A pricing measure to explain risk premium in power markets

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joint work with Fred S. Benth

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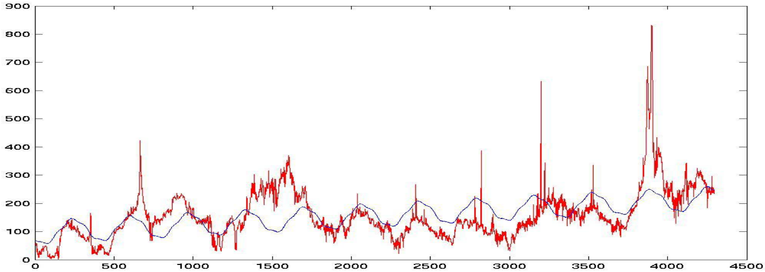
Stylized facts of the electricity spot prices

- Seasonal behaviour in yearly, weekly and daily cycles.
- Approximate stationary behaviour: Mean reversion.
- Non-Gaussianity and extreme spikes.
- Historical spot price at NordPool from the beginning in 1992 (NOK/MWh).



Factor models for the spot price

- We can consider two kind of models:
  - The arithmetic spot price model, defined by
$$S(t) = \Lambda_a(t) + X(t) + Y(t), \quad t \in [0, T^*].$$
  - The geometric spot price model, defined by
$$S(t) = \Lambda_g(t) \exp\left(X(t) + Y(t)\right), \quad t \in [0, T^*].$$
- $\Lambda_a(t)$ and $\Lambda_g(t)$ are assumed to be deterministic and they account for seasonalities in the prices.
- $X(t)$ has continuous paths and explains *normal variations*.
- $Y(t)$ has jumps and accounts for the *spikes*.
- $X(t)$ and $Y(t)$ are mean reverting stochastic processes.
- **Lucia and Schwartz** (2002), **Cartea and Figueroa** (2005) and **Benth et al.** (2008).

The (instantaneous) forward price

- In practice, electricity is a non-storable commodity.
- There is no buy and hold strategies $\Longrightarrow$ classical non-arbitrage arguments break down.
- **Incomplete market**: any probability measure Q equivalent to the historical measure P is valid.
- The forward price with time to delivery $0 < T < T^*$ at time $0 < t < T$ is given by

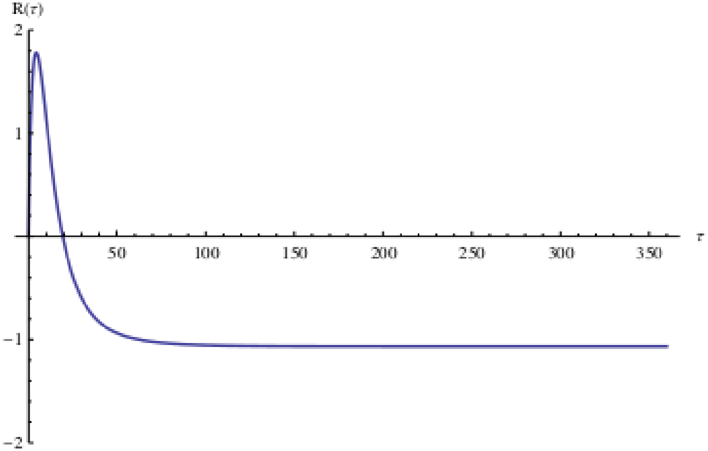
$$F_Q(t, T) = \mathbb{E}_Q[S(T)|\mathcal{F}_t]$$

  where $\mathcal{F}_t$ is the information in the market up to time t.
- The (theoretical) risk premium for forward prices is defined by the following expression

$$R_Q^F(t, T) \triangleq \mathbb{E}_Q[S(T)|\mathcal{F}_t] - \mathbb{E}_P[S(T)|\mathcal{F}_t].$$

Risk premium profile

- If $R_Q^F(t, T) > 0$, the market is in "contango".
- If $R_Q^F(t, T) < 0$, the market is in "normal backwardation".
- **Goal**: To reproduce the following risk premium profile.



Mathematical modeling of the factors

- Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, P)$ be a complete filtered probability space, where $T > 0$ is a fixed finite time horizon.
- Consider a standard Brownian motion W and a pure jump Lévy subordinator

$$L(t) = \int_0^t \int_0^\infty z N^L(ds, dz), t \in [0, T],$$

  where $N^L(ds, dz)$ is a Poisson random measure with Lévy measure ℓ satisfying $\int_0^\infty z\ell(dz) < \infty$.
- Let $\kappa_L(\theta) \triangleq \log \mathbb{E}_P[e^{\theta L(1)}]$ and

$$\Theta_L \triangleq \sup\{\theta \in \mathbb{R}_+ : \mathbb{E}[e^{\theta L(1)}] < \infty\}.$$

- A minimal assumption is that $\Theta_L > 0$.
- In the geometric spot model we also need $\Theta_L > 1$.

Mathematical modeling of the factors

- Consider the Ornstein-Uhlenbeck processes

$$X(t) = X(0) - \alpha_X \int_0^t X(s)ds + \sigma_X W(t),$$

$$Y(t) = Y(0) + \int_0^t (\kappa_L'(0) - \alpha_Y Y(s))ds + \int_0^t \int_0^\infty z\tilde{N}^L(ds, dz),$$

with $t \in [0, T], \alpha_X, \sigma_X, \alpha_Y > 0, X(0) \in \mathbb{R}, Y(0) \geq 0.$

- Using Itô formula one gets the following explicit representation

$$X(T) = X(t)e^{-\alpha_X(T-t)} + \sigma_X \int_t^T e^{-\alpha_X(T-s)} dW(s),$$

$$Y(T) = Y(t)e^{-\alpha_Y(T-t)} + \frac{\kappa_L'(0)}{\alpha_Y}(1 - e^{-\alpha_Y(T-t)}) + \int_t^T \int_0^\infty e^{-\alpha_Y(T-s)} z\tilde{N}^L(ds, dz),$$

where $0 \leq t \leq T$.

The change of measure

- For $t \in [0, T]$, consider the following family of Wiener and Poisson integrals

$$\tilde{G}_{\theta_1,\beta_1}(t) \triangleq \int_0^t \sigma_X^{-1} \left(\theta_1 + \alpha_X \beta_1 X(s)\right) dW(s),$$
$$\tilde{H}_{\theta_2,\beta_2}(t) \triangleq \int_0^t \int_0^\infty \left(e^{\theta_2 z} \left(1 + \frac{\alpha_Y \beta_2}{\kappa_L''(\theta_2)} z Y(s-)\right) - 1\right) \tilde{N}^L(ds, dz),$$

where $\bar{\beta} \in [0,1]^2, \bar{\theta} \in \bar{D}_L \triangleq \mathbb{R} \times D_L$ and $D_L \triangleq (-\infty, \Theta_L/2)$.

- The desired family of parametrised measure changes $Q_{\bar{\theta},\bar{\beta}}$ is given by the following Radon-Nykodim density

$$\left.\frac{dQ_{\bar{\theta},\bar{\beta}}}{dP}\right|_{\mathcal{F}_t} \triangleq \mathcal{E}(\tilde{G}_{\theta_1,\beta_1} + \tilde{H}_{\theta_2,\beta_2})(t) = \mathcal{E}(\tilde{H}_{\theta_2,\beta_2})(t)\mathcal{E}(\tilde{H}_{\theta_2,\beta_2})(t),$$

$t \in [0, T]$, where $\mathcal{E}(\cdot)$ denotes the stochastic exponential.

The change of measure

- Recall that, if M is a semimartingale, the stochastic exponential of M is given by

$$\mathcal{E}(M)(t) = \exp\left(M(t) - \frac{1}{2}[M^c, M^c](t)) \right) \times \exp\left(- \sum_{0 \leq s \leq t} \Delta M(s) - \log(1 + \Delta M(s) \right).$$

- If M is a local martingale, then $\mathcal{E}(M)$ is also a local martingale.
- If $\mathcal{E}(M)$ a positive local martingale, then $\mathcal{E}(M)$ is a supermartingale and $\mathbb{E}_P[\mathcal{E}(M)(t)] \leq 1, t \in [0, T]$.
- To have a well defined change of measure we need to ensure that $\mathbb{E}_P[\mathcal{E}(M)(T)] = 1$ and $\mathcal{E}(M)(t) > 0, t \in [0, T]$.
- Classical sufficient criteria do not provide sharp condidions.

The change of measure

Sketch of the proof that $\mathcal{E}(M)$ is a martingale with $M = \tilde{G}_{\theta_1,\beta_1}$ or $\tilde{H}_{\theta_2,\beta_2}$:

- Localise $\mathcal{E}(M)(t)$ using a reducing sequence $\{\tau_n\}_{n\geq 1}$.
- For any $n \geq 1$, $\{\mathcal{E}(M)(t)^{\tau_n}\}_{t\in[0,T]}$ is a true martingale and induces a change of measure .
- Test the uniform integrability of $\{\mathcal{E}(M)(T)^{\tau_n}\}_{n\geq 1}$ with $G(x) = x\log(x)$, i.e.

$$\sup_n \mathbb{E}_P[G(\mathcal{E}(M)(T)^{\tau_n})] < \infty.$$

- But this can be rewritten as

$$\sup_n \mathbb{E}_{Q^n}[\log(\mathcal{E}(M)(T)^{\tau_n})] < \infty.$$

- We can get rid off the ordinary exponential in $\mathcal{E}(M)(T)^{\tau_n}$.
- The problem is reduced to find a uniform bound for the second moment of X and Y under Q^n.

The dynamics under the new pricing measure

- Using a general version of Girsanov's theorem we get the dynamics for X and Y.
- The $Q_{\bar{\theta},\bar{\beta}}$-compensator measure of Y is given by

$$v^Y_{Q_{\bar{\theta},\bar{\beta}}}(dt, dz) = e^{\theta_2 z}\left(1 + \frac{\alpha_Y \beta_2}{\kappa_L''(\theta_2)} zY(t-)\right)\ell(dz)dt.$$

- Using Itô's formula again we get

$$X(T) = X(t)e^{-\alpha_X(1-\beta_1)(T-t)} + \frac{\theta_1}{\alpha_X(1-\beta_1)}(1 - e^{-\alpha_X(1-\beta_1)(T-t)})$$
$$+ \sigma_X \int_t^T e^{-\alpha_X(1-\beta_1)(T-s)} dW_{Q_{\bar{\theta},\bar{\beta}}}(s),$$
$$Y(T) = Y(t)e^{-\alpha_Y(1-\beta_2)(T-t)} + \frac{\kappa_L'(\theta_2)}{\alpha_Y(1-\beta_2)}(1 - e^{-\alpha_Y(1-\beta_2)(T-t)})$$
$$+ \int_t^T \int_0^\infty e^{-\alpha_Y(1-\beta_2)(T-s)} z\tilde{N}^L_{Q_{\bar{\theta},\bar{\beta}}}(ds, dz),$$

where $0 \leq t \leq T$.

Forward price formula for arithmetic spot model

- Recall that $S(t) = \Lambda_a(t) + X(t) + Y(t), \quad t \in [0, T^*]$.

Theorem

The forward price $F_Q(t, T)$ in the arithmetic spot model is given by

$$F_{Q_{\bar{\theta},\bar{\beta}}}(t, T) = \Lambda_a(T) + X(t)e^{-\alpha_X(1-\beta_1)(T-t)} + Y(t)e^{-\alpha_Y(1-\beta_2)(T-t)} + \frac{\theta_1}{\alpha_X(1-\beta_1)}(1 - e^{-\alpha_X(1-\beta_1)(T-t)}) + \frac{\kappa_L'(\theta_2)}{\alpha_Y(1-\beta_2)}(1 - e^{-\alpha_Y(1-\beta_2)(T-t)}).$$

- This pricing formula allows to model the spot price with stationary factors and obtain non-deterministic forward prices on the long end of the forward curve.

Risk premium formula for arithmetic spot model

Theorem

The risk premium for the forward price in the arithmetic spot model is given by

$$R^F_{a,Q_{\bar{\theta},\bar{\beta}}}(t,T) = X(t)e^{-\alpha_X(T-t)}(e^{\alpha_X\beta_1(T-t)} - 1)$$
$$+ Y(t)e^{-\alpha_Y(T-t)}(e^{\alpha_Y\beta_2(T-t)} - 1)$$
$$+ \frac{\theta_1}{\alpha_X(1-\beta_1)}(1 - e^{-\alpha_X(1-\beta_1)(T-t)})$$
$$+ \frac{\kappa_L'(\theta_2)}{\alpha_Y(1-\beta_2)}(1 - e^{-\alpha_Y(1-\beta_2)(T-t)}) - \frac{\kappa_L'(0)}{\alpha_Y}(1 - e^{-\alpha_Y(T-t)}).$$

- Analysis of possible risk profiles in **Benth and O.-L.** (2013).

Geometric spot model

- The problem is reduced to compute $\mathbb{E}_{Q_{\bar{\theta},\bar{\beta}}}[\exp(Y(T))|\mathcal{F}_t]$.
- Fortunately, $Y(t)$ is an affine process!!!.
- We have that

$$\mathbb{E}_{Q_{\bar{\theta},\bar{\beta}}}[\exp(Y(T))|\mathcal{F}_t] = \exp\left(Y(t)\Psi^1_{\theta_2,\beta_2}(T-t) + \Psi^0_{\theta_2,\beta_2}(T-t)\right),$$

  $t \in [0,T]$, where the pair of functions $\Psi^1_{\theta_2,\beta_2}$ and $\Psi^0_{\theta_2,\beta_2}$ is the solution of the so called generalised Riccati equation. See **Kallsen and Muhle-Karbe** (2010).
- Nonlinear ODE depending on the Lévy measure and the parameters.
- We classify the possible behaviour of the solutions in terms of the parameters, as we are only interested in global solutions.
- The expression for the forward price is obtained and a theoretical analysis of the possible risk profiles is performed in **Benth and O.-L.** (2013).

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