Pricing and Hedging Energy Quanto Options

Nina Lange [1]
Based on joint work with F.E. Benth[2] & T.Å. Myklebust[3]

[1]Department of Finance, Copenhagen Business School

[2]Department of Mathematics, University of Oslo

[3]Norwegian School of Economics / Norwegian Cancer Registry

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Hedging volume risk by weather?

- Volume risk is embedded in many contracts for delivery of energy, for instance electricity or gas
- Volume risk is a big concern for energy companies, especially since prices and volume tend to be correlated
- Volume and weather is somewhat correlated
- Weather derivatives have served as tools to hedge volume
- But weather derivatives are highly illiquid
- The recent decrease in traded volume (from 930.000 in 2007 to under 500.000 in 2009) is attributed to co-called "Energy quantos"

What are energy quantos

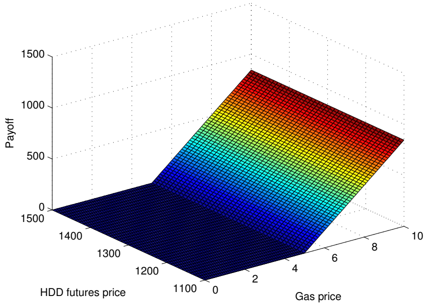
- A bit different from currency quantos as the price is not denominated in something else - it more has the structure of a "double option", e.g.,

$$\max(S_1 - K_1, 0) \times \max(S_2 - K_2, 0)$$

- Energy Quantos provide tailormade risk management products
- OTC trading is not that much of a compromise, since many weather derivatives are illiquid
- Counterparts can be insurance companies, reinsurance companies, other energy companies

Change in payoff compared to standard options

Univariate option vs. quanto option



Example of contract

Generally, Energy Quantos are tailormade. For instance:

	Nov	Dec	Jan	Feb	Mar
(a) High Strike (HDDs)	$\overline{K}_I^{11}$	$\overline{K}_I^{12}$	$\overline{K}_I^{1}$	$\overline{K}_I^{2}$	$\overline{K}_I^{3}$
(b) Low Strike (HDDs)	$\underline{K}_I^{11}$	$\underline{K}_I^{12}$	$\underline{K}_I^{1}$	$\underline{K}_I^{2}$	$\underline{K}_I^{3}$
(a) High Strike (Price/mmBtu)	$\overline{K}_E^{11}$	$\overline{K}_E^{12}$	$\overline{K}_E^{1}$	$\overline{K}_E^{2}$	$\overline{K}_E^{3}$
(b) Low Strike (Price/mmBtu)	$\underline{K}_E^{11}$	$\underline{K}_E^{12}$	$\underline{K}_E^{1}$	$\underline{K}_E^{2}$	$\underline{K}_E^{3}$
Volume (mmBtu)	200	300	500	400	250

Table : As an example the payoff for November will be: (a) In cold periods - $\max(H - \overline{K}_I, 0) \times \max(E - \overline{K}_E, 0)\times$ Volume. (b) In warm periods - $\max(\underline{K}_I - H, 0) \times \max(\underline{K}_E - E, 0)\times$ Volume. We see that the option pays out if both the underlying temperature and price variables exceed (dip below) the high strikes (low strikes).

Scarce litterature on Energy Quantos

- Paper by Caporin, Pres & Torró (Energy Economics 2012)
  - A bivariate time series model to capture the joint dynamics of energy prices and temperature.
  - Parameter-intensive econometric model
  - MC simulation to calculate prices.
  - Hedging goes unanswered
- Several papers showing the link between energy prices and temperature
- Several papers on seasonality, modelling of energy prices etc.

Contribution of Benth, Lange & Myklebust

- Both energy and weather futures are traded with delivery periods
- We convert the pricing problem by using futures contracts as underlying assets, rather than energy spot prices and temperature
- Typically, energy quanto options have a payoff which can be represented as an "Asian" structure on the energy spot price and the temperature index
- Hence, any "Asian payoff" on the spot and temperature for a quanto option can be viewed as a "European payoff" on the corresponding futures contracts
- Derive analytical solutions to the option pricing problem (in a log normal framework) as well as hedging strategies in terms of the traded underlying futures contracts
- Estimate our model using NYMEX futures data for gas and HDD

From Asian to European

- A futures contract $F^E(\tau_2, \tau_2)$ promises to deliver gas at a constant rate over the delievery period $[\tau_1, \tau_2]$.
- Defining an option on the average $\sum_{u=\tau_1}^{\tau_2} S_u$ therefore approximately corresponds to writing the option on the futures constract.
- The same argument holds for the temperature index, where we specifically use futures on Heating Degree Days (how much the temperature is below 18 degrees C)
- Specifically, we study a "double futures option"-structure:

$$\hat{p} = \max\left(F^E_{\tau_2}(\tau_1, \tau_2) - \overline{K}_E, 0\right) \times \max\left(F^I_{\tau_2}(\tau_1, \tau_2) - \overline{K}_I, 0\right)$$

General setup

We assume two futures prices are log-normal under the pricing measure $\mathbb{Q}$

$$F_T^E(\tau_1, \tau_2) = F_t^E(\tau_1, \tau_2) \exp(\mu_E + X) \tag{1}$$

$$F_T^I(\tau_1, \tau_2) = F_t^I(\tau_1, \tau_2) \exp(\mu_I + Y) \tag{2}$$

where

- (X, Y) is a bivariate normal random variable with mean zero, with covariance structure depending on t, T and $[\tau_1, \tau_2]$.
- $\sigma_X^2 = Var(X)$, $\sigma_Y^2 = Var(Y)$ and $\rho_{X,Y} = corr(X, Y)$.
- The futures price naturally is a martingale under the pricing measure $\mathbb{Q}$, so $\mu_E = -\sigma_X^2/2$ and $\mu_I = -\sigma_Y^2/2$.
- This structure encompasses many models, e.g., geometric Brownian motion models and multi-factor spot models.

Option price

The time t market price of an European energy quanto option with exercise at time τ_2 and payoff described on the previous slides

$$C_t = e^{-r(\tau_2 - t)} \bigg(F_t^E F_t^I e^{\rho_{X,Y}\sigma_X\sigma_Y} M(y_1^{***}, y_2^{***}; \rho_{X,Y}) \\ - F_t^E \overline{K}_I M\left(y_1^{**}, y_2^{**}; \rho_{X,Y}\right) \\ - F_t^I \overline{K}_E M\left(y_1^{*}, y_2^{*}; \rho_{X,Y}\right) + \overline{K}_E \overline{K}_I M\left(y_1, y_2; \rho_{X,Y}\right) \bigg)$$

$$y_1 = \frac{\log\left(\frac{F_t^E}{\overline{K}_E}\right) - \frac{1}{2}\sigma_X^2}{\sigma_X}, \quad y_2 = \frac{\log\left(\frac{F_t^I}{\overline{K}_I}\right) - \frac{1}{2}\sigma_Y^2}{\sigma_Y},$$

$$y_1^* = y_1 + \rho_{X,Y}\sigma_Y\,, y_1^{**} = y_1 + \sigma_X\,, y_1^{***} = y_1 + \rho_{X,Y}\sigma_Y + \sigma_X\,,$$

$$y_2^* = y_2 + \sigma_Y\,, y_2^{**} = y_2 + \rho_{X,Y}\sigma_X\,, y_2^{***} = y_2 + \rho_{X,Y}\sigma_X + \sigma_Y\,.$$

Here $M(x, y; \rho)$ denotes the standard bivariate normal cumulative distribution function with correlation ρ.

Temperature data

Temperature contracts

- Futures contracts on Heating Degree Days are traded for several cities for the months October to April
- The contract value is 20$ for each HDD in the month. The underlying is one month of accumulated HDD's for a specific location.
- The futures price is denoted by $F_t^I(\tau_1, \tau_2)$ and settled on the index $\sum_{u=\tau_1}^{\tau_2} HDD_u$.
- Estimation is done using the first seven contracts where the index period hasn't startet yet.
- The chosen locations are New York and Chicago, since these are located in an area with fairly large gas consumption.

Gas data

Gas contracts

- Futures contracts for delivery of gas is traded on NYMEX for each month ten years out.
- The underlying is delivery of gas throughout a month and the price is per unit.
- Estimation is done using the first 12 contracts for delivery at least one month later.

Two-factor futures price dynamics

- Joint dynamics of the futures price processes $F_t^E(\tau_1, \tau_2)$ and $F_t^I(\tau_1, \tau_2)$ under $\mathbb{Q}$ is given by (with $\eta_i(t, [\tau_1, \tau_2])$ exponential):

$$\frac{dF_t^i(\tau_1, \tau_2)}{F_t^i(\tau_1, \tau_2)} = \sigma_i dW_t^i + \eta_i(t, [\tau_1, \tau_2]) dB_t^i \,,$$

  for $i = E, I$ and thus log-normal.
- Two futures dynamics of this are connected by allowing the Brownian motions to be correlated across assets:

$$\rho_E = corr(W_1^E, B_1^E), \qquad \rho_I = corr(W_1^I, B_1^I)\,,$$
$$\rho_W = corr(W_1^E, W_1^I), \qquad \rho_B = corr(B_1^E, B_1^I)\,.$$

- Apply the parametrization by Sørensen (2002)

Fit of the model (New York and gas)

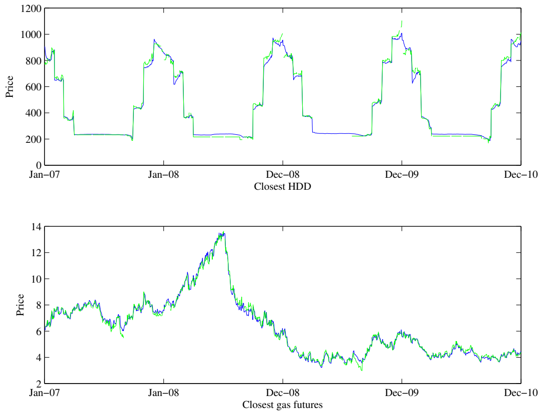
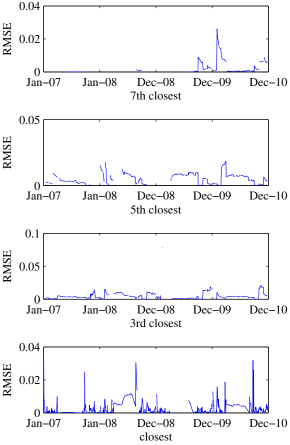
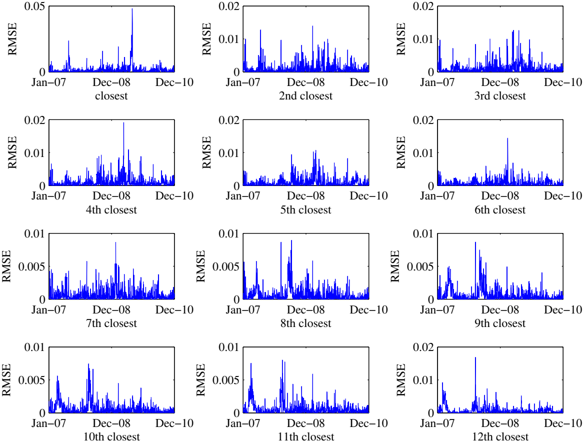


Figure : Model prices (blue) and observed prices (green) for the joint estimation of Natural Gas Futures and New York HDDs

RMSE for New York HDD futures



RMSE for gas



RMSE
0.05
0.02
0.01
0.005
0
Jan–07
Dec–08
Dec–10
closest
2nd closest
3rd closest
4th closest
5th closest
6th closest
7th closest
8th closest
9th closest
10th closest
11th closest
12th closest

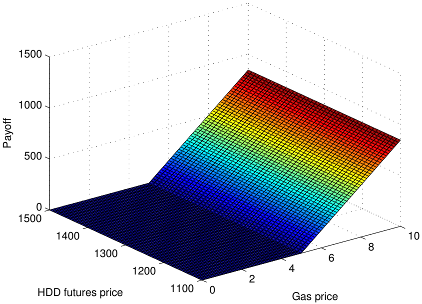
Example of derived option prices

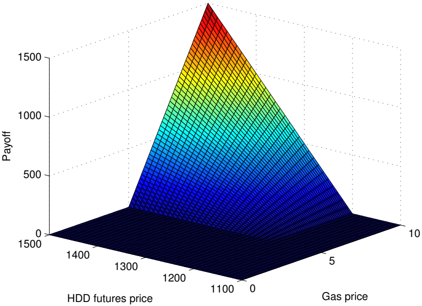
$\overline{K}_I \| \overline{K}_G$	3	4	5	6	7
1100	596 470	451 355	337 270	252 206	188 158
1150	443 325	338 246	254 187	191 142	143 110
1200	401 287	306 217	231 164	173 126	130 97
1250	362 252	277 191	210 145	158 111	118 85
1300	326 222	251 168	190 127	143 97	108 74

Table : Option prices for Chicago under the model (top) and under the assumption of no correlation (bottom). $r = 0.02$, τ_1=1-Dec-2011, τ_2=31-Dec-2011, t=31-Dec-2010

Change in payoff compared to pure gas options

Pure gas options vs. quanto option





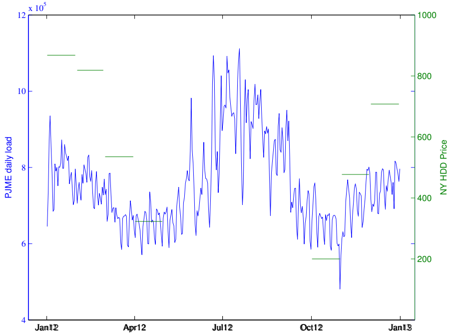
Work in progress

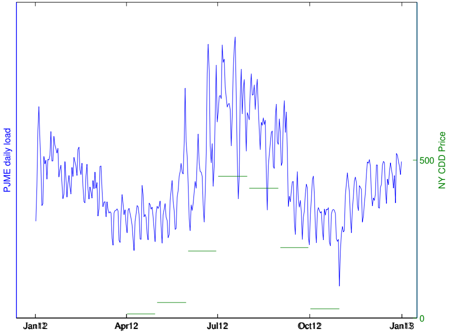
Building on Benth, Lange & Myklebust, we investigate how energy quantos hedge compared to “pure” hedges:

- The study is extended to electricity markets; PJM and Noodpool
- Includes both CDDs and HDDs to account for market characteristics
- To which extent is volume risk commoditized/weatherized
- How can the OTC contract be optimally structured given the available exchange traded contracts

PJM load and weather futures

Load and HDD – Load and CDD





Recap

- Quanto deals have decresed the volume of weather derivatives trading
- Quanto deals are good tools for tailoring the risk management of volume risk
- Derived closed form pricing and hedging formulas in log normal model
- A two asset two factor model was applied to NYMEX gas data
- Currently working on hedge performance and contract design

Comments

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