Calculation of Monte Carlo Sensitivities for a portfolio of time coupled options

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Wolfgang Raabe

Lehrstuhl für Energiehandel und Finanzdienstleistungen
Universität Duisburg-Essen

About the author

Wolfgang Raabe

Education

- Diploma in Physics at WWU Münster (main focus: nonlinear physics, computational geophysics)
- Certificate in Quantitative Finance (CQF)
- Phd studies at University of Duisburg-Essen (Prof. Dr. Rüdiger Kiesel)

Working experience

- Internship at d-fine
- Risk Controlling at RWE Supply & Trading
- Risk Management at RWE AG
- Strategy at RWE Innogy GmbH

Motivation and main targets

- Monte Carlo simulation is a popular method for valuation of complex real options like power plants for important reasons:
  - Simple implementation
  - Very flexible
  - High acceptance (even) by top management
- However, the calculation of MC sensitivities (Delta, Gamma) is a well-known challenge
- Little confidence in existing frameworks apart from intrinsic valuation

Main targets:

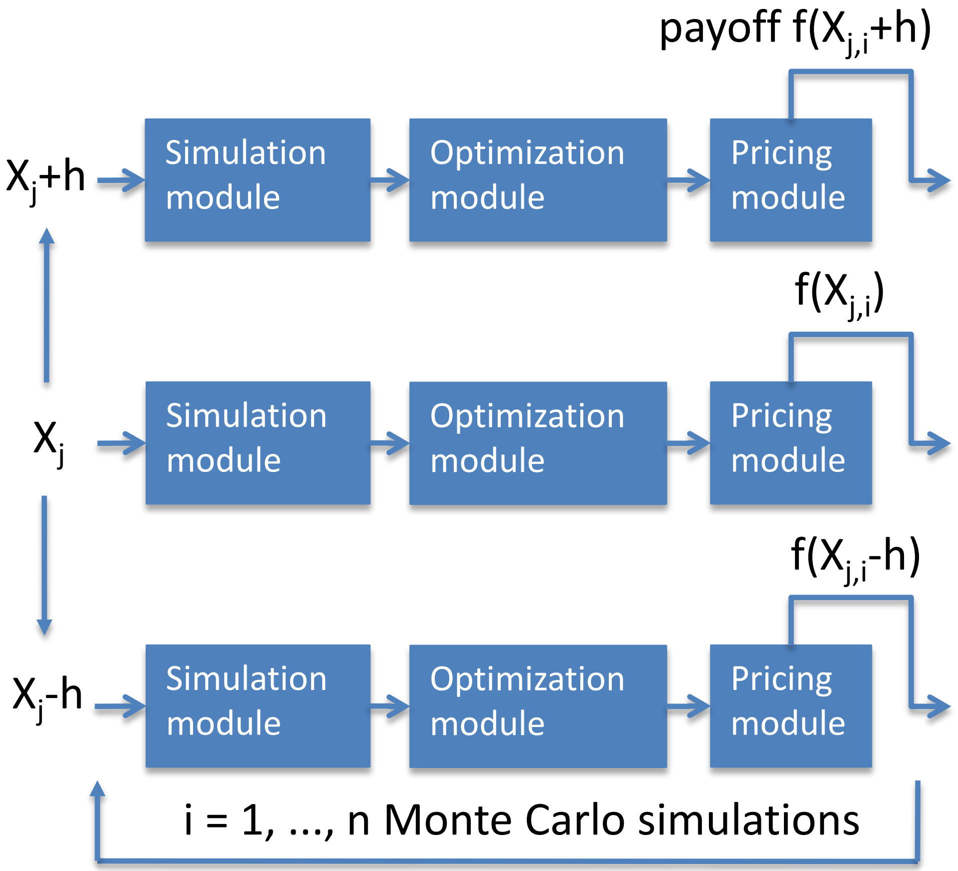
1. Combine numerical advantages of a MC method from the fixed income community with an energy related valuation framework for time coupled options
2. Give evidence that method allows to calculate robust sensitivities including Delta and Gamma
3. Apply framework for valuation and risk assessment of stylized reserve requirements in a portfolio context

Due to technical constraints power plant options difficult to evaluate

- A power plant can be interpreted as a strip of hourly options subject to technical constraints
- Within the most prominent constraints:
  - minimum up-/down time (t_{up}, t_{down})
  - min-/max load (P_{min}, P_{max})
  - reserve requirements (Res_{pos}, Res_{neg})
- Constraints lead to interdependencies between exercise decisions at different points of time ("time coupling effects")
- Potentially significant impact on power plant dispatch

⇒ Power plant cannot be replicated by a simple strip of hourly options and typically no closed pricing formula given

Typical field of application for standard Monte Carlo simulation



Value: $V(X_j) \approx \frac{1}{n}\sum_{i=1}^{n} f(X_{j,i})$

Finite Difference (FD) approximation of sensitivities Delta and Gamma:

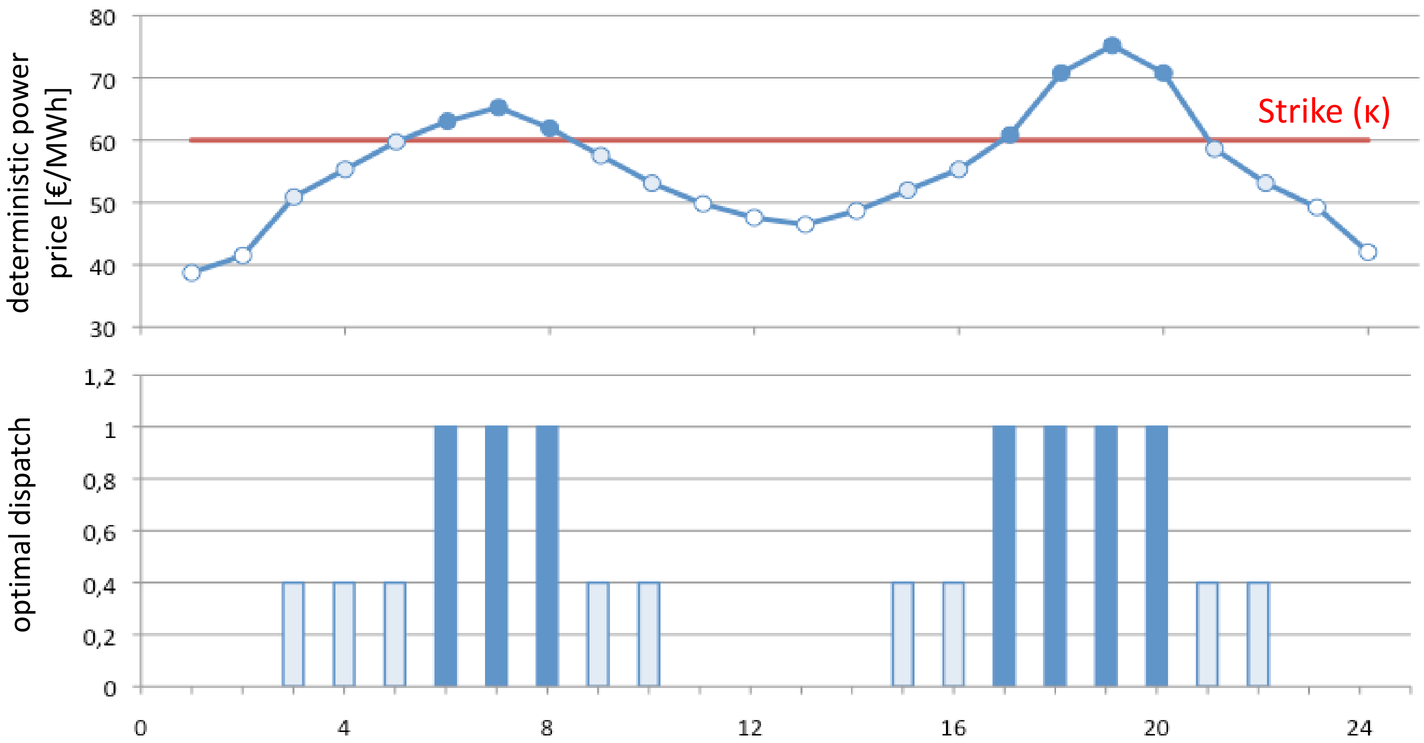
$$\Delta_j \approx \frac{1}{n}\sum_{i=1}^{n} \frac{f(X_{j,i}+h) - f(X_{j,i}-h)}{2h}$$

$$\Gamma_j \approx \frac{1}{n}\sum_{i=1}^{n} \frac{f(X_{j,i}+h) + f(X_{j,i}-h) - 2f(X_{j,i})}{h^2}$$

⇒ (2m+1) x n simulations needed for the calculation of m 2nd order sensitivities (central differences)

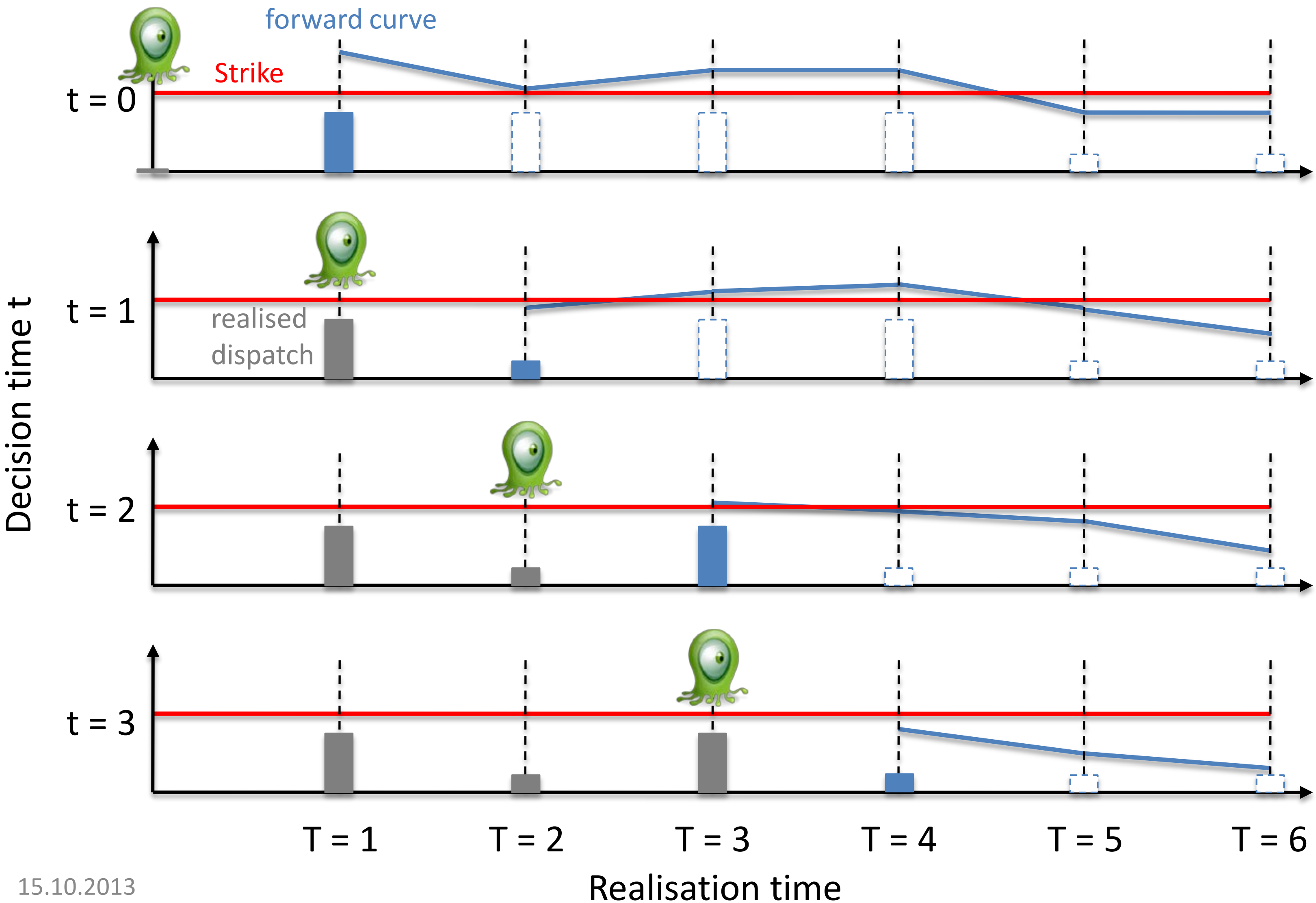
But: time coupled options require large computational costs in standard MC

1. Mixed-integer optimization to find optimal dispatch (P_{max}=1, P_{min}=0.4, t_{up}=8):



2. Avoid “perfect foresight error” with rolling intrinsic valuation

Rolling intrinsic valuation (t_{up} = 6)



Also well-known direct MC methods can usually not be applied

1. Pathwise method (Papatheodorou, 2005; Giles, 2007):
   - ✓ Unbiased estimators
   - ✓ No resimulation needed
   - ✗ Typically not applicable for 2^{nd} order derivatives
   - ✗ Requires expression of pathwise derivatives
2. Likelihood ratio method (Broadie and Glasserman, 1996; Glasserman, 2003):
   - ✓ Unbiased estimators
   - ✓ No resimulation needed
   - ✓ Applicable to 2^{nd} order derivatives
   - ✗ Explicit probability density required

$\Rightarrow$ Demand for 2^{nd} order derivatives (Gamma) or realistic price processes conflict with requirements of these methods (or their combinations)

Possible solution: Proxy Simulation Scheme (PSS) method of Fries & Kampen

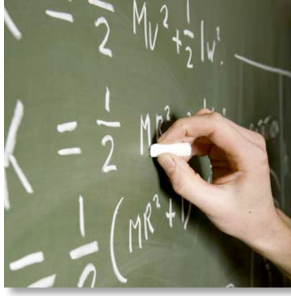
- In 2005 Christian P. Fries & Jörg Kampen proposed a new MC method with application for the Libor Market Model: Proxy Simulation Scheme method

Fundamental idea:

1. Take MC realisations which have been used for valuation of an option and reinterpret them as realisations of virtually shifted forward curves
2. Apply method on the level of numerical implementation, i.e. after time discretisation of the underlying stochastic differential equation (main difference to likelihood method)

⇒ Calculation of m 2nd order sensitivities by using only n simulations instead of (2m+1) x n simulations

Time discretization of underlying stochastic differential equation



- Price process **X** with drift **μ**, volatility **σ** and correlated brownian motions W_j:

$$\boxed{dX_j(t) = \mu_j(t)X_j(t)dt + \sigma_j(t)X_j(t)dW_j(t)}$$

$$dW_j(t) \cdot dW_k(t) = \rho_{jk}(t)dt$$

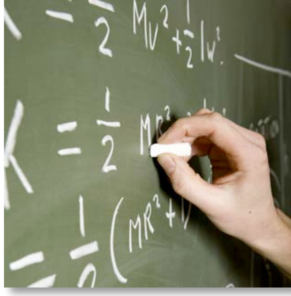
- Using log-coordinates and de-coupling via Cholesky decomposition:

$$d\mathbf{K}(t) = -\frac{1}{2}\mathbf{\Sigma}^2(t)\mathbf{1}dt + \mathbf{\Sigma}(t)\mathbf{L}(t)\mathbf{dU}(t)\sqrt{dt}$$

- Time discretization (simple Euler scheme):

$$\boxed{\mathbf{K}^*(t_{q+1}) \approx \mathbf{K}^*(t_q) - \frac{1}{2}\mathbf{\Sigma}^{*2}(t_q)\mathbf{1}\Delta t + \mathbf{\Sigma}^*(t_q)\mathbf{L}^*(t_q)\mathbf{\Delta U}(t_q)\sqrt{\Delta t}}$$

Use Proxy Simulation Scheme to calculate expected payoff



$$Y := (\mathbf{X}(t)) \quad \text{with} \quad t \in [T_1, T_2]$$

Proxy scheme Φ_{Y^0} does not depend on parameter Θ!

$$\begin{aligned}
E[f(Y(\Theta))] \approx E[f(Y^*(\Theta))] &= \int f(y)\Phi_{Y^*(\Theta)}(y)dy \\
&= \int f(y)\frac{\Phi_{Y^*(\Theta)}(y)}{\boxed{\Phi_{Y^0}(y)}}\boxed{\Phi_{Y^0}(y)}dy \\
&= \int f(y)\cdot\boxed{w(\Theta)}\cdot\Phi_{Y^0}(y)dy \\
= E\left[f(Y^0)\cdot w(\Theta)\right] &\approx \hat{E}\left[f(Y^0)\cdot w(\Theta)\right] \\
&= \frac{1}{n}\sum_{i=1}^{n}\boxed{f(Y_i^0)}\cdot\boxed{w_i(\Theta)}.
\end{aligned}$$

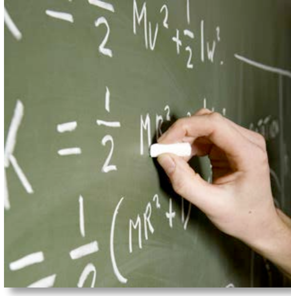
time discretization

MC approximation

weights

MC simulations now use weighted realizations of the proxy scheme

Use PSS to calculate general derivatives



$$
\begin{aligned}
\frac{\partial}{\partial \Theta} E[f(Y(\Theta))] &\approx \frac{\partial}{\partial \Theta} E[f(Y^*(\Theta))] \\
&\approx \frac{1}{2h}\left(E[f(Y^*(\Theta+h))] - E[f(Y^*(\Theta-h))]\right) \\
&= \int f(y)\frac{1}{2h}\left(\Phi_{Y^*(\Theta+h)}(y) - \Phi_{Y^*(\Theta-h)}(y)\right) dy \\
&= \int f(y)\frac{1}{2h}\frac{\left(\Phi_{Y^*(\Theta+h)}(y) - \Phi_{Y^*(\Theta-h)}(y)\right)}{\Phi_{Y^0}(y)}\Phi_{Y^0}(y)dy \\
&= \int f(y)\cdot\frac{1}{2h}\left(w(\Theta+h) - w(\Theta-h)\right)\cdot\Phi_{Y^0}(y)dy \\
&= E\left[f(Y^0)\cdot\frac{1}{2h}\left(w(\Theta+h) - w(\Theta-h)\right)\right] \\
&\approx \frac{1}{n}\sum_{i=1}^{n} \boxed{f(Y_i^0)}\cdot \boxed{\frac{1}{2h}\left(w_i(\Theta+h) - w_i(\Theta-h)\right)}.
\end{aligned}
$$

time discretization

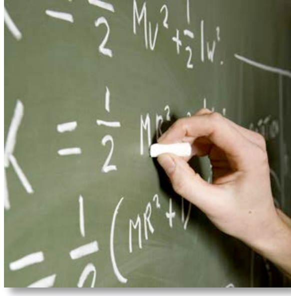
FD approximation

MC approximation

Same realizations – only weights have been changed!

modified weights

Calculation of Δ and Γ with the Proxy Simulation Scheme



- Θ is one initial forward price: $\Theta := X_k^*(t_0)$

$\Rightarrow$ Only difference between simulation schemes are initial conditions

- Use original simulation scheme X* as “proxy scheme” or “basis scheme”:

$$\frac{\partial}{\partial\Theta}E[f(Y(\Theta))] \approx \frac{1}{n}\sum_{i=1}^{n} f(Y_i^*)\cdot\frac{1}{2h}\left(w_i(\Theta+h)-w_i(\Theta-h)\right)$$

- Consider (virtual) upwards shifted scheme X^{up} to derive $w_i(\Theta+h)$:

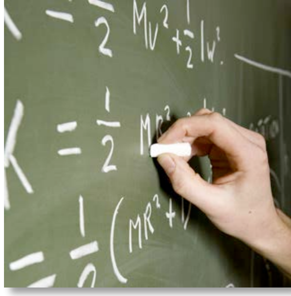
$$\mathbf{X}^{up}(t_0) = \mathbf{X}^*(t_0) + h\cdot\mathbf{e}_k$$

$$\mathbf{K}^{up}(t_0) = \mathbf{K}^*(t_0) + \delta_k^{up}\cdot\mathbf{e}_k \qquad \delta_k^{up} = \log(X_k(t_0)+h) - \log(X_k(t_0))$$

- Associated weight $w_i(\Theta+h)$: pathwise relation between probability densities:

$$w_i(\Theta+h) = \frac{\Phi_{Y^{up}}(Y_i^*)}{\Phi_{Y^*}(Y_i^*)}$$

Deriving PSS weight $w_i(\Theta + h) = \frac{\Phi_{Y^{up}}(Y_i^*)}{\Phi_{Y^*}(Y_i^*)}$: at first the denominator



- The denominator is equal to the realisation probability of Y_i^* in the original simulation scheme

⇒ Fully determined by standard normal distributed random numbers **ΔU** as applied in MC simulation i:

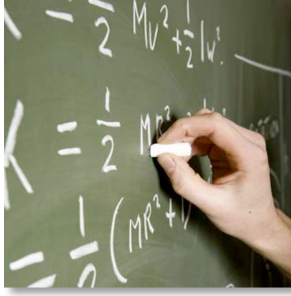
$$\boxed{\Phi_{Y^*}(Y_i^*) = \prod_{q=1}^{m} \frac{1}{\sqrt{2\pi}^{m-(q-1)}} \exp\left(-\frac{1}{2}(\Delta \mathbf{U}_i)^T(t_q) \cdot \Delta \mathbf{U}_i(t_q)\right)}$$

- $\Phi_{yup}(Y_i^*)$ is the probability of realisation Y_i^* in a virtually shifted scheme Y^{up}
- For derivation of the numerator consider an upshift of the first price $X_1^*(t_0)$

⇒ $\Phi_{yup}(Y_i^*)$ needs to be the probability of a vector of random numbers $\mathbf{\Delta U}^{up}$ which generate a jump that aligns both schemes within the first time step

$$\begin{aligned} \mathbf{K}^{up}(t_0) &= \mathbf{K}^*(t_0) + \delta_1^{up} \cdot \mathbf{e}_1 \\ \mathbf{K}^{up}(t_q) &\overset{!}{=} \mathbf{K}^*(t_q) \quad \text{for} \quad q = 1, 2, \ldots, m \end{aligned}$$

Deriving PSS weight $w_i(\Theta + h) = \frac{\Phi_{Y^{up}}(Y_i^*)}{\Phi_{Y^*}(Y_i^*)}$: secondly the numerator



- Easy to show that the required jump is generated by the following vector of standard normal distributed random numbers $\mathbf{\Delta U}^{up}$:

$$\mathbf{\Delta U}^{up}(t_0) \stackrel{!}{=} \mathbf{\Delta U}(t_0) - \frac{1}{\sqrt{\Delta t}} \mathbf{L}^{*-1}(t_0) \mathbf{\Sigma}^{*-1}(t_0) \delta_1^{up} \cdot \mathbf{e}_1$$

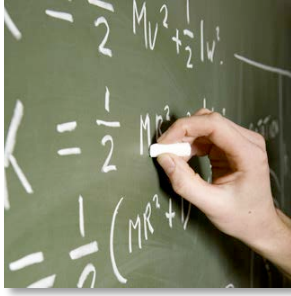
- In case of a perturbation of the k^th^ element of $\mathbf{X}^*(t_0)$ the jump can be spread over k time steps leading to

$$\mathbf{\Delta U}^{up}(t_q) \stackrel{!}{=} \mathbf{\Delta U}(t_q) - \frac{1}{\sqrt{\Delta t}} \mathbf{L}^{*-1}(t_q) \mathbf{\Sigma}^{*-1}(t_q) \frac{\delta_k^{up}}{k} \cdot \mathbf{e}_k \quad \text{for} \quad q < k$$

- The numerator is given by the associated probability:

$$\boxed{\Phi_{Y^{up}}(Y_i^*) = \prod_{q=1}^{k} \frac{1}{\sqrt{2\pi}^{k-(q-1)}} \exp\left(-\frac{1}{2} (\mathbf{\Delta U}_i^{up})^T(t_q) \cdot \mathbf{\Delta U}_i^{up}(t_q) \right)}$$

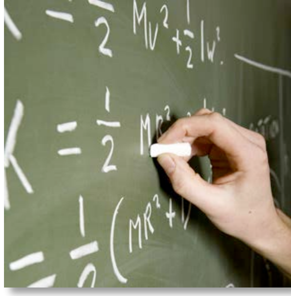
Finally all parts of the expressions for Δ and Γ are available

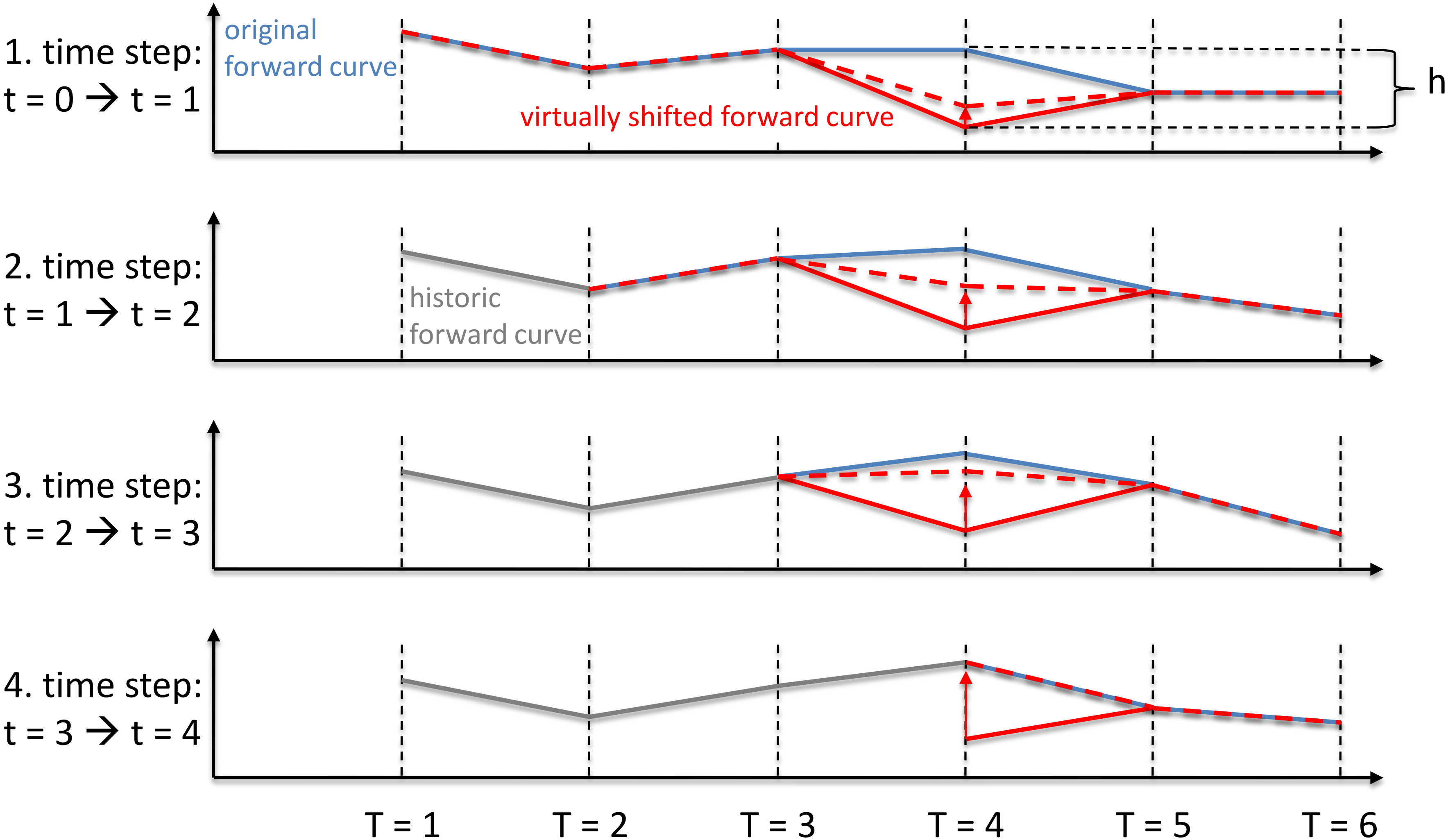


- Δ and Γ can be simulated by using the same realisations as used for valuation and associated weights are computed on-the-fly

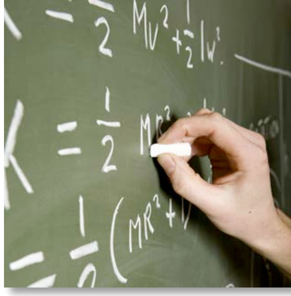
$$
\begin{aligned}
\frac{\partial}{\partial\Theta}E[f(Y(\Theta))] &\approx \frac{1}{n}\sum_{i=1}^{n} f(Y_i^*)\cdot\frac{1}{2h}\left(w_i(\Theta+h)-w_i(\Theta-h)\right)\\
&= \frac{1}{n}\sum_{i=1}^{n} f(Y_i^*)\cdot\frac{1}{2h}\left(\frac{\Phi_{Y^{up}}(Y_i^*)}{\Phi_{Y^*}(Y_i^*)}-\frac{\Phi_{Y^{down}}(Y_i^*)}{\Phi_{Y^*}(Y_i^*)}\right)\\
\frac{\partial^2}{\partial\Theta^2}E[f(Y(\Theta))] &\approx \frac{1}{n}\sum_{i=1}^{n} f(Y_i^*)\cdot\frac{1}{h^2}\left(w_i(\Theta+h)+w_i(\Theta-h)-2w_i(\Theta)\right)\\
&= \frac{1}{n}\sum_{i=1}^{n} f(Y_i^*)\cdot\frac{1}{h^2}\left(\frac{\Phi_{Y^{up}}(Y_i^*)}{\Phi_{Y^*}(Y_i^*)}+\frac{\Phi_{Y^{down}}(Y_i^*)}{\Phi_{Y^*}(Y_i^*)}-2\right)
\end{aligned}
$$

PSS: stepwise convergence between real and shifted scheme





Four fundamental advantages of the PSS method



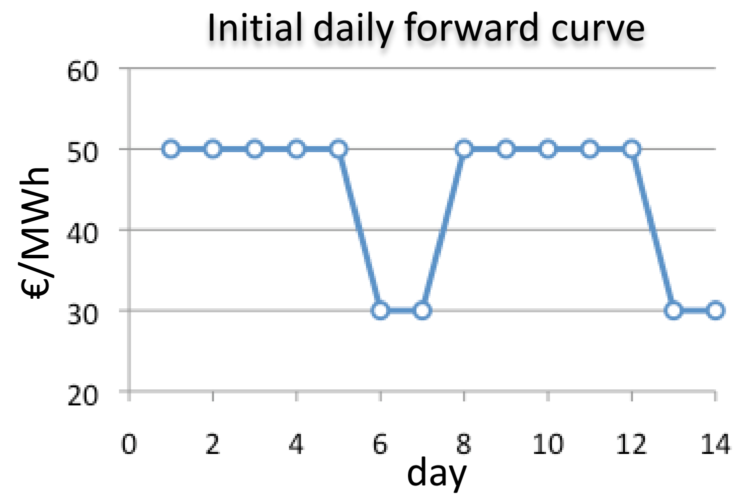
1. Realisations for valuation are recycled for calculation. Thereby significant computational savings.

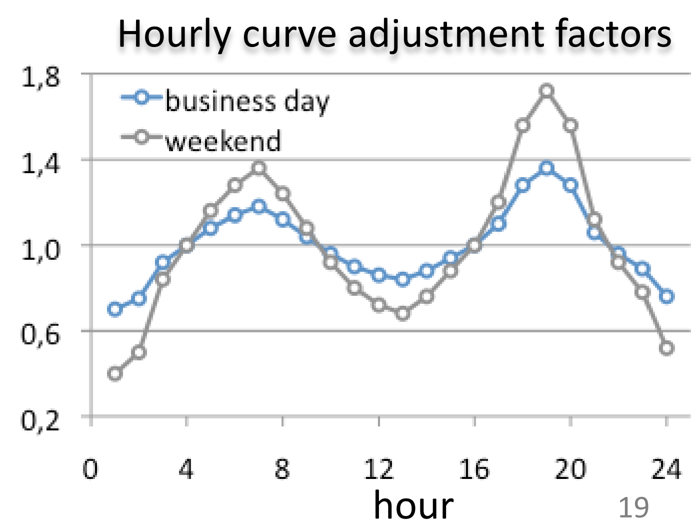
$$
\begin{aligned}
E[f(Y(\Theta))] &\approx \frac{1}{n}\sum_{i=1}^{n} f(Y_i^0)\cdot w_i(\Theta) \\
\frac{\partial}{\partial\Theta}E[f(Y(\Theta))] &\approx \frac{1}{n}\sum_{i=1}^{n} f(Y_i^0)\cdot \frac{1}{2h}\left(w_i(\Theta+h)-w_i(\Theta-h)\right) \\
\frac{\partial^2}{\partial\Theta^2}E[f(Y(\Theta))] &\approx \frac{1}{n}\sum_{i=1}^{n} f(Y_i^0)\cdot \frac{1}{h^2}\left(w_i(\Theta+h)+w_i(\Theta-h)-2w_i(\Theta)\right)
\end{aligned}
$$

2. PSS is a FD method applied on the probability density, typically a continuous function. Thereby small shiftsize h possible (small bias).
3. Easy to implement as an add-on to any existing MC pricing algorithm.
4. PSS works on the level of the time discretized price evolution scheme and not on the level of the exact model sde. Consequently, all required probabilities are known per construction and can be calculated on the fly.

Putting pieces together: Setup of the valuation model

- Calculation of daily option values and sensitivities (Δ, Γ) for 14 days (also 7 days)
- Underlying risk factor: daily baseload power price forward curve (rolled from day to day)
- Hourly curve adjustment (hca) factors used to derive hourly forward prices from daily profile (strong photovoltaics impact assumed)
- Dispatch calculated on a day ahead basis (using hourly prices for remaining valuation tenor)
- Volatility based on Benth and Koekebakker (2005), fitted to two factor price model of Börger (2007)
- Regular parametric correlation matrix Schoenmakers and Coffey (2002)
- Fortran 90 program on Ubuntu Linux CPUs





Putting pieces together: mip to find optimal dispatch

- Currently valuation of a single option using GLPK (open source, C)
- Maximization of value function V…

$$\boxed{V(\beta, s, \gamma) = \sum_{i=1}^{N} (\gamma_i P_{max} + P_{min}(\beta_i - \gamma_i)) \cdot (X_i - \kappa)}$$

$$\beta_i \in \mathbb{N},\ \ 0 \leq \beta_i \leq 1 \qquad s_i \in \mathbb{R},\ \ 0 \leq s_i \leq 1 \qquad \gamma_i \in \mathbb{R},\ \ 0 \leq \gamma_i \leq 1$$

…subject to technical constraints (min/max load, min up/down times)…

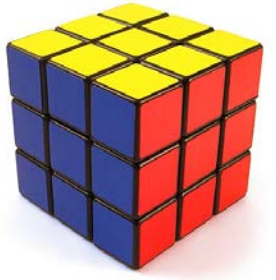
$$\begin{aligned} \beta_i - \beta_{i-1} &\leq s_i \\ \beta_i &\geq \gamma_i \end{aligned} \qquad \beta_i \geq \sum_{t=i-t_{up}+1}^{i} s_t \qquad \beta_i \leq 1 - \sum_{t=i+1}^{i+t_{down}} s_t$$

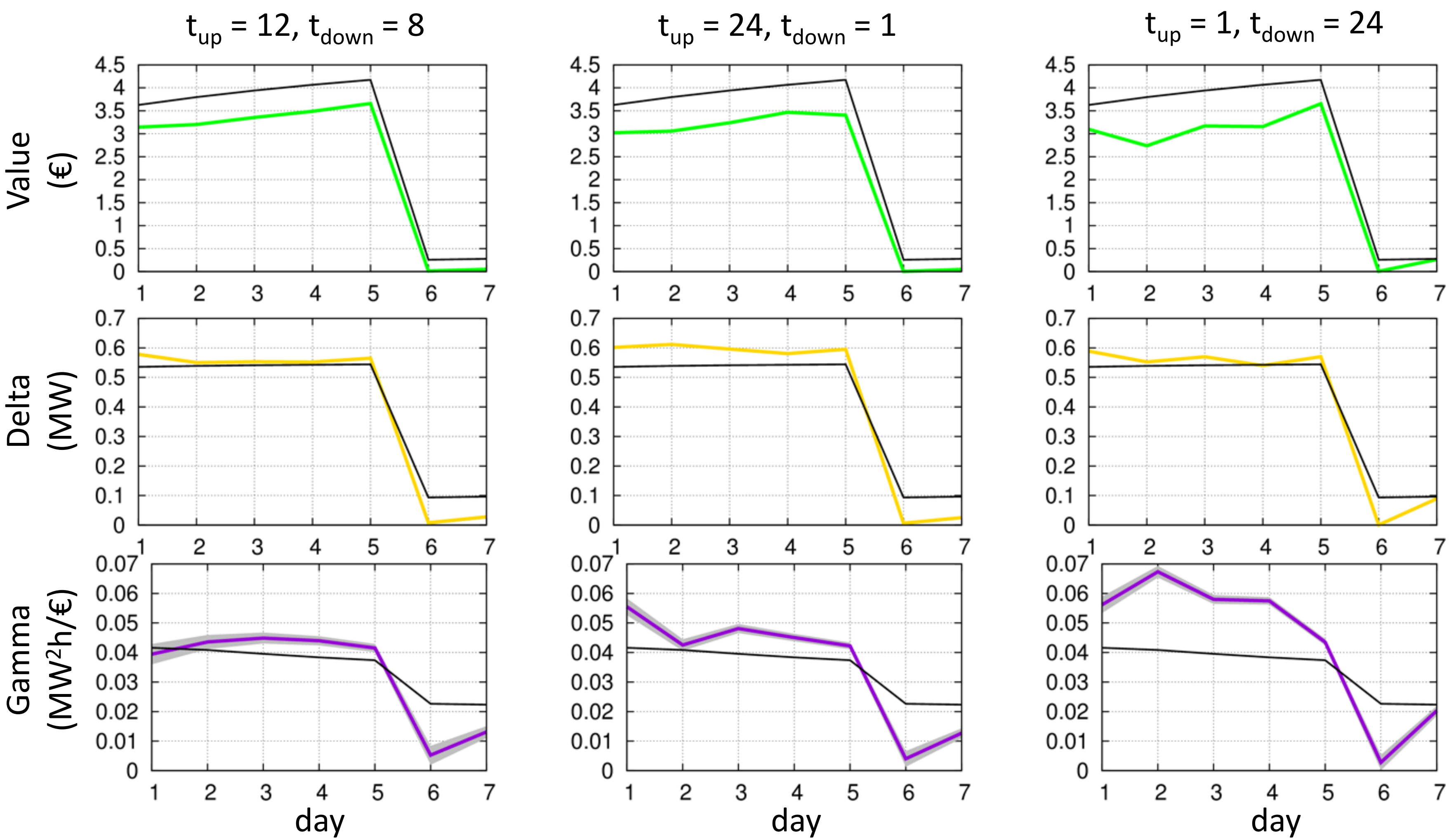
…and reserve requirements

$$\beta_i P_{max} - (\gamma_i P_{max} + (\beta_i - \gamma_i) P_{min}) \geq \mathrm{Res}_{pos}$$

$$(\gamma_i P_{max} + (\beta_i - \gamma_i) P_{min}) - \beta_i P_{min} \geq \mathrm{Res}_{neg}$$

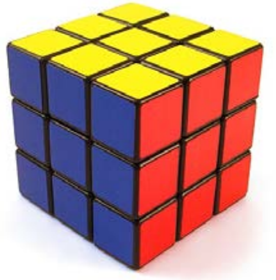
Technical constraints can lead to increase of „flexibility“

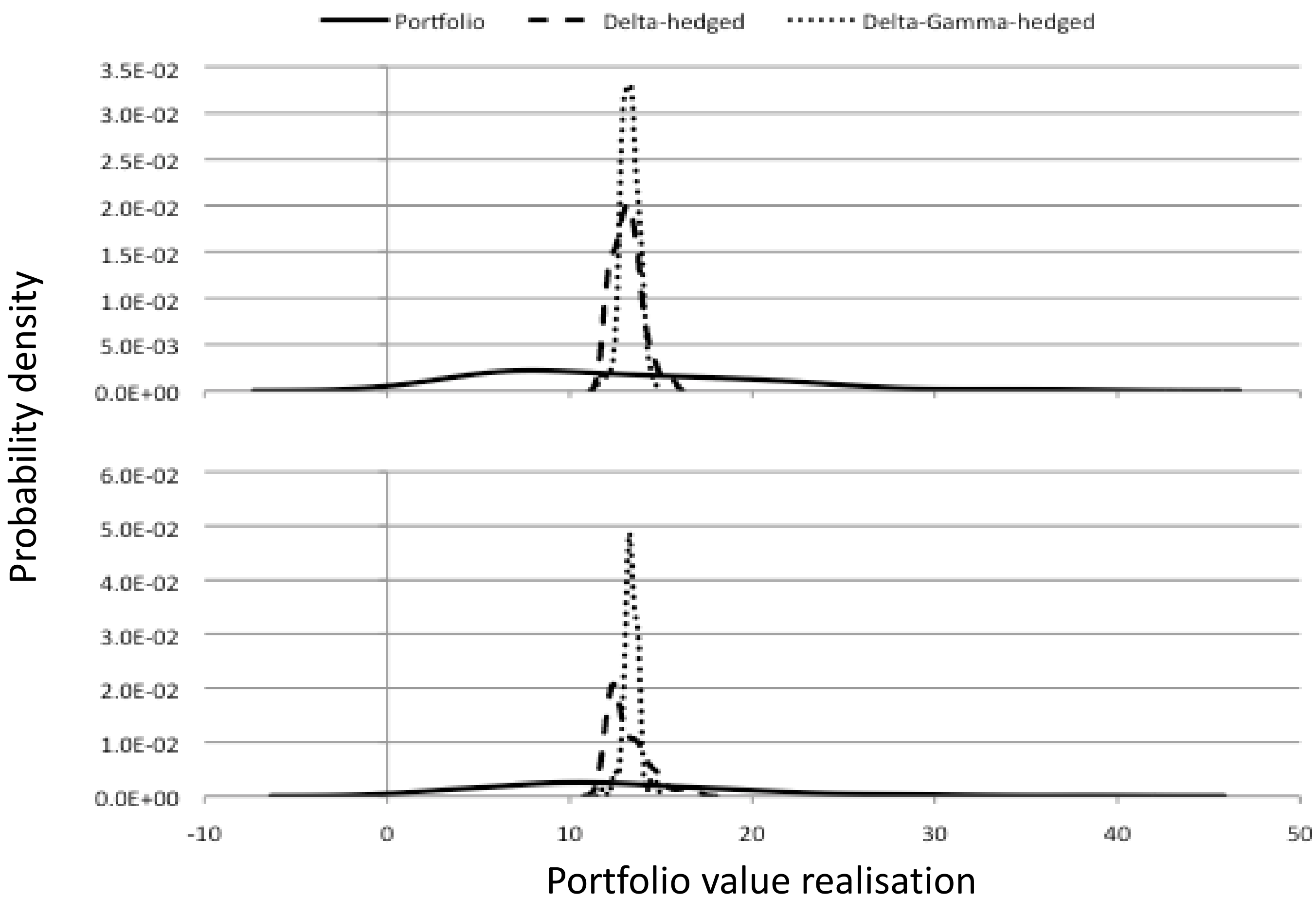




Increase of weekday Gamma due to clustering of itm/otm blocks

Prove hedge effectiveness via Delta-Gamma hedging framework





100k inner simulations

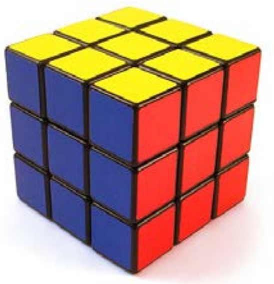
Stdev(Portfolio): 187.6
Stdev(Δ-hedged): 19.5
Stdev(Δ-Γ-hedged): 11.7

500k inner simmulations

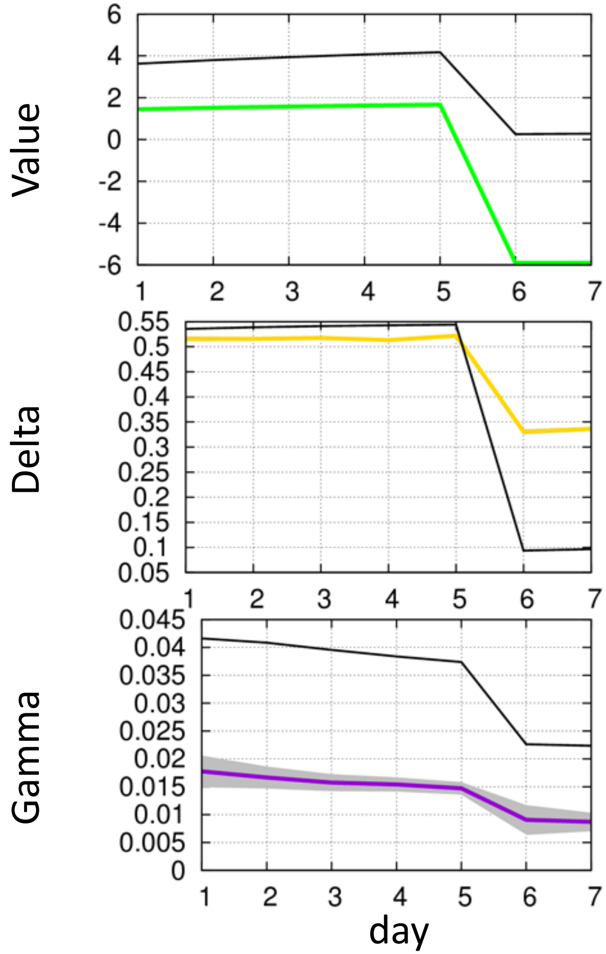
Stdev(Portfolio): 172.4
Stdev(Δ-hedged): 26.9
Stdev(Δ-Γ-hedged): 10.4

Simulation of 4 weekdays, t_{up} = 12, t_{down} = 8, 100 outer sims

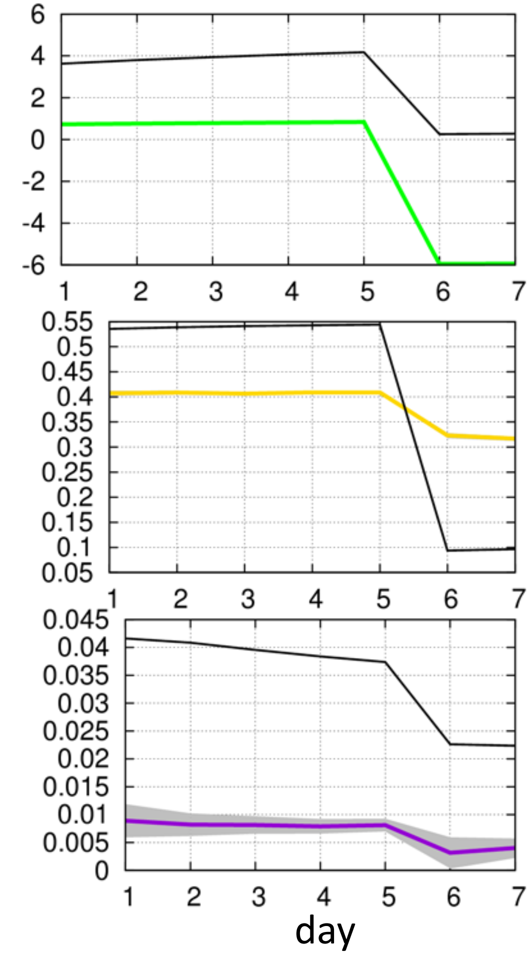
Positive reserve requirement reduces portfolio flexibility



$t_{up} = 12$, $t_{down} = 8$, $Res_{pos} = 0.3$

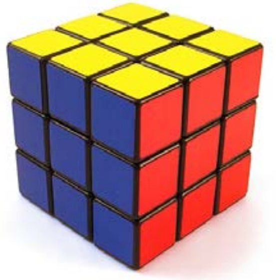


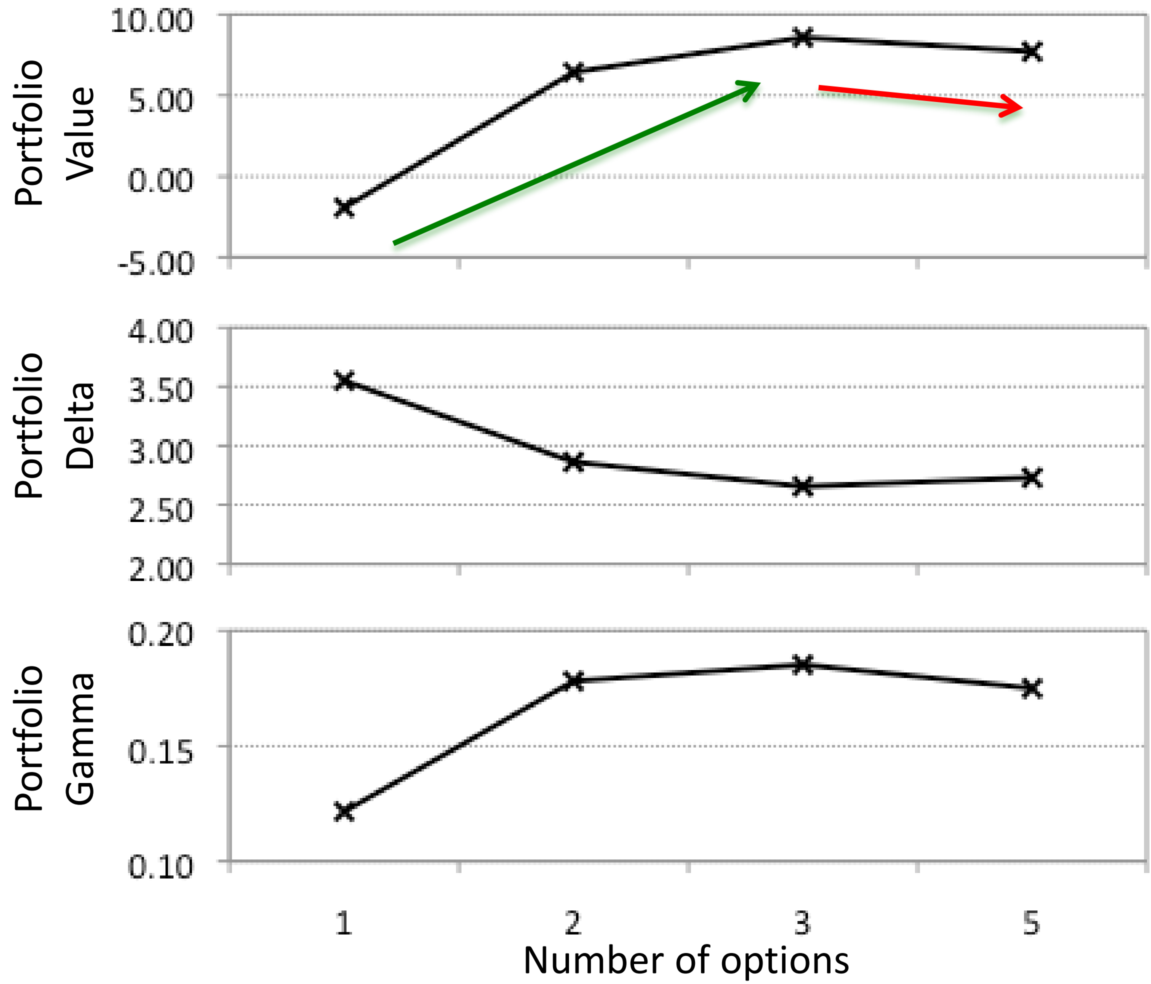
$t_{up} = 12$, $t_{down} = 8$, $Res_{pos} = 0.5$



Some results are intuitive

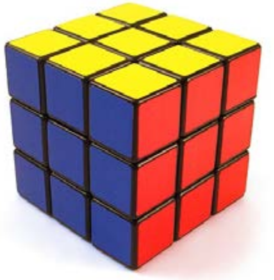
Portfolio effect: split of option into 2, 3, and 5 smaller parts

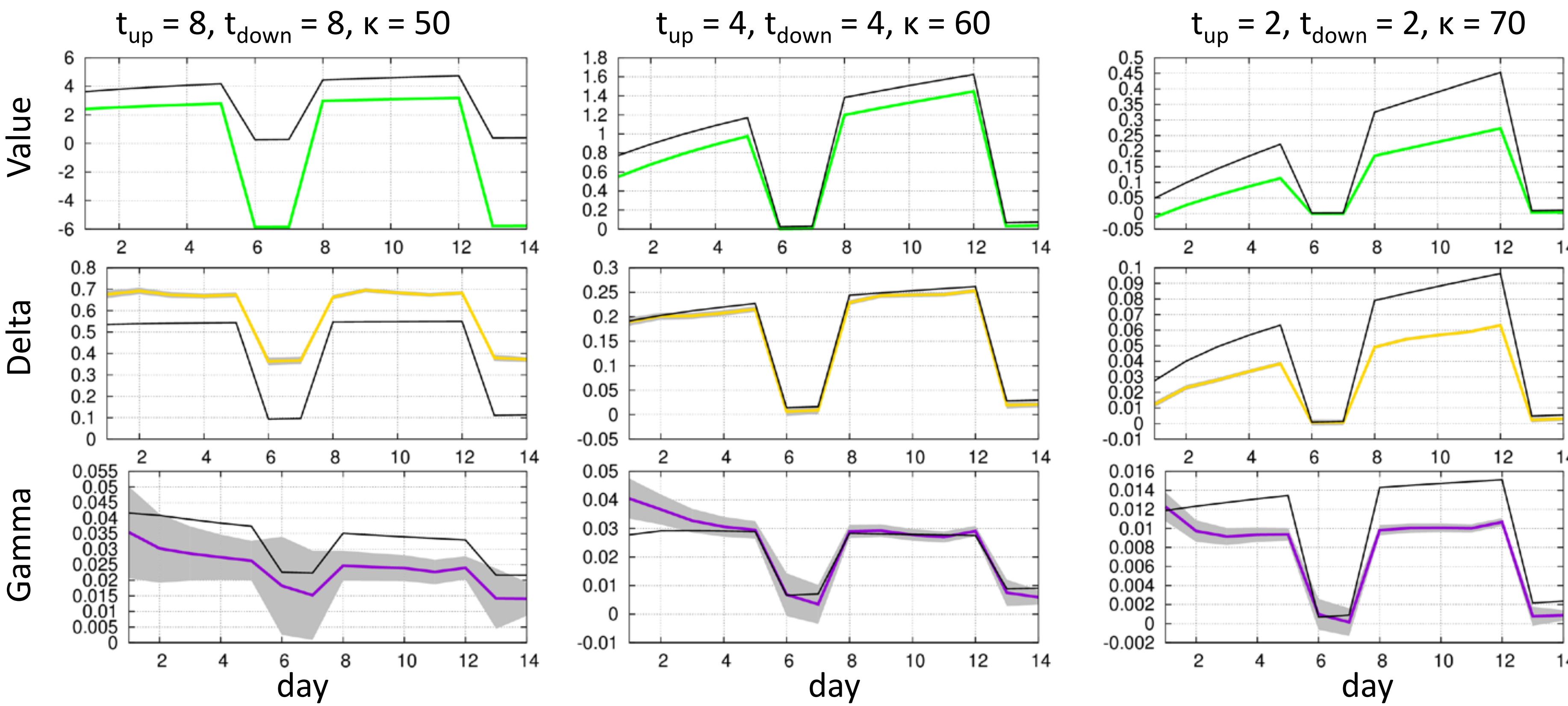




7 days, $Res_{pos} = 0.2$, initial: $t_{up} = 12$, $t_{down} = 8$, $P_{min} = 0.3$, $P_{max} = 1.0$

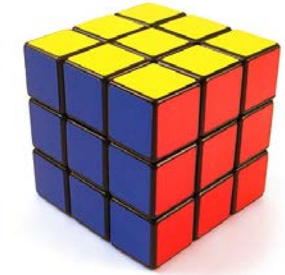
Portfolio of 3 options with low reserve requirement ($Res_{pos} = 0.3$)

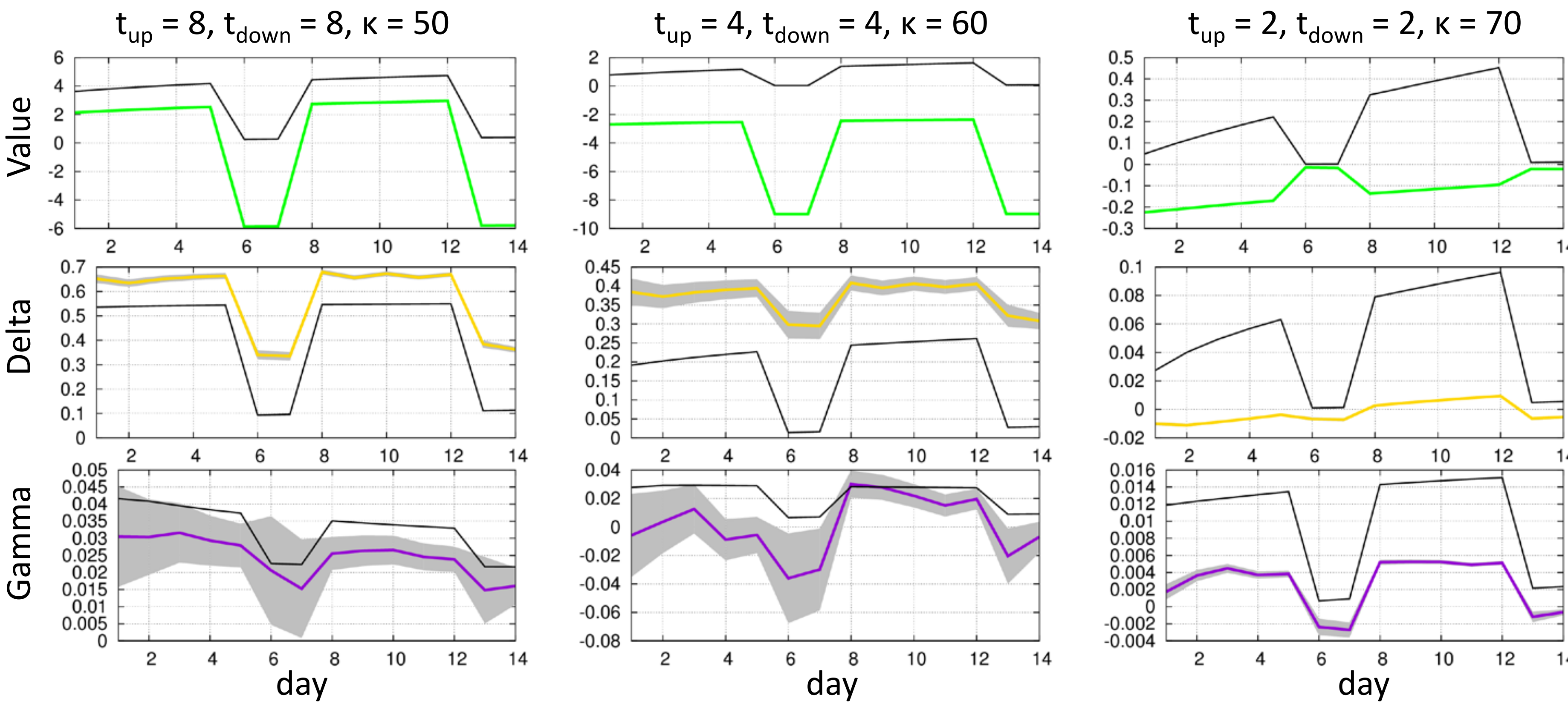




WE: power plant 1 provides reserve, WD: all options affected

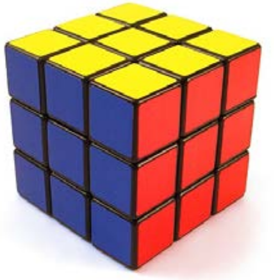
Portfolio of 3 options with high reserve requirement (Res_{pos} = 1.0)

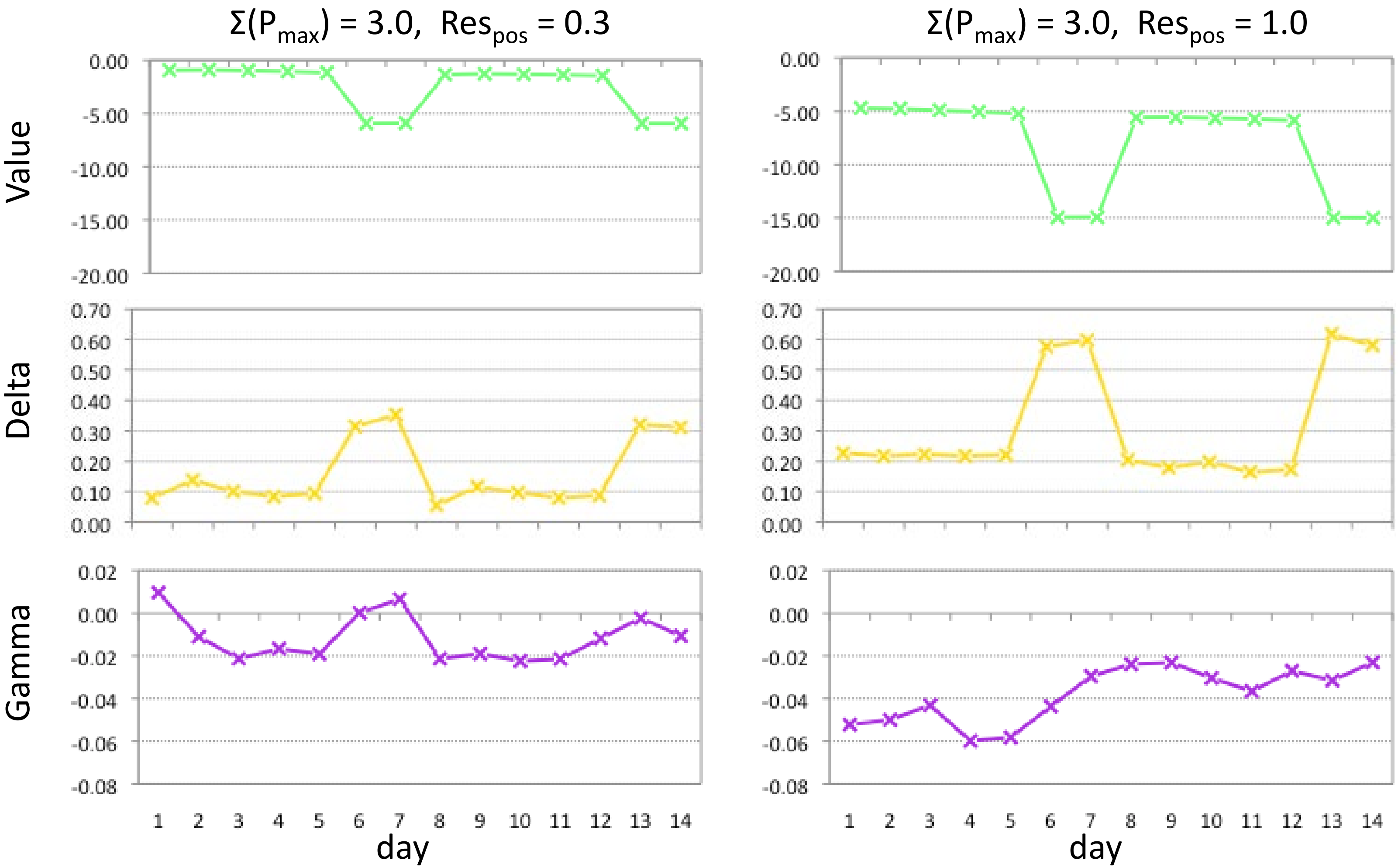




Two options needed in parallel for reserve requirement, especially on WEs power plant 2 serves as „option writer“

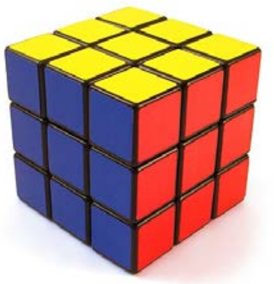
Valuation of reserve requirements in portfolio context (3 options)





Neg. Γ but pos. Δ: reserve not necessarily a „short power plant“!

Reserve impact on different option portfolios (14 days)



Young (minor tech. constraints):

(1) $t_{up/down}$ = 8/8,
$P_{min/max}$ = 0.3/1.0, κ = 50

(2) $t_{up/down}$ = 4/4
$P_{min/max}$ = 0.3/1.0 , κ = 60

(3) $t_{up/down}$ = 2/2
$P_{min/max}$ = 0.3/1.0 , κ = 70

Young (split options):

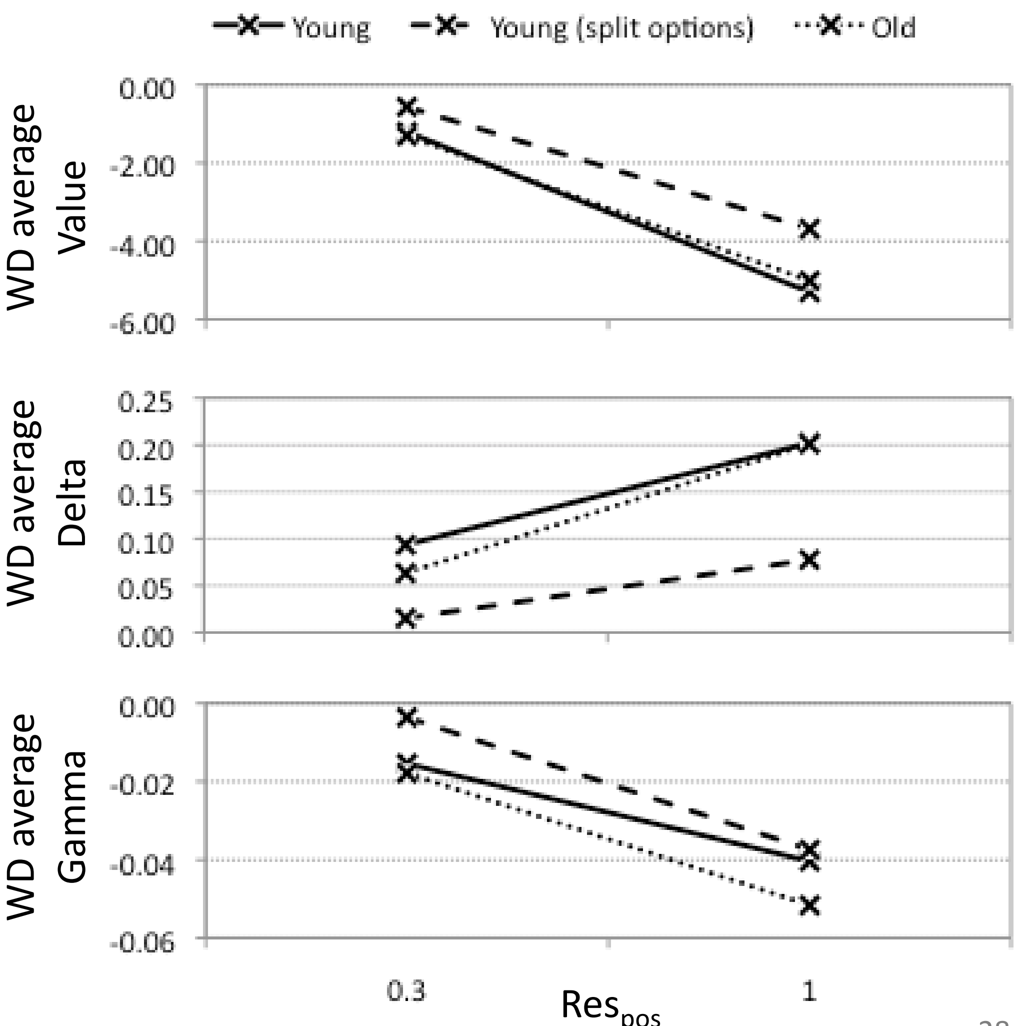
Options from „Young" portfolio each one split in two parts

Old (heavy tech. constraints):

(1) $t_{up/down}$ = 20/12
$P_{min/max}$ = 0.3/1.0 , κ = 50

(2) $t_{up/down}$ = 18/10
$P_{min/max}$ = 0.3/1.0 , κ = 60

(3) $t_{up/down}$ = 12/8
$P_{min/max}$ = 0.3/1.0 , κ = 70



Conclusion & outlook

- Broad range of numerical results for single options and portfolios available
- PSS seems to be an option for the calculation of Monte Carlo sensitivities (Delta and Gamma) in an energy related framework
- Method allows to value reserve contracts in a portfolio context and to derive associated hedge parameters

- Compare „rules of thumb“ for dispatch calculation with full rolling intrinsic approach (due performance increase high relevance for practitioners)