We have explicit decision trees for stylized examples
- If liquidity does not improve soon, then it is optimal to trade only at 0 or T.
- If liquidity does improve soon, then it is optimal to close the position when the marginal cost saving $=$ marginal add. risk.

Thank you!