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Medium-term planning for thermal electricity production

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Outlook

- We aim at a simplified model for **mid-term planning for thermal electricity production** that can be used for repetitive calculation

- **Optimization model:**
  - Costs: fuel, fixed and variable operating costs
  - Different fuels are bought at the spot market and stored to produce electricity
  - We allow for trading at CO2 spot market (emission certificates)
  - Production is sold at the spot market
  - **Maximization of the asset value** (cash + value of stored fuels) at the end of the planning horizon

Production

- Consider **time periods** $t \in 0, 1, \ldots, T$ with length Δ_t.
- We model **thermal generators** i which may use different **fuels** j to produce energy $x_{t,i,j}$ and are characterized by **efficiencies** $\eta_{i,j}$ and **maximum power** β_i, in particular
- We consider Δ_t as **weeks**. If Δ_t smaller, integer decisions related to switching, ramping, minimum power production constraints etc. become relevant
- A **cost model** for the generators:
  - **Fuel costs** (spot markets) are given by $P^f_{t+1,j}(\omega) \cdot x_{t,i,j}/\eta_{i,j}$.
  - **Variable operating costs** are estimated by $\gamma_i \cdot \sum_{j=1}^{J} x_{t,i,j} / (\beta_i \Delta_t)$
  - In addition we consider **fixed operating costs** κ_i per time unit.

Storage

- We model **storage** s_t, cumulated CO_2**-emissions** e_t, **cumulated** CO_2**-certificates** a_t and a **cash position** w_t.
- With $f_{t,j}$ denoting the amount of fuel j bought at time t storage develops as

$$s_{0,j} = s_j^0 \tag{1}$$

$$s_{t,j} = s_{t-1,j} - \sum_{i=1}^{I} \frac{x_{t-1,i,j}}{\eta_{i,j}} + f_{t,j} \ \forall t > 0, j \tag{2}$$

$$0 \leq s_{t,j} \leq \bar{s}_j \ \forall t, j, \tag{3}$$

and production is restricted by

$$\sum_{i=1}^{I} \frac{x_{t,i,j}}{\eta_{i,j}} \leq s_{t,j} \ \forall t, j. \tag{4}$$

CO_2-accounting

- If ε_{ij} denotes the CO_2**-emissions** (t per MWH) of fuel j if burned by generator i, the amount e_t of CO_2 emitted is

$$e_0 = e^0. \tag{5}$$

$$e_t = e_{t-1} + \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\varepsilon_{ij}}{\eta_{i,j}} \cdot x_{t-1,i,j} \quad \forall t > 0.$$

- At each time it is possible to buy ($c_t \geq 0$) or sell ($c_t < 0$) certificates at the market for CO_2 **allowances** at prices P_t^c. Hence the accumulated amount of pollution covered by certificates is

$$a_0 = a^0$$
$$a_t = a_{t-1} + c_t \quad \forall t > 0.$$

Cash accounting

- The cash position starts with $w_0 = w^0 - \sum_{j=1}^{J} P^f_{0,j} f_{0,j}$.

and develops by

$$
\begin{aligned}
w_t &= (1+\rho_l) w^+_{t-1} - (1+\rho_b) w^-_{t-1} \\
&+ P^x_t \cdot \sum_{i=1}^{I} \sum_{j=1}^{J} x_{t-1,i,j} \\
&- \sum_{j=1}^{J} P^f_{t,j} \sum_{i=1}^{I} f_{t,j} \\
&- P^c_t c_t \\
&- \sum_{j=1}^{J} \zeta_j \frac{(s_{t,j} + s_{t-1,j})}{2} \\
&- \sum_{i=1}^{I} \frac{\gamma_i}{\beta_i} \cdot \sum_{j=1}^{J} x_{t-1,i,j} - \kappa_i \cdot \Delta_{t-1} \quad 0 < t < T
\end{aligned}
$$

- At time T no fuel is bought anymore, but a penalty has to be payed if certificates are not sufficient: $(\theta + P^c_T)(e_T - a_T)^+$

Optimization problem: Objective

- The producer aims at the asset value (excluding the value of generating units) at the end of the planning horizon

$$v_T = w_T + \sum_{j=1}^{J} s_{T,j} \cdot P^f_{T,j}. \tag{6}$$

- All prices are stochastic processes. Decisions at time t have to be taken with information available at time t. Hence the decision variables are also stochastic. The equations and inequalities have to be understood as "holds almost surely".
- Our objective is a mixture of expectation and $AV@R$ with a mixing factor $0 \leq \lambda \leq 1$

$$\begin{aligned} \max_{x,f,c,(s,w,v,a,e)} \quad & (1-\lambda) \cdot \mathbb{E}\left[v_T\right] + \lambda \cdot AV@R_\alpha(v_T) \\ s.t. \quad & \text{all constraints} \\ & x, f, c \lhd \Sigma \\ & s, w, v, a, e \lhd \Sigma \quad . \end{aligned} \tag{7}$$

Modeling the risk factors

- We look at daily European commodity prices:
  - Gas prices: Gaspool (GPL), April 2007-December 2011
  - Crude oil prices: Brent Crude oil, May 2003-December 2011
  - EUA: April 2008-December 2011
  - Coal: North West Europe(NWE) steam coal marker, December 2005-May 2012
  - Electricity prices: EEX Phelix, September 2008-December 2011
- We employ a common model for simulating commodity prices: gas, oil, coal and emissions allowances (EUA)
  - Similar patterns among commodity prices: leptokurtic distribution, negatively skewed returns, non-stationary variation are described by **Geometric Brownian Motion with Jump Process (GBMPJ)/Merton model**
- Spot electricity prices behave considerably different from other commodities and need a separate modeling approach: **Regime Switching Model**

Regime switching model for electricity prices

$$MCP_t := \begin{cases} f_t^L - Spike_t^- & \text{with } p_t^- \\ f_t \cdot \exp(r_t) & \text{with } 1 - p_t^- - p_t^+ \\ f_t^U + Spike_t^+ & \text{with } p_t^+ \end{cases}$$

with

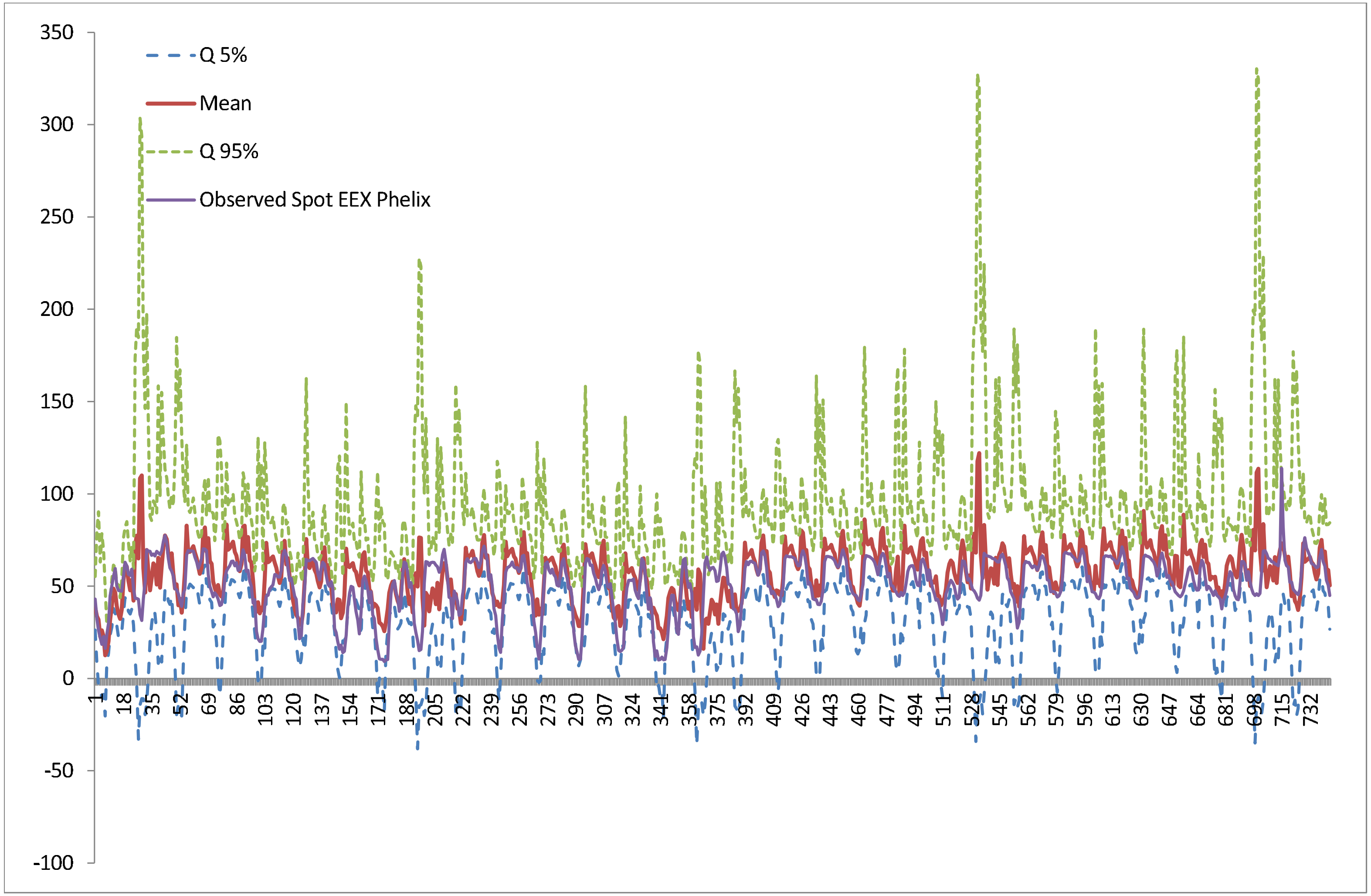
$$\begin{aligned} Spike_t^+ &\sim \mathrm{Exp}(1/\lambda_t^+) \\ Spike_t^- &\sim \mathrm{Exp}(1/\lambda_t^-) \\ r_t &\sim N(0, \sigma_t^2) \\ f_t^L &= f_t * \exp\left(\alpha_L * \sigma_t\right) \\ f_t^U &= f_t * \exp\left(\alpha_U * \sigma_t\right) \end{aligned}$$

Energy prices: Results

	Sample	Parameter estimation					
		α	σ	λ	μ	δ	ML
Crude oil (monthly)	01.05.2003-01.12.2011	0.325	0.259	80.373	-0.0017	0.027	-5314.05
		(0.141)	(0.013)	(19.490)	(0.0017)		
	01.05.2003-01.12.2010	0.283	0.271	68.981	-0.0013	0.028	-4705.27
		(0.149)	(0.013)	(17.156)	(0.0020)		
Heating oil (monthly)	01.05.2003-01.12.2011	0.218	0.245	99.953	-0.0005	0.028	-5405.37
		(0.134)	(0.011)	(18.398)	(0.0013)		
	01.05.2003-01.12.2010	0.158	0.253	103.751	0.0000	0.028	-4781.89
		(0.149)	(0.013)	(20.784)	(0.0014)		
EUA (monthly)	01.04.2008-01.12.2011	0.178	0.254	81.165	-0.0051	0.036	-2152.87
		(0.202)	(0.016)	(18.070)	(0.0029)		
	01.04.2008-01.12.2010	0.327	0.268	78.921	-0.0057	0.036	-1595.28
		(0.246)	(0.020)	(21.967)	(0.0037)		
Gas (monthly)	01.04.2007-01.12.2011	0.321	0.379	99.790	-0.0006	0.068	-2015.13
		(0.281)	(0.019)	(14.135)	(0.0038)		
	01.04.2008-01.12.2010	0.316	0.423	105.479	0.0003	0.071	-1514.65
		(0.361)	(0.025)	(17.925)	(0.0045)		
Coal (weekly)	09.12.2005-01.12.2011	0.308	0.170	21.749	-0.0082	0.053	-552.264
		(0.117)	(0.020)	(7.506)	(0.0068)		
	09.12.2005-01.12.2010	0.437	0.172	25.860	-0.0098	0.052	-450.484
		(0.140)	(0.024)	(10.157)	(0.0071)		

Table 1: *ML Estimation results of the GMBJ model for oil, EUA, gas and coal spot prices. Standard errors are in paranthesis.*

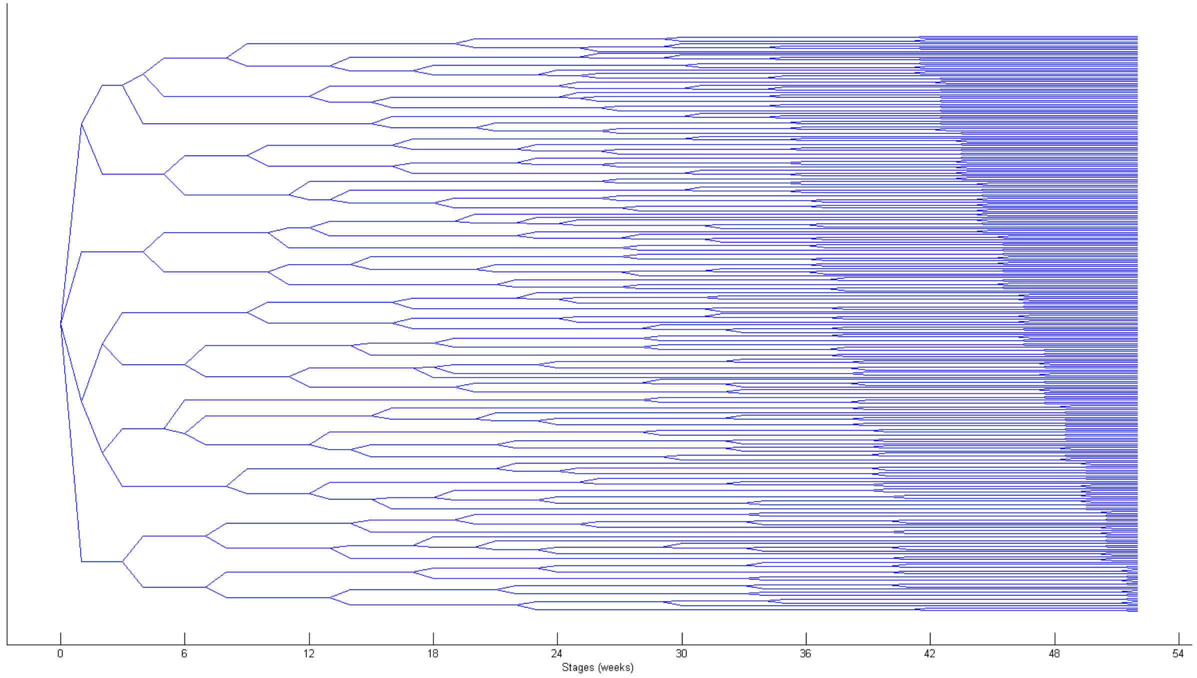
Electricity prices: Out of sample results



The structure

Reference for tree reduction method:

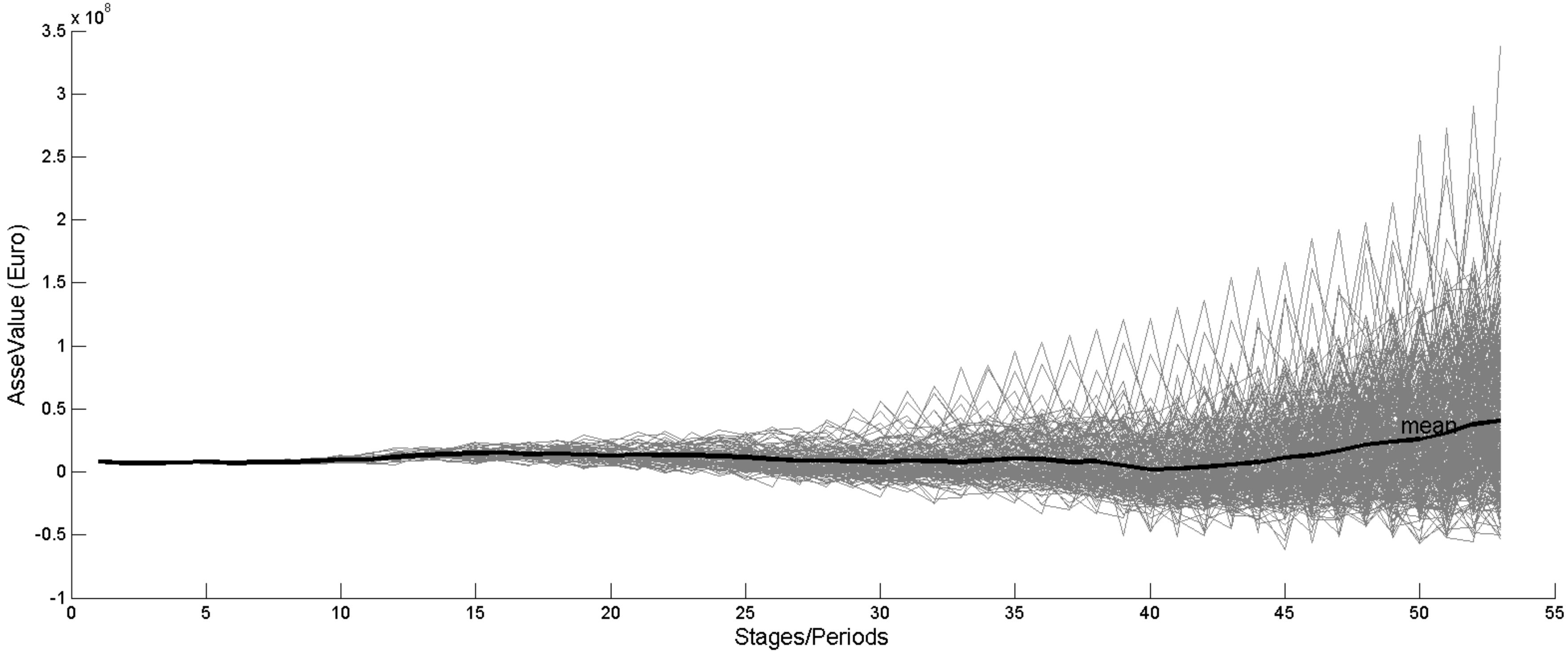
- Pflug/Pichler (2012) introduced and analyzed a generalization of the well known Wasserstein distance
- Kovacevic/Pichler (2012) propose an algorithm for improving the distance between the trees



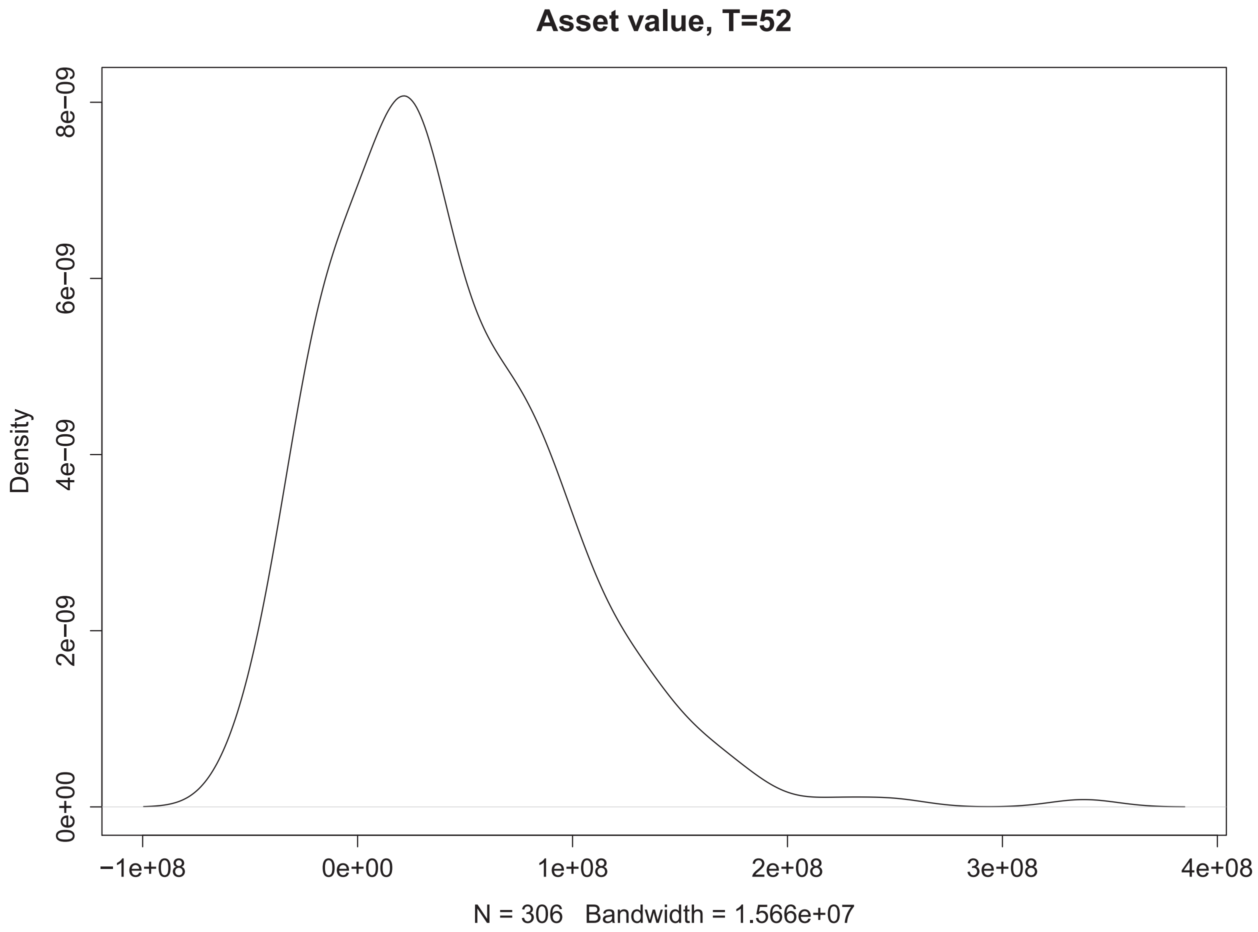
System specification

- The thermal system consists of:
  - Two combined cycle plants (gas/oil)
  - Three combustion turbines (gas/oil)
  - One steam turbine (coal)
- Premises:
  - We start with a small amount of small fuel
  - Cash position: 1 million EUR
  - Interest on cash: 2.5%; on debt: 12.5%
  - AV@R calculated at level $\alpha = 0.05$
  - Mixture parameter λ is set to 0.5 in the standard case
- Implementation: AIMMS 3.12, solver GUROBI 4.6

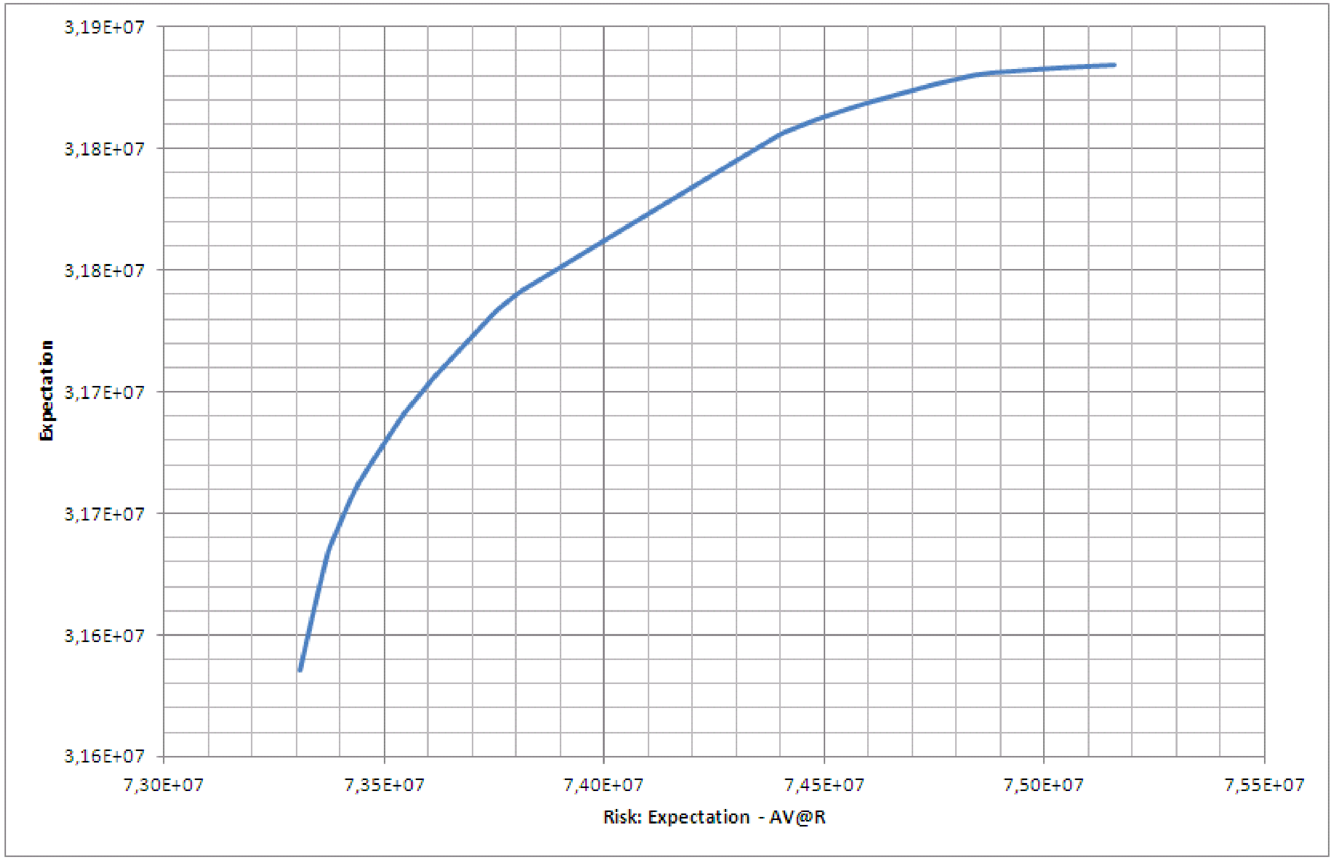
Development of the asset value



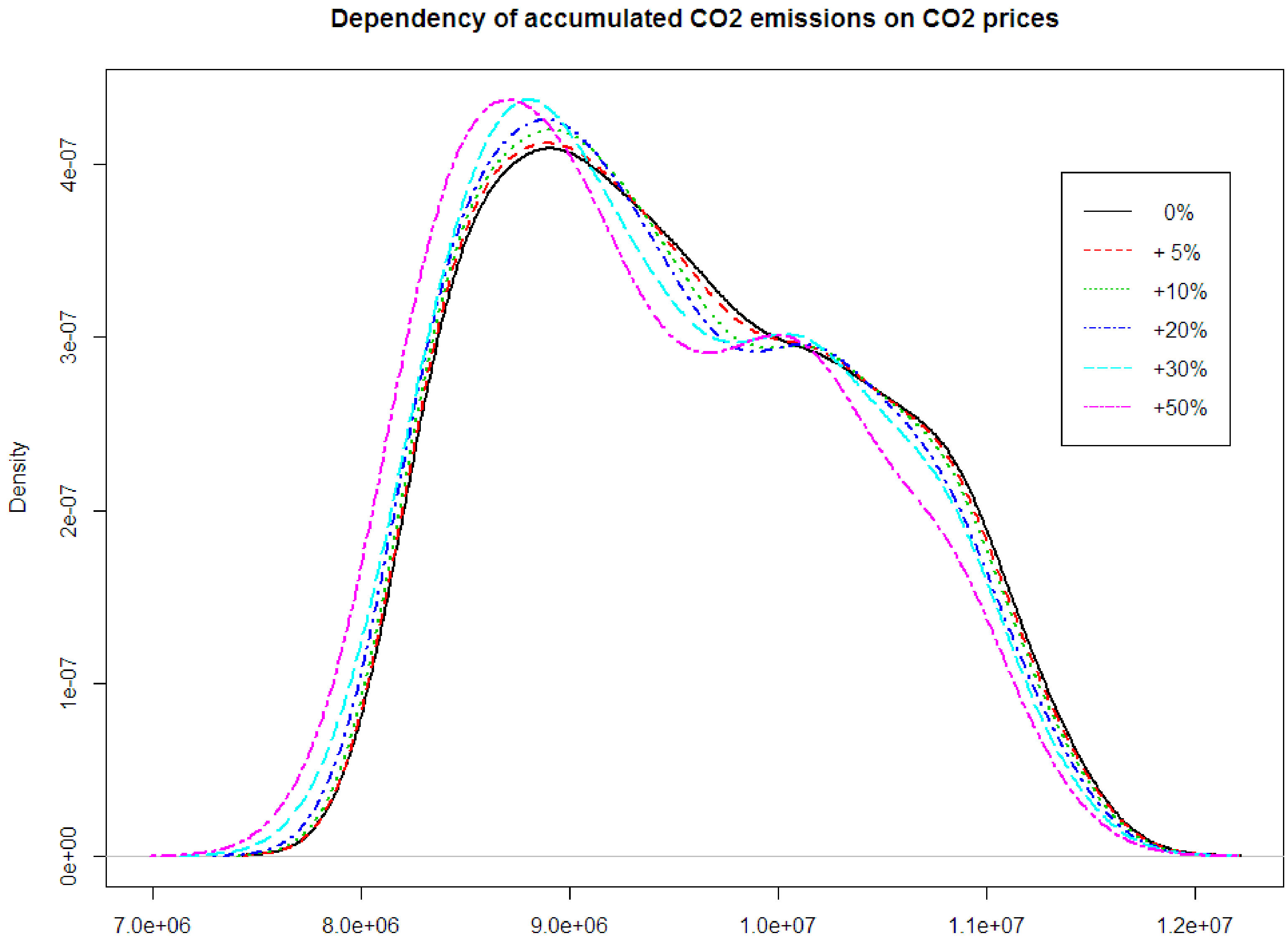
Distribution of the asset value - end of the planning horizon



Efficient frontier. Tradeoff expected end value vs. riskiness of the end value



Effect of increases in CO2 prices on the accumulated CO2 emissions



Indifference pricing

- Given the thermal system as described above, consider in addition an electricity **delivery contract**: A fixed amount **E** of electricity has to be delivered (produced) during all weeks (52) of the planning horizon at a fixed price **K**.
- **Which price is the minimum price such that the producer is interested to sign the contract?**
- **Solve with indifference pricing:**

$$\min_{K,(\dots)} K \tag{8}$$

$$s.t. \lambda \cdot \mathbb{E}\left[v_T\right] + (1-\lambda) \cdot AV@R_\alpha(v_T) \geq v^* \tag{9}$$

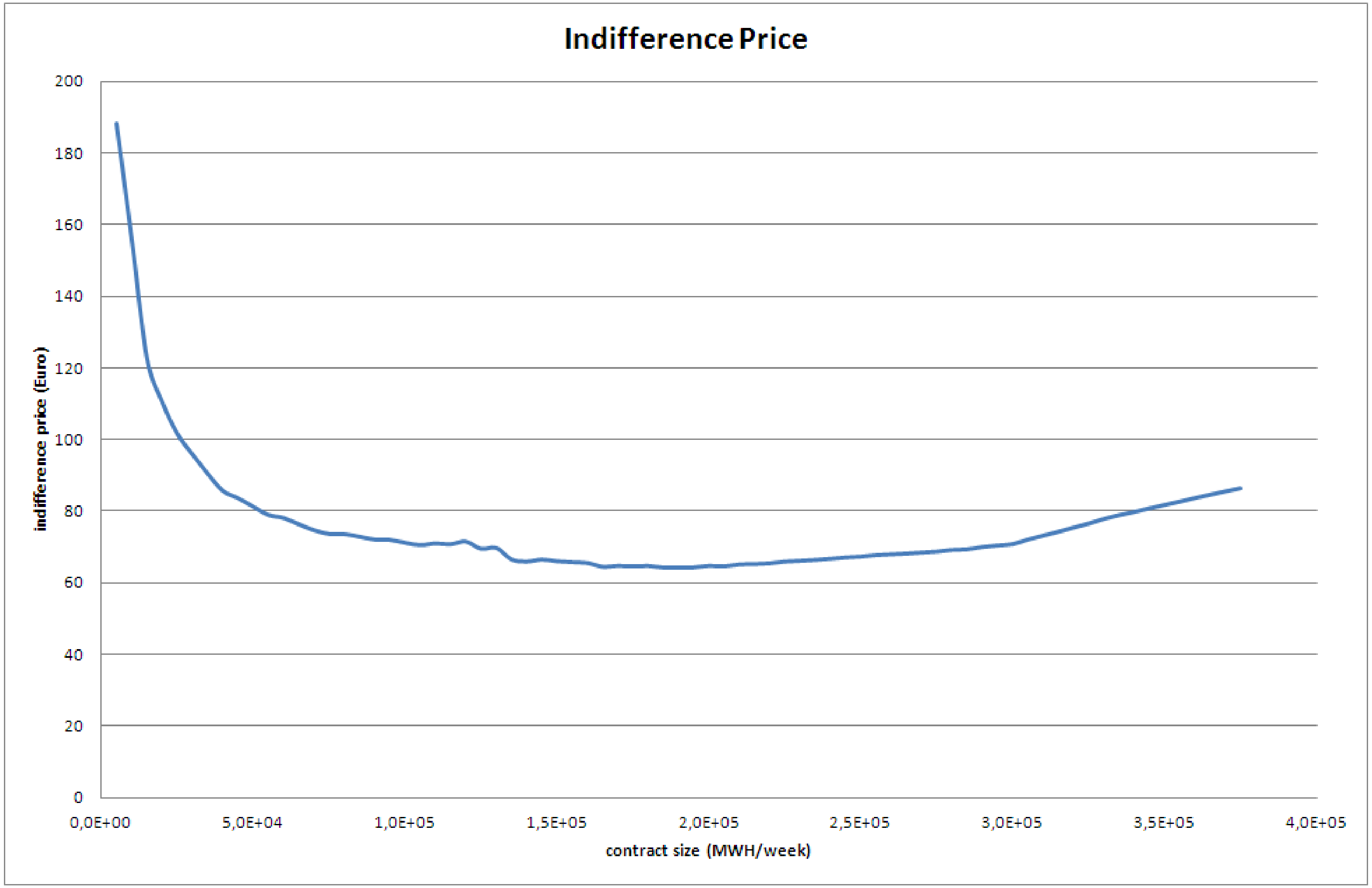
- All constraints of the original problem, except

  * It is possible to buy electricity y_t at the spot market,

  * $\sum_{i \in I, j \in J} x_{t,i,j} + y_t \geq E$

  * The cash calculation has to be corrected: $P_t^x \cdot (\sum_{i=1}^{I} \sum_{j=1}^{J} x_{t-1,i,j} - E) + K \cdot E$.

Indifference pricing



Conclusion

- We specified a flexible model for mid-term planning, such that iterative analysis – repeatedly using the optimization model can be done in reasonable time
- We simulated the risk factors: oil, gas, coal and CO2 emissions by a GBMJ process and electricity prices by a spot-forward model
- Simulated hourly/daily commodity prices were aggregated to weekly average price scenarios and reduced to stochastic trees suitable for multistage optimization
- We show the sensitivity of the asset value and of CO2 emissions to increases in the prices of the CO2 allowances
- We investigated the pricing of electricity delivery contracts with fixed amount and price in the framework of indifference pricing