UNIVERSITÄT DUISBURG ESSEN

A Structural Model for Coupled Electricity Markets

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Michael M. Kustermann | Chair for Energy Trading and Finance | University of Duisburg-Essen

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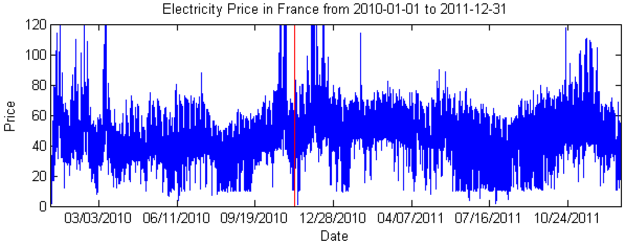
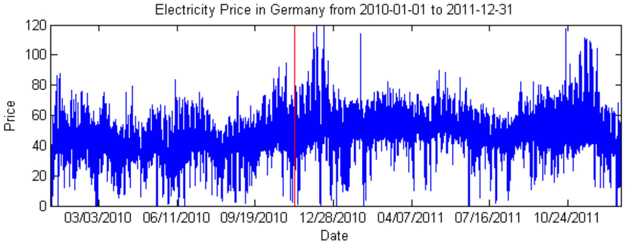
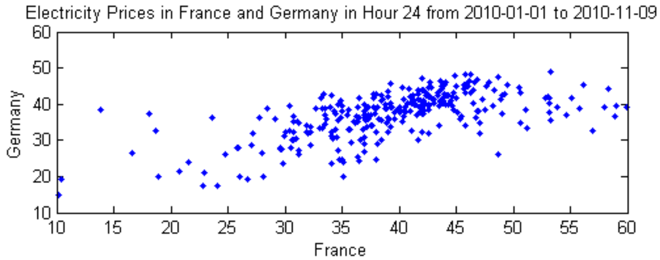
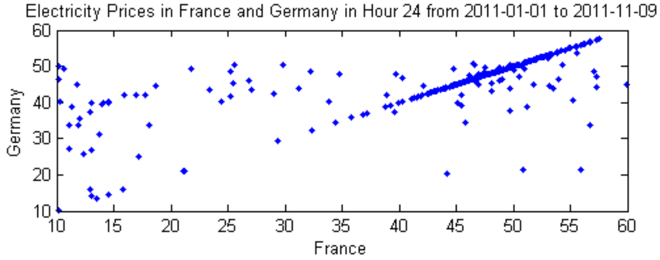
German and French Power Prices from Jan 2010 to Dec 2011





German vs French Power Prices - 2010 and 2011

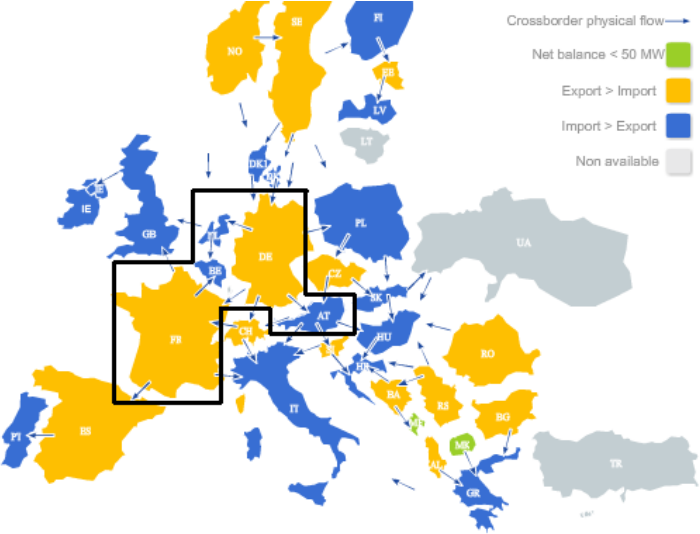




Definitions

- A Market Area is a set of nodes and edges in an electric network, for which a unique energy price is calculated ('spot' i.e. day-ahead).
- Two market areas A and B are interconnected, if there exists an edge, which connects a node in A with a node in B.
- An edge which connects two market areas is called interconnector.
- The sum over the available capacities of all interconnectors between A and B is called available (cross boarder) transmission capacity.
- 'Market coupling uses implicit auctions in which players do not actually receive allocations of cross-border capacity themselves but bid for energy on their exchange. The exchanges then use the available cross-border transmission capacity to minimize the price difference between two or more areas.' (EPEX SPOT)

CWE Region


Crossborder physical flow
Net balance < 50 MW
Export > Import
Import > Export
Non available
NO
SE
FI
LV
LT
DK1
IE
GB
NL
BE
DE
PL
UA
CZ
SK
AT
HU
FR
CH
SI
RO
HR
BA
RS
IT
BG
ES
PT
MK
AL
GR
TR

Requirements for a Structural Model for Coupled Markets

We focus on the two market case. A structural model for coupled markets should

- be simple.
- be tractable.
- lead to a closed form formula for the cdf of spot prices.
- lead to closed form formulae for futures prices.

Economic Assumptions

Starting point for our model is the following structure of a hybrid model

- price independent demand
- market supply curve has exponential shape
- fuels prices shift market supply curve multiplicatively
- market clearing price is given as intersection of supply and demand

Model for Demand and Fuel

We assume Demand in Country $i \in \{1, 2\}$ to be given by

$$
\begin{aligned}
D_t^i &= f_t^i + \tilde{D}_t^i \\
d\tilde{D}_t^i &= -k^i \tilde{D}_t^i dt + \sigma_i dW_t^i \\
dW_t^i dW_t^j &= \rho_{i,j} dt
\end{aligned}
$$

where

$$
\begin{aligned}
f_t^i = \beta_1^i &+ \beta_2^i \cos(2\pi \frac{t}{24} + \beta_3^i) \\
&+ \beta_4^i \cos(2\pi \frac{t}{168} + \beta_5^i) + \beta_6^i \cos(2\pi \frac{t}{8760} + \beta_7^i)
\end{aligned}
$$

denotes the deterministic seasonal component.

Model for Demand and Fuel II

Moreover, we assume that only one fuel might be marginal and is given by

$$
\begin{aligned}
d\ln(S_t) &= k^S(\theta^S - \ln(S_t))dt + \sigma_S dW_t^S \\
dW_t^S dW_t^1 &= \rho_{S,1} \\
dW_t^S dW_t^2 &= \rho_{S,2}.
\end{aligned}
$$

It follows

$$
\begin{pmatrix} D_t^1 \\ D_t^2 \\ \ln(S_t) \end{pmatrix} |_{\mathcal{F}_s} \sim N\left(\mu(s,t), \Sigma(s,t)\right)
$$

The parameters $\mu(s,t)$ and $\Sigma(s,t)$ are explicitely given in terms of the parameters of the SDEs.

Model for the Market Supply Curve

We assume the Market Supply Curve in Country $i \in \{1, 2\}$, C^i, to be given as a function of demand D and fuels price S:

$$C^i(D, S) = Se^{a_i + b_i D} + c.$$

I.e. we assume

- constant production capacities
- production costs consist of fuels cost and fuel price independent costs (labour costs,...).
- exponential dependence of the market clearing price on demand.

Cross Border physical Flows

We denote the physical flow from country 2 to country 1 by E_t. The maximum capacity is restricted and depends on the direction of the flow:

$$E_t \in [E_{min}, E_{max}], \ E_{min} \leq 0, \ E_{max} \geq 0.$$

Note that, if

- $E_{min} = E_{max} = 0$, markets are not connected and thus, pricing might be done independently.
- $E_{max} = -E_{min} \to \infty$, the interconnector is never congested and thus, one unique market price for both markets exists at all hours.

Cross Border physical Flows in case of coupled markets

In interconnected markets, only the electricity which is not imported has to be produced. Thus, the electricity price is determined as

$$P_t^1(D_t^1, E_t, S_t) = C^1(D_t^1 - E_t, S_t) = S_t e^{a_1 + b_1(D_t^1 - E_t)} + c.$$

Here, E_t is the imported amount and $D_t^1 - E_t$ is the residual demand which has to be satisfied by local production. Define:

$$\begin{aligned}
A_1 &= \{\omega \in \Omega : P_t^1(D_t^1, E_{max}, S_t) \geq P_t^2(D_t^2, -E_{max}, S_t)\} \\
A_2 &= \{\omega \in \Omega : P_t^1(D_t^1, E_{min}, S_t) \leq P_t^2(D_t^2, -E_{min}, S_t)\} \\
A_3 &= \Omega \setminus (A_1 \cup A_2)
\end{aligned}$$

Then, the cross border flow in case of coupled markets is

$$E_t(\omega) = \begin{cases} E_{max} & \text{, if } \omega \in A_1 \\ E_{min} & \text{, if } \omega \in A_2 \\ \frac{a_1 - a_2}{b_1 + b_2} + \frac{b_1}{b_1 + b_2} D_t^1(\omega) - \frac{b_2}{b_1 + b_2} D_t^2(\omega) & \text{, if } \omega \in A_3 \end{cases}$$

Market Clearing Prices

Given the cross border physical flow which minimizes price differences between countries, the resulting electricity price for country 1 may be stated as:

$$P_t^1(\omega) = P_t^1(D_t^1, E_t, S_t) = \begin{cases} C^1(D_t^1(\omega) - E_{max}, S_t(\omega)) & \text{, if } \omega \in A_1 \\ C^1(D_t^1(\omega) - E_{min}, S_t(\omega)) & \text{, if } \omega \in A_2 \\ C^m(D_t^1(\omega) + D_t^2(\omega), S_t(\omega)) & \text{, if } \omega \in A_3 \end{cases}$$

The function C^m can be viewed as the aggregated market supply curve for both countries and is given by

$$C^m(D, S) = Se^{a_m + b_m D} + c$$

with $a_m = \frac{a_1 b_2 + a_2 b_1}{b_1 + b_2}$ and $b_m = \frac{b_1 b_2}{b_1 + b_2}$. Equivalent results hold for P_t^2 in country 2.

Distribution of the market clearing prices - limiting cases

Define the generalized lognormal distribution $logN(\mu, \sigma^2, c)$ as the distribution with density

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma(x-c)} e^{-\frac{1}{2}\left(\frac{\ln(x-c)-\mu}{\sigma}\right)^2}, \forall x \in (c, \infty).$$

Then it obviously holds that

$$P_t^i|_{\mathcal{F}_s} \xrightarrow{d} logN\left(a_i + \overline{b_i}^T \mu, \overline{b_i}^T \Sigma \overline{b_i}, c\right) \quad , \text{ if } \; E_{max} = -E_{min} \to 0^+.$$

Here, $\overline{b_i} = \begin{cases} (b_1, 0, 1)^T & \text{if } \; i = 1 \\ (0, b_2, 1)^T & \text{if } \; i = 2 \end{cases}$.

And

$$P_t^i|_{\mathcal{F}_s} \xrightarrow{d} logN(a_m + \overline{b}_m \mu, \overline{b}_m^T \Sigma \overline{b}_m, c) \;, \text{ if } \; E_{max} = -E_{min} \to \infty.$$

Here, $\overline{b}_m = (b_m, b_m, 1)^T$.

Distribution of the market clearing prices

Defining $P_t = \left(P_t^1, P_t^2\right)^T$ we find the distribution function:

$$F_{P_t|\mathcal{F}_s}(x) = \mathbb{Q}(P_t \leq x|\mathcal{F}_s) =$$
$$\mathbb{Q}\left(\{P_t \leq x\} \cap A_1|\mathcal{F}_s\right) + \mathbb{Q}\left(\{P_t \leq x\} \cap A_2|\mathcal{F}_s\right) + \mathbb{Q}\left(\{P_t \leq x\} \cap A_3|\mathcal{F}_s\right).$$

We are able to calculate above Probabilities. It turns out

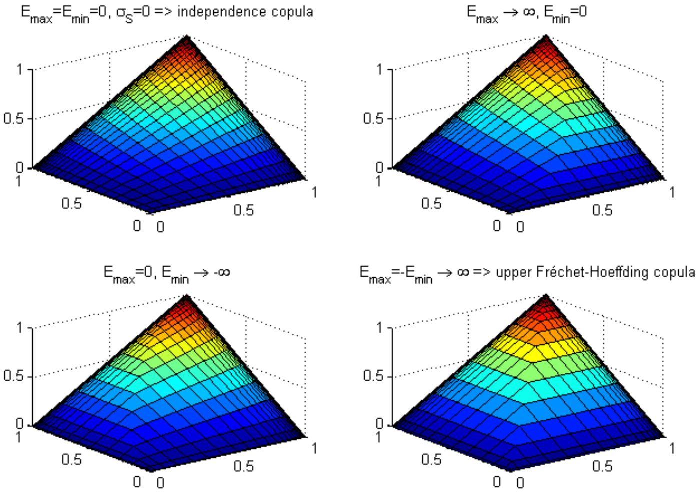
$$\mathbb{Q}(\{P_t^1 \leq x_1\} \cap \{P_t^2 \leq x_2\} \cap A_1|\mathcal{F}_s) = \Phi_3\left(d(x); B\mu; B\Sigma B^T\right)$$

where $\Phi_3(y;\mu;\Sigma)$ denotes the cdf at y of the (degenerate) normal distribution with mean μ and covariance Σ. The parameters are

$$d(x) = \begin{pmatrix} \ln(x_1 - c) - a_1 + b_1 E_{max} \\ \ln(x_2 - c) - a_2 - b_2 E_{max} \\ a_1 - a_2 - (b_1 + b_2) E_{max} \end{pmatrix}, \quad B = \begin{pmatrix} b_1 & 0 & 1 \\ 0 & b_2 & 1 \\ -b_1 & b_2 & 0 \end{pmatrix}.$$

Similar expressions can be found for the other 2 terms.

Example of the resulting Copulae



Futures prices in the structural model

We consider futures with hourly delivery. Denote by $F^i(s,t)$ the futures price of electricity in country i at time s for delivery in t. Under the risk-neutral measure we have

$$\begin{aligned} F^1(s,t) = \mathbb{E}_s^Q[P_t^1] &= \int_\Omega P_t^1(\omega)\mathbb{Q}(d\omega) \\ &= \int_{A_1} C^1\left(D_t^1(\omega) - E_{max}, S_t(\omega)\right)\mathbb{Q}(d\omega) \\ &+ \int_{A_2} C^1\left(D_t^1(\omega) - E_{min}, S_t(\omega)\right)\mathbb{Q}(d\omega) \\ &+ \int_{A_3} C^m\left(D_t^1(\omega) + D_t^2(\omega), S_t(\omega)\right)\mathbb{Q}(d\omega) \end{aligned}$$

and equivalent for country 2.

Futures prices in the structural model II

Similar to the calculation for the cdf, we find

$$\int_{A_1} C^1\left(D_t^1(\omega) - E_{max}, S_t(\omega)\right)\mathbb{Q}(d\omega) =$$

$$c \cdot \Phi\left(\frac{a_1 - a_2 - (b_1 + b_2)E_{max} - \overline{b}_3^T \mu}{\sqrt{\overline{b}_3^T \Sigma \overline{b}_3}}\right) +$$

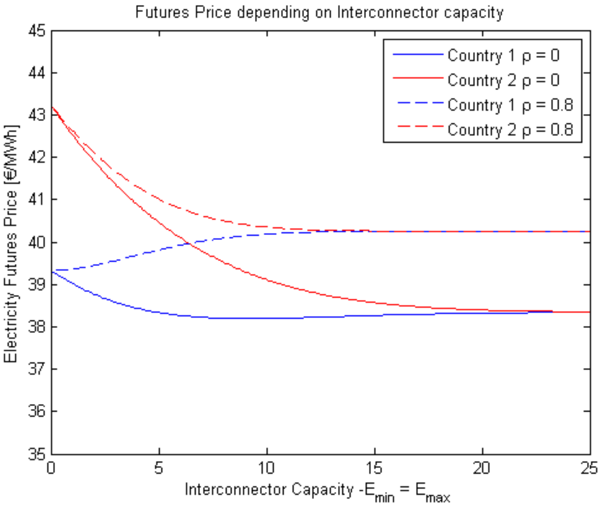
$$e^{a_1 - b_1 E_{max} + \overline{b}_1^T \mu + \frac{1}{2}\overline{b}_1^T \Sigma \overline{b}_1}\Phi\left(\frac{a_1 - a_2 - (b_1 + b_2)E_{max} - \overline{b}_3^T \mu - \overline{b}_3^T \Sigma \overline{b}_3}{\sqrt{\overline{b}_3^T \Sigma \overline{b}_3}}\right)$$

where $\overline{b}_3 = (-b_1, b_2, 0)^T$.

Futures prices for futures with delivery in a set $\mathbb{T}$ of hours are then given by

$$F^i(s, \mathbb{T}) = \frac{1}{|\mathbb{T}|}\sum_{t \in \mathbb{T}} F^i(s, t).$$

Example of futures prices



Plain Vanilla Calls

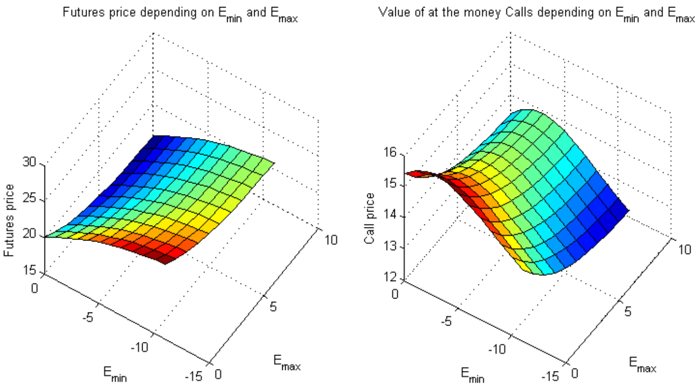
Define the following sets

$$
\begin{aligned}
B_{max} &= \{\omega \in \Omega : C^1(D_t^1 - E_{max}, S_t) \geq K\} \\
B_{min} &= \{\omega \in \Omega : C^1(D_t^1 - E_{min}, S_t) \geq K\} \\
B_{mid} &= \{\omega \in \Omega : C^m(D_t^1 + D_t^2, S_t) \geq K\}.
\end{aligned}
$$

Then, the price of a Call with Strike K is given by

$$
\begin{aligned}
\mathbb{E}_s^{\mathbb{Q}}\left[\left(P_t^1 - K\right)^+\right] = &\int_{A_1 \cap B_{max}} C^1(D_t^1 - E_{max}, S_t) d\mathbb{Q} \\
&+ \int_{A_2 \cap B_{min}} C^1(D_t^1 - E_{min}, S_t) d\mathbb{Q} + \int_{A_3 \cap B_{mid}} C^m(D_t^1 + D_t^2, S_t) d\mathbb{Q} \\
&- K\left(\mathbb{Q}\left(A_1 \cap B_{max}\right) + \mathbb{Q}\left(A_2 \cap B_{min}\right) + \mathbb{Q}\left(A_3 \cap B_{mid}\right)\right)
\end{aligned}
$$

Example of a Plain Vanilla Call



Export: The price of an at the money option increases with increasing futures price, but increasing export capacities further reduces variablility of demand and thus the call loses value.

Thank you for your attention...

References

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Appendix

Price Convergence FR - GER (hourly basis)

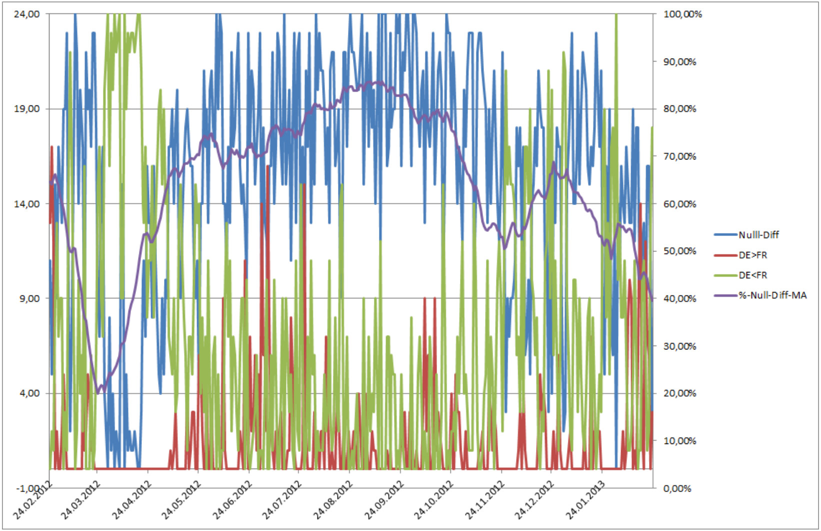

Figure: Price Convergence between France and Germany