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Medium-term planning for thermal electricity production

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Outlook

- We aim at a simplified model for **mid-term planning for thermal electricity production** that can be used for repetitive calculation

- **Optimization model:**
  - Costs: fuel, fixed and variable operating costs
  - Different fuels are bought at the spot market and stored to produce electricity
  - We allow for trading at CO2 spot market (emission certificates)
  - Production is sold at the spot market
  - **Maximization of the asset value** (cash + value of stored fuels) at the end of the planning horizon
Production

- Consider **time periods** \( t \in 0, 1, \ldots, T \) with length \( \Delta_t \).

- We model **thermal generators** \( i \) which may use different **fuels** \( j \) to produce energy \( x_{t,i,j} \) and are characterized by **efficiencies** \( \eta_{i,j} \) and **maximum power** \( \beta_i \), in particular.

- We consider \( \Delta_t \) as **weeks**. If \( \Delta_t \) smaller, integer decisions related to switching, ramping, minimum power production constraints etc. become relevant.

- **A cost model** for the generators:
  - **Fuel costs** (spot markets) are given by \( P_{t+1,j}(\omega) \cdot x_{t,i,j} / \eta_{i,j} \).
  - **Variable operating costs** are estimated by \( \gamma_i \cdot \sum_{j=1}^{J} x_{t,i,j} / (\beta_i \Delta_t) \)
  - In addition we consider **fixed operating costs** \( \kappa_i \) per time unit.
Storage

- We model storage $s_t$, cumulated CO$_2$-emissions $e_t$, cumulated CO$_2$-certificates $a_t$ and a cash position $w_t$.
- With $f_{t,j}$ denoting the amount of fuel $j$ bought at time $t$ storage develops as

$$s_{0,j} = s_j^0$$  \hspace{1cm} (1)$$

$$s_{t,j} = s_{t-1,j} - \sum_{i=1}^{I} \frac{x_{t-1,i,j}}{\eta_{i,j}} + f_{t,j} \forall t > 0, j$$  \hspace{1cm} (2)$$

$$0 \leq s_{t,j} \leq \bar{s}_j \forall t, j,$$  \hspace{1cm} (3)$$

and production is restricted by

$$\sum_{i=1}^{I} \frac{x_{t,i,j}}{\eta_{i,j}} \leq s_{t,j} \forall t, j.$$  \hspace{1cm} (4)$$
**CO₂-accounting**

- If $\varepsilon_{ij}$ denotes the CO₂-emissions (t per MWH) of fuel $j$ if burned by generator $i$, the amount $e_t$ of CO₂ emitted is

$$e_0 = e^0. \tag{5}$$

$$e_t = e_{t-1} + \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\varepsilon_{ij}}{\eta_{i,j}} \cdot x_{t-1,i,j} \quad \forall t > 0.$$ 

- At each time it is possible to buy ($c_t \geq 0$) or sell ($c_t < 0$) certificates at the market for CO₂ allowances at prices $P_t^c$. Hence the accumulated amount of pollution covered by certificates is

$$a_0 = a^0$$

$$a_t = a_{t-1} + c_t \quad \forall t > 0.$$
Cash accounting

- The cash position starts with $w_0 = w^0 - \sum_{j=1}^{J} P_{0,j}^f f_{0,j}$.

and develops by

$$w_t = (1 + \rho_t)w_{t-1}^+ - (1 + \rho_b)w_{t-1}^- + P_t^x \cdot \sum_{i=1}^{I} \sum_{j=1}^{J} x_{t-1,i,j} - \sum_{j=1}^{J} P_{t,j}^f \sum_{i=1}^{I} f_{t,j}$$

$$- P_t^c c_t$$

$$- \sum_{j=1}^{J} \zeta_j \frac{(s_{t,j} + s_{t-1,j})}{2}$$

$$- \sum_{i=1}^{I} \frac{\gamma_i}{\beta_i} \sum_{j=1}^{J} x_{t-1,i,j} - \kappa_i \cdot \Delta_{t-1} \quad 0 < t < T$$

- At time $T$ no fuel is bought anymore, but a penalty has to be payed if certificates are not sufficient: $(\theta + P_T^c)(e_T - a_T)^+$
Optimization problem: Objective

- The producer aims at the asset value (excluding the value of generating units) at the end of the planning horizon

\[
v_T = w_T + \sum_{j=1}^{J} s_{T,j} \cdot P^f_{T,j}.
\]  

(6)

- All prices are stochastic processes. Decisions at time \( t \) have to be taken with information available at time \( t \). Hence the decision variables are also stochastic. The equations and inequalities have to be understood as “holds almost surely”.

- Our objective is a mixture of expectation and \( AV@R \) with a mixing factor \( 0 \leq \lambda \leq 1 \)

\[
\max_{x,f,c,(s,w,v,a,e)} (1 - \lambda) \cdot \mathbb{E}[v_T] + \lambda \cdot AV@R_\alpha(v_T)
\]

(7)

\[
s.t. \quad \text{all constraints}
\]

\[
\begin{align*}
x, f, c & \leq \Sigma \\
s, w, v, a, e & \leq \Sigma
\end{align*}
\]
Modeling the risk factors

- We look at daily European commodity prices:
  - Gas prices: Gaspool (GPL), April 2007-December 2011
  - Crude oil prices: Brent Crude oil, May 2003-December 2011
  - EUA: April 2008-December 2011
  - Coal: North West Europe (NWE) steam coal marker, December 2005-May 2012
  - Electricity prices: EEX Phelix, September 2008-December 2011

- We employ a common model for simulating commodity prices: gas, oil, coal and emissions allowances (EUA)
  - Similar patterns among commodity prices: leptokurtic distribution, negatively skewed returns, non-stationary variation are described by Geometric Brownian Motion with Jump Process (GBMPJ)/Merton model

- Spot electricity prices behave considerably different from other commodities and need a separate modeling approach: Regime Switching Model
Regime switching model for electricity prices

\[ MCP_t := \begin{cases} 
  f_t^L - \text{Spike}_t^- & \text{with } p_t^- \\
  f_t \cdot \exp(r_t) & \text{with } 1 - p_t^- - p_t^+ \\
  f_t^U + \text{Spike}_t^+ & \text{with } p_t^+
\end{cases} \]

with

\[ \text{Spike}_t^+ \sim \text{Exp}(1/\lambda_t^+) \]
\[ \text{Spike}_t^- \sim \text{Exp}(1/\lambda_t^-) \]
\[ r_t \sim N(0, \sigma_t^2) \]
\[ f_t^L = f_t \ast \exp(\alpha_L \ast \sigma_t) \]
\[ f_t^U = f_t \ast \exp(\alpha_U \ast \sigma_t) \]
## Energy prices: Results

<table>
<thead>
<tr>
<th>Sample</th>
<th>Parameter estimation</th>
<th>α</th>
<th>σ</th>
<th>λ</th>
<th>μ</th>
<th>δ</th>
<th>ML</th>
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<tbody>
<tr>
<td>Crude oil (monthly)</td>
<td>01.05.2003-01.12.2011</td>
<td>0.325</td>
<td>0.259</td>
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<td>(19.490)</td>
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<td>(0.013)</td>
<td>(17.156)</td>
<td>(0.0020)</td>
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<td>Heating oil (monthly)</td>
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<td>(0.134)</td>
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<td>(0.011)</td>
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<td>(0.013)</td>
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<td>EUA (monthly)</td>
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<td>(0.202)</td>
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<td>(0.016)</td>
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<td>Gas (monthly)</td>
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<td>(17.925)</td>
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<td>(0.024)</td>
<td>(10.157)</td>
<td>(0.0071)</td>
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</table>

Table 1: **ML Estimation results of the GMBJ model for oil, EUA, gas and coal spot prices. Standard errors are in parenthesis.**
Electricity prices: Out of sample results
The structure

Reference for tree reduction method:

- Pflug/Pichler (2012) introduced and analyzed a generalization of the well known Wasserstein distance
- Kovacevic/Pichler (2012) propose an algorithm for improving the distance between the trees
System specification

- The thermal system consists of:
  - Two combined cycle plants (gas/oil)
  - Three combustion turbines (gas/oil)
  - One steam turbine (coal)

- Premises:
  - We start with a small amount of small fuel
  - Cash position: 1 million EUR
  - Interest on cash: 2.5%; on debt: 12.5%
  - AV@R calculated at level $\alpha = 0.05$
  - Mixture parameter $\lambda$ is set to 0.5 in the standard case

- Implementation: AIMMS 3.12, solver GUROBI 4.6
Development of the asset value
Distribution of the asset value - end of the planning horizon

Asset value, T=52

Density

$N = 306$  Bandwidth $= 1.566e+07$
Efficient frontier. Tradeoff expected end value vs. riskiness of the end value
Effect of increases in CO2 prices on the accumulated CO2 emissions

Dependency of accumulated CO2 emissions on CO2 prices
Indifference pricing

- Given the thermal system as described above, consider in addition an electricity delivery contract: A fixed amount $E$ of electricity has to be delivered (produced) during all weeks (52) of the planning horizon at a fixed price $K$.
- **Which price is the minimum price such that the producer is interested to sign the contract?**
- **Solve with indifference pricing:**

$$\min_{K,(\ldots)} K \quad (8)$$

$$\text{s.t.} \lambda \cdot \mathbb{E}[v_T] + (1 - \lambda) \cdot AV@R_{\alpha}(v_T) \geq v^* \quad (9)$$

- All constraints of the original problem, except

  * It is possible to buy electricity $y_t$ at the spot market,
  $$\sum_{i \in I, j \in J} x_{t,i,j} + y_t \geq E$$

  * The cash calculation has to be corrected: $P_t^x \cdot (\sum_{i=1}^I \sum_{j=1}^J x_{t-1,i,j} - E) + K \cdot E$.  

Indifference pricing
Conclusion

- We specified a flexible model for mid-term planning, such that iterative analysis – repeatedly using the optimization model can be done in reasonable time.

- We simulated the risk factors: oil, gas, coal and CO2 emissions by a GBMJ process and electricity prices by a spot-forward model.

- Simulated hourly/daily commodity prices were aggregated to weekly average price scenarios and reduced to stochastic trees suitable for multistage optimization.

- We show the sensitivity of the asset value and of CO2 emissions to increases in the prices of the CO2 allowances.

- We investigated the pricing of electricity delivery contracts with fixed amount and price in the framework of indifference pricing.