Design of efficient cap-and-trade systems - An equilibrium analysis

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Relevant issues with EU ETS

- The main objective of the EU ETS is a socially sustainable and economically efficient **reduction of the emissions** of greenhouse gases. Are we on the right way?
- **Grandfathering** of the certificates has been finally strongly reduced, especially for the electricity generation industry
- In a relatively recent document [1], the European Commission recognizes the urgency to cope with the problem of an **excess of allowances circulating** in the market
- Some criticisms have also raised from the political side towards EU ETS ([2]), and some exponents have even doubted **if this system is still worth to survive**

General objectives

- Getting a clear understanding of the **full potential benefits** of the environmental markets based on a cap-and-trade principle
- Getting a clear vision of what is at the **origin of the drawbacks experienced so far**, trying to gain insights on how to calibrate appropriate measures

Basic Facts

1. A common result shared by all the analyses developed so far is that cap-and-trade systems indeed represent the **most efficient way to reduce and control the environmental damage** generated by the industrial activity
2. The **efficiency properties** of environmental markets have been first addressed in [3] and [4]
3. Montgomery shows that the equilibrium price for a certificate must be driven by the **cost of the most virtuous companies** to abate its marginal unit of pollutant

Risk aversion

- Only two papers (to our knowledge) have considered explicitly **risk averse decision makers**, [6] and [7]
- In this work we assume that agents are risk averse and our main purpose is to show several **properties of the joint equilibrium on the markets of electricity and emissions**

A toy model for electricity and emission markets

- Finite number of agents, $I = \{1, ..., N\}$
- Two times, 0 and T
- Agents schedule their production plans at $t = 0$ and emission reports are surrendered at $t = T$
- Agent i decision, $i \in I$, consists of **production plan** $\xi_0^i \in \Xi^i$ and **allowance trading** $\left(\vartheta_0^i, \vartheta_T^i\right)$. No capacity expansion is considered
- For each production plan $\xi_0^i \in \Xi^i$ the functions $V^i(\xi_0^i)$, $C^i(\xi_0^i)$, and $E^i(\xi_0^i)$ stand for produced volume, total production costs, and total CO2 emissions
- Uncertainty is modeled by random variables, realized on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$

Production

Given energy demand D_0, electricity price P_0, and the emission allowance price A_0, each agent decides on its production plan $\xi_0^i \in \Xi^i$. In this way he/she determines as well the accumulated **production costs**

$$C^i(\xi_0^i),$$

and the **total income** from the sold electricity

$$P_0 V^i(\xi_0^i)$$

Allowance trading

- Allowances can be exchanged between agents by trading at the prices A_0 and A_T
- Denote by ϑ_0^i and ϑ_T^i the change of the allowance number held by agent i at times 0 and T, respectively
- Trading generates a cost

$$\vartheta_0^i A_0 + \vartheta_T^i A_T. \tag{1}$$

Note that ϑ_0^i and A_0 are deterministic, whereas ϑ_T^i and A_T are modeled as random variables

Allowance allocation

- It is natural to assume that each agent is confronted with emissions, which can not be predicted with certainty at time 0
- The allowances γ^i effectively available for compliance are calculated by withdrawing unpredictable emissions from the initial allocation
- The available allowance amount γ^i is modeled by a random variable. Hence the total number $\gamma = \sum_{i \in I} \gamma^i$ of allowances, effectively available for compliance is also random
- $$P(\gamma = z) = 0 \qquad (2)$$

Penalty payment

- Penalty $\pi \in [0, \infty[$ must be paid at maturity T for each unit of pollutant not covered by allowances
- Penalty payment is given by

$$\pi(E^i(\xi_0^i) - \vartheta_0^i - \vartheta_T^i - \gamma^i)^+$$

Individual revenue

- The revenue of agent i following trading and production strategy $(\vartheta^i, \xi^i) = (\vartheta_0^i, \vartheta_T^i, \xi_0^i)$ depends on the market prices of allowances and electricity $(A, P) = (A_0, A_T, P_0)$ and is given by

$$L^{A,P,i}(\vartheta^i, \xi^i) = P_0 V^i(\xi_0^i) - C^i(\xi_0^i) - \vartheta_0^i A_0 - \vartheta_T^i A_T \\ - \pi(E^i(\xi_0^i) - \vartheta_0^i - \vartheta_T^i - \gamma^i)^+ \tag{3}$$

Risk aversion and rational behavior

Agent i rational behavior is targeted on the maximization of the functional

$$(\vartheta^i, \xi^i) \mapsto E(U^i(L^{A,P,i}(\vartheta^i, \xi^i)))$$

over all the possible trading and production strategies $(\vartheta^i, \xi^i) = (\vartheta^i_0, \vartheta^i_T, \xi^i_0)$, where U^i is a pre-specified strictly increasing concave utility function

Market equilibrium

Definition

Given energy demand $D_0 \in \mathbb{R}_+$, the prices $(A^*, P^*) = (A_0^*, A_T^*, P_0^*)$ are called equilibrium prices, if, for each agent $i \in I$ there exists a strategy $(\vartheta^{i*}, \xi^{i*}) = (\vartheta_0^{*i}, \vartheta_T^{*i}, \xi_0^{*i})$ such that:

(i) the energy demand is covered

$$\sum_{i \in I} V^i(\xi_0^{i*}) = D_0,$$

(ii) the emission certificates are in zero net supply

$$\sum_{i \in I} \vartheta_t^{*i} = 0 \quad \text{almost surely for } t = 0 \text{ and } t = T, \tag{4}$$

(iii) each agent $i \in I$ is satisfied by his own policy in the sense that

$$E(U^i(L^{A^*,P^*,i}(\vartheta^{*i}, \xi^{*i}))) \geq E(U^i(L^{A^*,P^*,i}(\vartheta^i, \xi^i)) \quad \text{for any } (\vartheta^i, \xi^i)$$

Opportunity costs

- When facing energy (electricity) generation, producers consider **a profit, which could be potentially realized when, instead of production, unused emission allowances were sold to the market**.
- For instance, if the price of the emission certificates is 12 euros per tonne of CO2 and the production of one Megawatt-hour (MWh) emits two tonnes of CO2, the producer compares:
  - produce and sell one MWh to the market,
  - do not produce this MWh and sell allowances covering two tonnes of CO2
- In this situation, the opportunity cost of producing one MWh is $24 = 2 \times 12$ euros

Opportunity costs

- Clearly, **both** the production and the opportunity costs must be considered in the formation of the electricity market price
- Thereby, if the production costs of electricity are 30 euros per MWh, the energy will be produced only if its price covers both the production and the opportunity costs. Thus electricity can only be delivered at a price exceeding $54 = 30 + 2 \times 12$ euros.
  To trigger the electricity production, the **opportunity costs must be added** to the production costs
- Notice: this would be true even if a producer had received the allowances for **free** (i.e. grandfathering of initial allowances)

Opportunity costs

- It turns out that adding opportunity costs is nothing but the **core mechanism**, responsible for the emission savings. Namely, due to the opportunity costs, **clean technologies appear cheaper than CO2 emission-intense** production strategies
- For instance, a gas turbine, which yields 1 MWh at a cost of 40 euros and emits only one tonne of CO2, will hardly be preferred to the coal-fired steam turbine of the previous example (prod. cost $= 30$ euros) without emission regulation
- However, given an emission regulation, the opposite is true. If the allowance price is equal to 12 euro per tonne of CO2 as above:
  - full operating costs for gas turbine $52 = 40 + 1 \times 12$ euros,
  - full operating costs for coal-fired steam turbine $54 = 30 + 2 \times 12$ euros
- **Merit order**: the gas turbine **will be scheduled first**, followed by the coal-fired steam turbine, which runs only if the installed gas turbine capacity does not meet energy demand

Definition

Consider a given energy amount $d \in \mathbb{R}_+$ and allowance price $a \in \mathbb{R}_+$.

Introduce the *individual opportunity merit order* costs of agent $i \in I$ as

$$\mathcal{C}^i(d, a) = \inf\{C^i(\xi_0^i) + aE^i(\xi_0^i) : \xi_0^i \in \Xi^i, \quad V^i(\xi_0^i) = d\}.$$

An individual production plan $\xi_0^i \in \Xi^i$ is called conform with opportunity costs at emission price $a \in \mathbb{R}_+$, if

$$\mathcal{C}^i(V^i(\xi_0^i), a) = C^i(\xi_0^i) + aE^i(\xi_0^i).$$

Cumulative opportunity merit order

Definition

Introduce the *cumulative opportunity merit order* costs as

$$\mathcal{C}(d,a) = \inf\{\sum_{i\in I}(C^i(\xi_0^i) + aE^i(\xi_0^i)) \,:\, \xi_0^i \in \Xi^i, \quad i \in I, \quad \sum_{i\in I} V^i(\xi_0^i) = d\}.$$

The production plans $\xi_0^i \in \Xi^i$, $i \in I$, are called conform with opportunity costs at emission price a, if

$$\mathcal{C}(\sum_{i\in I} V^i(\xi_0^i), a) = \sum_{i\in I}\left(C^i(\xi_0^i) + aE^i(\xi_0^i)\right).$$

Merit order electricity price

Definition

Any price $p \in \mathbb{R}_+$ with the property that

$$-\mathcal{C}(\tilde{d}, a) + p\tilde{d} \leq -\mathcal{C}(d, a) + pd \quad \text{for all } \tilde{d} \in \mathbb{R}_+$$

is referred to as an opportunity merit order electricity price at (d, a).

Properties of equilibrium - Production plans and electricity price

Proposition (1)

Given energy demand D_0*, let* $(A^*, P^*) = (A_0^*, A_T^*, P_0^*)$ *be the equilibrium prices with the corresponding strategies* $(\vartheta^{i*}, \xi^{i*})$*,* $i \in I$*, then*

i) For each agent $i \in I$*, the* **individual** *production plan* ${\xi_0^*}^i$ *is conform with the opportunity costs at the emission price* A_0^*

$$\mathcal{C}^i(V^i({\xi_0^*}^i), A_0^*) = C^i({\xi_0^*}^i) + A_0^* E^i({\xi_0^*}^i). \tag{5}$$

ii) The **market** *production schedule* ${\xi_0^*}^i$*,* $i \in I$*, is conform with opportunity costs at the emission price* A_0^*

$$\mathcal{C}(\sum_{i \in I} V^i({\xi_0^*}^i), A_0^*) = \sum_{i \in I}(C^i({\xi_0^*}^i) + A_0^* E^i({\xi_0^*}^i)). \tag{6}$$

iii) P_0^* *is an opportunity merit order* **energy price** *at* $(\sum_{i \in I} V^i({\xi_0^*}^i), A_0^*)$*.*

Proof of proposition (1)

Proof.

i) Assume that an agent deviates from the optimal strategy following an alternative production plan $\xi_0^i \in \Xi^i$. The difference $E^i(\xi_0^i) - E^i(\xi_0^{*i})$ is traded at the market in addition to ϑ_0^{i*}:

$$\vartheta_0^i = \vartheta_0^{*i} + E^i(\xi_0^i) - E^i(\xi_0^{*i}), \qquad \vartheta_T^i = \vartheta_T^{i*}.$$

A direct calculation shows that the profit of this alternative strategy $(\vartheta^i, \xi^i) = (\vartheta_0^i, \vartheta_T^i, \xi_0^i)$ differs from the original profit

$$L^{A^*,P^*,i}(\vartheta^i, \xi^i) = L^{A^*,P^*,i}(\vartheta^{*i}, \xi^{*i}) + R(\xi_0^i, \xi_0^{*i})$$

by the amount

$$R(\xi_0^i, \xi_0^{*i}) = P_0^*(V^i(\xi_0^i) - V^i(\xi_0^{*i})) + (C^i(\xi_0^{*i}) - C^i(\xi_0^i)) + A_0^*(E^i(\xi_0^{*i}) - E^i(\xi_0^i)).$$

Proof of proposition (1)

Proof.

Note that the difference $R(\xi_0^i, \xi_0^{*i})$ is entirely determined in $t = 0$ and it can not be positive, since otherwise

$$L^{A^*,P^*,i}(\vartheta^i, \xi^i) > L^{A^*,P^*,i}(\vartheta^{*i}, \xi^{*i}).$$

Now, from $R(\xi_0^i, \xi_0^{*i}) \leq 0$ we have

$$-C^i(\xi_0^{*i}) - A_0^* E^i(\xi_0^{*i}) + P_0^* V^i(\xi_0^{*i}) \geq -C^i(\xi_0^i) - A_0^* E^i(\xi_0^i) + P_0^* V^i(\xi_0^i) \quad (7)$$

for each $\xi_0^i \in \Xi^i$. With this, we observe that any alternative production plan ξ_0^i which produces $V^i(\xi_0^i)$ at least equal to $V^i(\xi_0^{*i})$ necessarily satisfies

$$C^i(\xi_0^{*i}) + A_0^* E^i(\xi_0^{*i}) \leq C^i(\xi_0^i) + A_0^* E^i(\xi_0^i),$$

which proves the property i). □

Proof.

ii) Summing up (7) over $i \in I$ yields for arbitrary $\xi_0^i \in \Xi^i$

$$-\sum_{i \in I} \left(C^i(\xi_0^{*i}) + A_0^* E^i(\xi_0^{*i}) \right) + P_0^* \sum_{i \in I} V^i(\xi_0^{*i}) \geq$$
$$\geq -\sum_{i \in I} \left(C^i(\xi_0^{i}) + A_0^* E^i(\xi_0^{i}) \right) + P_0^* \sum_{i \in I} V^i(\xi_0^{i}). \tag{8}$$

From this, we obtain that, for any production plan $\xi_0^i \in \Xi^i$ satisfying $\sum_{i \in I} V^i(\xi_0^i) \geq \sum_{i \in I} V^i(\xi_0^{*i})$, $i \in I$:

$$\sum_{i \in I} \left(C^i(\xi_0^{*i}) + A_0^* E^i(\xi_0^{*i}) \right) \leq \sum_{i \in I} \left(C^i(\xi_0^{i}) + A_0^* E^i(\xi_0^{i}) \right),$$

which implies the desired assertion (6). □

Proof of proposition (1)

Proof.

iii) We need to prove that, for any $\tilde{d} \in \mathbb{R}_+$,

$$-\mathcal{C}(\tilde{d}, A_0^*) + P_0^* \tilde{d} \leq -\mathcal{C}(\sum_{i \in I} V^i(\xi_0^{i*}), A_0^*) + P_0^* \sum_{i \in I} V^i(\xi_0^{i*}).$$

For each choice of production strategies $\xi_0^i \in \Xi^i$, $i \in I$, the estimate (8), combined with (6) yields

$$-C(\sum_{i \in I} V^i(\xi_0^{*i}), A_0^*) + P_0^* \sum_{i \in I} V^i(\xi_0^{*i}) \geq$$
$$\geq -\sum_{i \in I} \left(C^i(\xi_0^i) + A_0^* E^i(\xi_0^i)\right) + P_0^* \sum_{i \in I} V^i(\xi_0^i).$$

In particular, if the strategies are chosen from

$$\{(\xi_0^i)_{i \in I} : \xi_0^i \in \Xi^i, \quad i \in I, \qquad \sum_{i \in I} V^i(\xi_0^i) \geq \tilde{d}\}, \tag{9}$$

Proof of proposition (1)

Proof.

then it holds

$$-\mathcal{C}(\sum_{i\in I} V^i(\xi_0^{*i}), A_0^*) + P_0^* \sum_{i\in I} V^i(\xi_0^{*i}) \geq -\sum_{i\in I}\big(C^i(\xi_0^i) + A_0^* E^i(\xi_0^i)\big) + P_0^* \tilde{d}.$$

Passing on the right-hand side of this inequality to

$$\mathcal{C}(\tilde{d}, A_0^*) := \inf\{\sum_{i\in I}(C^i(\xi_0^i) + A_0^* E^i(\xi_0^i)) \; : \; \xi_0^i \in \Xi^i, i \in I, \sum_{i\in I} V^i(\xi_0^i) \geq \tilde{d}\}$$

yields the desired assertion

$$-\mathcal{C}(\sum_{i\in I} V^i(\xi_0^{*i}), A_0^*) + P_0^* \sum_{i\in I} V^i(\xi_0^{*i}) \geq -\mathcal{C}(\tilde{d}, A_0^*) + P_0^* \tilde{d}.$$

□

Properties of equilibrium

These results are relevant for different reasons:

- they show that **cap-and-trade systems are cost efficient** (individually and globally) also in the presence of **risk averse decision makers**
- they **extend** the cost efficiency property of emission markets, which was originally proved only under risk neutrality
- they also show that in equilibrium the price of energy guarantees the highest level of (competitive) profit for the producers, to satisfy a known level of demand
- coupled with cost-pass-through (of opportunity costs) and grandfathering of emission allowances, they tell us that the **overall cost** for reducing CO2 emissions **is minimum for the society, but it does not imply that it is fairly distributed** between society sectors (i.e. producers and consumers)

No arbitrage pricing of emission allowances

- We now show another natural property of cap-and-trade emission markets: absence of arbitrage. It turns out that the terminal allowance price is digital.

Proposition (2)

Given energy demand D_0, let $(A^*, P^*) = (A_0^*, A_T^*, P_0^*)$ be the equilibrium prices with the corresponding strategies $(\vartheta^{i*}, \xi^{i*})$, $i \in I$, then it holds:

i) There exists a risk neutral measure $\mathbb{Q}^* \sim \mathbb{P}$ such that $A^* = (A_0^*, A_T^*)$ follows a martingale with respect to $\mathbb{Q}^*$.

ii) The terminal allowance price in equilibrium is digital

$$A_T^* = \pi 1_{\{\sum_{i \in I} E^i(\xi_0^{i*}) - \gamma \geq 0\}}. \tag{10}$$

Proof of proposition (2)

Proof.

i) According to the first fundamental theorem of asset pricing ([8]), the existence of the so-called equivalent martingale measure, satisfying

$$A_0^* = E^{\mathbb{Q}^*}(A_T)$$

is ensured by absence of arbitrage, which is in turn implied by equilibrium. We verify that the equilibrium rules out all arbitrage opportunities, through an indirect proof. Suppose that ν_0 is an allowance trading arbitrage:

$$\mathbb{P}(\nu_0(A_T^* - A_0^*) \geq 0) = 1, \quad \mathbb{P}(\nu_0(A_T^* - A_0^*) > 0) > 0. \tag{11}$$

Then we obtain a contradiction. Indeed suppose each agent i can change his/her original policy $(\vartheta^{*i}, \xi^{*i})$ to an improved strategy $(\tilde{\vartheta}^i, \xi^{*i})$ satisfying

$$E(U^i\left(L^{A^*,i}(\vartheta^{*i}, \xi^{*i})\right) < E\left(U^i(L^{A^*,i}(\tilde{\vartheta}^i, \xi^{*i})\right). \tag{12}$$

Proof of proposition (2)

Proof.

The improvement is achieved by incorporating arbitrage ν_0 into a trading of allowances as follows

$$\tilde{\vartheta}_0^i := \vartheta_0^{*i} + \nu_0, \qquad \tilde{\vartheta}_T^i := \vartheta_T^{*i} - \nu_0.$$

Indeed, the revenue improvement is

$$-\tilde{\vartheta}_0^i A_0^* - \tilde{\vartheta}_T^i A_T^* = -\vartheta_0^{*i} A_0^* - \vartheta_T^{*i} A_T^* + \nu_0 (A_T^* - A_0^*),$$

which we combine with (11) to see that

$$\mathbb{P}\left(L^{A,i}(\vartheta^{*i}, \xi^{*i}) \leq L^{A,i}(\tilde{\vartheta}^i, \xi^{*i})\right) = 1, \mathbb{P}\left(L^{A,i}(\vartheta^{*i}, \xi^{i}) < L^{A,i}(\tilde{\vartheta}^i, \xi^{*i})\right) > 0,$$

which implies (12), therefore contradicting the optimality of $(\vartheta^{*i}, \xi^{*i})$. □

Proof of proposition (2)

Proof.

ii) From the equilibrium property it follows that for almost each $\omega \in \Omega$ the terminal allowance position adjustment $\vartheta_T(\omega)$ is a maximizer on $\mathbb{R}$ to

$$z \mapsto -zA_T^*(\omega) - \pi(E^i(\xi_0^{i*}) - \vartheta_0^{i*} - \gamma^i(\omega) - z)^+. \tag{13}$$

Note that a maximizer of this mapping exists only if $0 \leq A_T^*(\omega) \leq \pi$. That is, the terminal allowance price in equilibrium must be within the interval $A_T^* \in [0, \pi]$ almost surely. Let us first show that the price actually attains only boundary values almost surely, i.e.

$$A_T^* \in \{0, \pi\}, \quad \text{almost surely.} \tag{14}$$

Suppose that an intermediate $A_T^*(\omega) \in]0, \pi[$ value is taken, then the unique maximizer of the function (13) is attained on

$$E^i(\xi_0^{i*}) - \vartheta_0^{i*} - \gamma^i(\omega).$$

Proof of proposition (2)

Proof.

This implies that $\vartheta_T^{i*}(\omega) = E^i(\xi_0^{i*}) - \vartheta_0^{i*} - \gamma^i(\omega)$ holds for each $i \in I$, and a summation over i yields

$$\sum_{i \in I} \vartheta_T^{i*}(\omega) = \sum_{i \in I} \left(E^i(\xi_0^{i*}) - \vartheta_0^{i*} - \gamma^i \right) = \sum_{i \in I} E^i(\xi_0^{i*}) - \gamma(\omega).$$

Note that the equilibrium property (4) ensures that the random variable on the left-hand side of the above equality is zero almost surely. Thus, the inclusion

$$\{A_T^* \in]0, \pi[\} \subseteq \{\sum_{i \in I} E^i(\xi_0^{i*}) - \gamma = 0\} \tag{15}$$

holds almost surely. Because of (2), the probability of the event on the right-hand side of the above inclusion is zero, which shows (14). If $A_T^*(\omega) = 0$, then a maximizer $\vartheta_T^{i*}(\omega)$ to (13) is attained on $[E^i(\xi_0^{i*}) - \vartheta_0^{i*} - \gamma^i(\omega), \infty[$.

Proof of proposition (2)

Proof.

Hence

$$\{A_T^* = 0\} \subseteq \{E^i(\xi_0^{i*}) - \vartheta_0^{i*} - \gamma^i \leq \vartheta_T^{i*}\}$$

holds almost surely for each $i \in I$, which implies that

$$\{A_T^* = 0\} \subseteq \{\sum_{i \in I} E^i(\xi_0^{i*}) - \gamma \leq \sum_{i \in I} \vartheta_T^{i*}\}$$

holds almost surely. Then, because of the equilibrium property (4), we obtain an almost sure inclusion

$$\{A_T^* = 0\} \subseteq \{\sum_{i \in I} E^i(\xi_0^{i*}) - \gamma \leq 0\}$$

Proof of proposition (2)

Proof.

Now, using the dichotomy (14) and, since the probability of the exact coincidence on the right-hand side of (15) is zero, we conclude for the complementary event that

$$\{A_T^* = \pi\} \supseteq \{\sum_{i \in I} E^i(\xi_0^{i*}) - \gamma \geq 0\} \tag{16}$$

holds almost surely. Let us show the opposite inclusion. If $A_T^*(\omega) = \pi$, then a maximizer ϑ_T^{i*} to (13) is attained on $]-\infty, E^i(\xi_0^{i*}) - \vartheta_0^{i*} - \gamma^i(\omega)]$. Hence

$$\{A_T^* = \pi\} \subseteq \{E^i(\xi_0^{i*}) - \vartheta_0^{i*} - \gamma^i \geq \vartheta_T^{i*}\}$$

holds almost surely for each $i \in I$, which implies that

$$\{A_T^* = \pi\} \subseteq \{\sum_{i \in I} E^i(\xi_0^{i*}) - \gamma \geq \sum_{i \in I} \vartheta_T^{i*}\} \text{ holds almost surely.}$$

Proof of proposition (2)

Proof.

Now, because of the equilibrium property (4), we obtain

$$\{A_T^* = \pi\} \subseteq \{\sum_{i \in I} E^i(\xi_0^{i*}) - \gamma \geq 0\}. \tag{17}$$

Finally, combine the inclusions (16) and (17) to obtain the assertion (10). □

Social optimality

- Cost efficiency is of course a fundamental property of economic systems. However **reduction of environmental damage** and the **fair distribution of costs** among the different sectors of a society are also relevant aspects to the purpose of social and environmental sustainability
- Here we combine social costs of production, C, and interpret $\pi(E(\xi_0)-\gamma)^+$ as a proxy of the environmental damage associated to the production plan ξ_0
- Let us agree that

$$B(\xi_0) = C(\xi_0) + \pi(E(\xi_0)-\gamma)^+$$

expresses the **social burden** caused by the overall production plan $\xi_0 \in \times_{i \in I} \Xi^i$

Social optimality

It turns out that the equilibrium strategy ξ_0^* also minimizes the social burden, among all the production strategies covering a given demand.

Proposition (3)

Given energy demand D_0, let $(A^*, P^*) = (A_0^*, A_T^*, P_0^*)$ be the equilibrium prices with the corresponding strategies $(\vartheta^{i*}, \xi^{i*})$, $i \in I$. Let $\mathbb{Q}^*$ be a risk neutral measure whose existence is shown in Proposition (2). Then

$$E^{\mathbb{Q}^*}(B(\xi_0^*)) \leq E^{\mathbb{Q}^*}(B(\xi_0)) \tag{18}$$

holds for each overall production schedule $\xi_0 = (\xi_0^i)_{i \in I} \in \times_{i \in I} \Xi^i$ which yields at least the same production volume $V(\xi_0) \geq V(\xi_0^*)$.

- Notice that social optimality results with respect to a **risk neutral measure**.

Proof of proposition (3)

Proof.

For each convex function f holds $f(x) + \nabla f(x) \cdot h \leq f(x+h)$, where $\nabla f(x)$ stands for one of the sub-gradients of f at the point x. In particular, for convex function $f : \mathbb{R} \to \mathbb{R}_+$, $x \mapsto x^+$, we obtain $x^+ + 1_{\{x \geq 0\}} h \leq (x+h)^+$ for all $x, h \in \mathbb{R}$. With the equilibrium overall production strategy $\xi_0^* = (\xi_0^{i*})_{i \in I}$, we conclude that

$$(E(\xi_0^*) - \gamma)^+ + 1_{\{E(\xi_0^*) - \gamma \geq 0\}} (E(\xi_0) - E(\xi_0^*)) \leq (E(\xi_0) - \gamma)^+$$

holds almost surely for any overall production strategy $\xi_0 \in \times_{i \in I} \Xi^i$. Calculating expectations with respect to $\mathbb{Q}^*$ on both sides, we obtain

$$\mathbb{E}^{\mathbb{Q}^*} \left((E(\xi_0^*) - \gamma)^+ \right) + \mathbb{E}^{\mathbb{Q}^*} \left(1_{\{E(\xi_0^*) - \gamma \geq 0\}} \right) (E(\xi_0) - E(\xi_0^*)) \leq$$
$$\leq \mathbb{E}^{\mathbb{Q}^*} \left((E(\xi_0) - \gamma)^+ \right).$$

Proof of proposition (3)

Proof.

Using martingale property and digital terminal value of the equilibrium allowance prices, shown in Proposition (2), we finally obtain

$$\pi \mathbb{E}^{\mathbb{Q}^*}\left((E(\xi_0^*) - \gamma)^+\right) + A_0^*(E(\xi_0) - E(\xi_0^*)) \le \pi \mathbb{E}^{\mathbb{Q}^*}\left((E(\xi_0) - \gamma)^+\right). \tag{19}$$

Now, from Proposition (1) we got that, for every equilibrium strategy such that $V(\xi_0) \ge V(\xi_0^*)$,

$$C(\xi_0^*) - C(\xi_0) \le A_0^*(E(\xi_0) - E(\xi_0^*)).$$

Combining it with (19), gives

$$C(\xi_0^*) + \pi \mathbb{E}^{\mathbb{Q}^*}\left((E(\xi_0^*) - \gamma)^+\right) \le C(\xi_0) + \pi \mathbb{E}^{\mathbb{Q}^*}\left((E(\xi_0) - \gamma)^+\right),$$

which proves our claim (18). □

Conclusions

- We have shown that **opportunity costs must be priced into the final product price**. This is the issue which lets clean technologies to appear cheaper than dirty technologies, and which triggers the overall shift towards a cleaner production
- We have shown that, under cap-and-trade emission systems, the **same plans which are optimal for risk averse** agents to reduce production costs are **also optimal to minimize the social burden under a risk neutral** probability

Conclusions

- As for the **fair distribution of the social burden** over the different components of the society, our model shows at which point the cap-and-trade system becomes vulnerable. Namely, if the **merit order does not provide a sufficient flexibility** (e.g. there are only few technologies and/or the certificate price is not able to change the merit order), then the consumers just pay the opportunity costs while **no emission savings is gained**
- An improved merit order flexibility can result from combining emission trading with **emission taxation** and by **subsidizing** clean production technologies
- The (risk neutral) **social optimality interpretation** of the market equilibrium allows using the machinery of stochastic optimal control for construction and more detailed quantitative analysis of equilibrium-like market situations.

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