Spatial Risk Premium on Weather and Hedging Weather Exposure in Electricity

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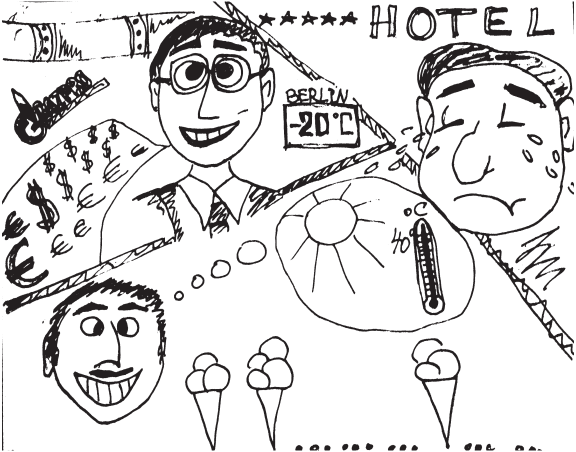
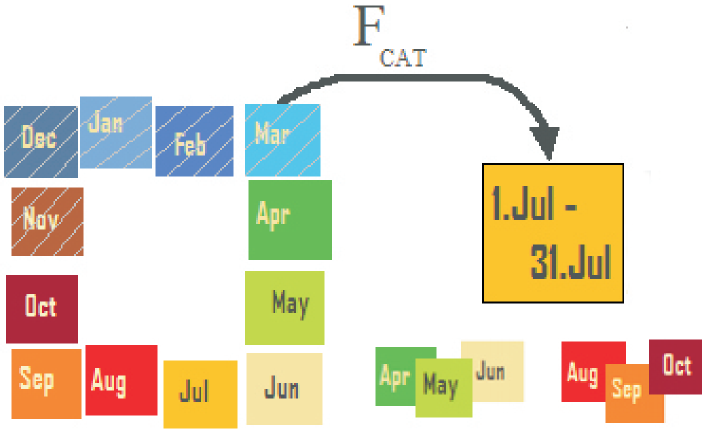


"Everybody talks about the weather but nobody does anything about it." Mark Twain


F_{CAT}
Dec
Jan
Feb
Mar
Nov
Apr
Oct
May
Sep
Aug
Jul
Jun
1.Jul -
31.Jul
Apr
May
Jun
Aug
Sep
Oct

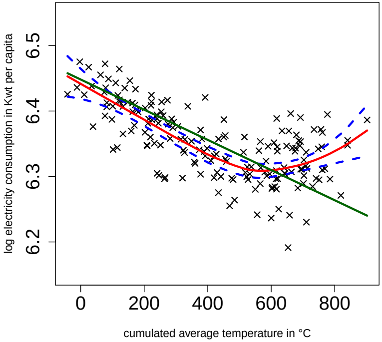


Figure 1: Estimated dependency of log electricity consumption in Germany on cumulative average temperature: 19960101-20100930, effects of prices and income removed (from Akdeniz Duran et al., 2011).

Future Contracts on Temperature

- transfer weather related risks,
- number of agents limited,
- weather non-tradable, insurance nature,
- geographically separated markets, spatial relationship.

Figure 2: Contracts on 11 cities in Europe are traded on the CME.

Pricing Model

- Econometrics of temperature (Benth et al., 2007)
  - seasonal function

$$\Lambda(t) = a + bt + \sum_{j=1}^{k} \left\{ c_{2j-1} \sin\left(\frac{2j\pi t}{365}\right) + c_{2j} \cos\left(\frac{2j\pi t}{365}\right) \right\}, \tag{1}$$

  - p-dimensional Ornstein-Uhlenbeck process $\mathbf{X}(t)$:

$$d\mathbf{X}(t) = A\mathbf{X}(t)dt + \mathbf{e}_p \sigma(t) dB(t),$$

  - $B(t)$ is Brownian motion, $\mathbf{e}_p$ pth column of I_p, $\sigma(t)$ seasonal variation.

Pricing Model

Price of a future on Cumulative Average Temperature index (CAT): (Benth et al., 2007)

$$
\begin{aligned}
F_{CAT(t,\tau_1,\tau_2)} = & \int_{\tau_1}^{\tau_2} \Lambda(u)du + \mathbf{a}(t)\mathbf{X}(t) \\
& + \int_{t}^{\tau_1} \theta(u)\sigma(u)\mathbf{a}(u)\mathbf{e}_p du \\
& + \int_{\tau_1}^{\tau_2} \theta(u)\sigma(u)\mathbf{e}_1^\top A^{-1} \left[\exp\left\{A(\tau_2 - u)\right\} - I_p\right] \mathbf{e}_p du.
\end{aligned}
\tag{2}
$$

with $\mathbf{a}(t) = \mathbf{e}_1^\top A^{-1} \left[\exp\left\{A(\tau_2 - t)\right\} - \exp\left\{A(\tau_1 - t)\right\}\right]$

- Market price of risk (MPR) $\theta(t)$ unknown for specific locations.

Pricing Temperature around the Globe

- $\theta(t)$ varies across locations.
- Connect $\theta(t)$ to a known risk factor!
- Spatial model to interpolate for other locations.

How can FPCA help pricing an arbitrary location?

Outline

1. Motivation ✓
2. Spatial Model for Risk Premium
   - Functional Principal Components (FPCA)
   - Geographically Weighted Regression (GWR)
3. Empirical Risk Premia and Hedging Weather Exposure
4. Outlook

Risk Premium

Given (2) the RP at $t = \tau_1$ for ith location and jth contract (RP_{ij}):

$$\begin{aligned} RP_{ij}(t = \tau_1, \tau_2) &= F_{CAT,ij}(\theta^i, t = \tau_1, \tau_2) - \widehat{F}_{CAT,ij}(0, t = \tau_1, \tau_2) + \varepsilon_{ij}, \\ &= \int_{\tau_1}^{\tau_2} \theta^i(u) \sigma_{ij}(u) \mathbf{e}_1^\top A_i^{-1} \\ &\quad \times \left[\exp\left\{A_i(\tau_2 - u)\right\} - I_p\right] \mathbf{e}_p du + \varepsilon_{ij}. \end{aligned}$$

$\widehat{F}_{CAT,ij}(0, t = \tau_1, \tau_2)$ estimated price for jth contract in ith location, zero MPR, A_i is the matrix of O-U-Process coefficients for the ith location and ε_{ij} noise.

- a functional regression set up with scalar response:
  $w^i(t) \stackrel{\text{def}}{=} \mathbf{e}_1^\top A^{-1} \left[\exp\left\{A(\tau_2 - t)\right\} - I_p\right] \mathbf{e}_p$ and
  $\theta^i_w(t) = \theta(t) w^i(t)$:

$$RP_{ij}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \theta^i_w(u)\sigma_{ij}(u)du + \varepsilon_{ij}.$$

- Regress RP on PC scores of $\sigma(t)$ for dimension reduction.
- Note $\theta^i_w(t)$ contains location dependent parameters, need spatial setting.

Functional Principal Components

1. Decompose

   $$\sigma_{ij}(t) = \{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} + \bar{\sigma}_i(t),$$

   $\sigma_{ij}(t)$ variation curve for ith location and jth month,

   $\bar{\sigma}_i(t)$ average curve for ith location.

$$\begin{aligned} RP_{ij} &= \int_{\tau_1}^{\tau_2} \theta_w^i(u)\bar{\sigma}_i(u)du \\ &\quad + \int_{\tau_1}^{\tau_2} \theta_w^i(u) \underbrace{\{\sigma_{ij}(u) - \bar{\sigma}_i(u)\}}_{\text{FPCA for } \sigma_{ij}} du. \end{aligned}$$

2. Perform FPCA: derive scores for temperature variation

PC Scores

- PC scores for functions $\sigma_{ij}(t)$ $i = 1, \ldots, 9$ (9 cities), $j = 1, \ldots, 7$ (7 traded months):

$$c_{ijk} = \int_{\tau_1}^{\tau_2} \xi_{ik}(t) \left\{ \sigma_{ij}(t) - \bar{\sigma}_i(t) \right\} dt,$$

  c_{ijk} scores for K largest eigenvalues, $\xi_{ik}(t)$ orthonormal eigenfunctions of $Cov\{\sigma(\cdot)\}$ operator.
- Collect scores capturing the variance in the data in matrix C.
- Parametrize the relationship to RP by geographically weighted regression.

3. Regress the response RP_{ij} at $t = \tau_1$ on the PC scores (Ramsay & Silverman, 2008):

$$RP_{ij} = \beta_{i0} \; + \; \sum_{k=1}^{K} \beta_{ik} \int_{\tau_1}^{\tau_2} \xi_{ik}(t)\{\sigma_{ij}(t) - \bar{\sigma}_i(t)\}dt + \widetilde{\varepsilon}_{ij}$$

for ith location and jth month, with $\widetilde{\varepsilon}_{ij}$ containing ε_{ij} and the truncation error resulting from taking first K PC scores.

- Functional form of MPR: $\theta_w^i(t) = \sum_k \beta_{ik}\xi_{ik}(t)$ APPENDIX
- Need spatial model for regression on PC scores.

Spatial Specification: GWR

Why GWR (Fotheringham et al., 2002)?

- distance based weights,
- local nature of spatial dependence,
- regression coefficients are functions of the geo locations,

$\rightarrow$ forecast to non-sampled locations.

GWR: the Model

$$
\begin{aligned}
W_i^{\frac{1}{2}} RP &= W_i^{\frac{1}{2}} C\beta_i + e_i, e_i \text{ vector of } \textit{iid} \text{ errors},\\
RP &= (RP_{1,1}, RP_{2,1}, \ldots, RP_{n,1}, RP_{1,2}, \ldots\ \ldots, RP_{n,7})^\top,\\
C &= \begin{pmatrix} c_{1,1,1} & \cdots & c_{1,1,K} \\ c_{2,1,1} & \cdots & c_{1,1,K} \\ \cdots & \cdots & \cdots \\ c_{n,7,1} & \cdots & c_{n,7,K} \end{pmatrix}\\
n &- \text{ total number of locations}\\
K &- \text{ number of PC scores}
\end{aligned}
$$

GWR: the Model

$$
\begin{aligned}
W_i &= diag(w_i), \; i = 1, \ldots, n \\
w_i &= \operatorname{diag}\left[\exp\left\{-\frac{1}{2}\left(\frac{d_{i1}}{h^*}\right)^2\right\}, \ldots, \exp\left\{-\frac{1}{2}\left(\frac{d_{in}}{h^*}\right)^2\right\}\right], \\
h^* &= \arg\min_{h \in H} \sum_{m=1}^{7n} \left\{RP_m - \widehat{RP}_{\neq m}(h)\right\}^2,
\end{aligned}
$$

with $d_{il}, \; l = 1, \ldots, n$ euclidean distances to ith city, $\widehat{RP}_{\neq m}(h)$ estimated RP without the mth value using h.

Temperature Data

City	First Date	Last Date	First F_{CAT} Trade
Amsterdam	19730101	20101231	20030401
Berlin	19480101	20101231	20030401
Barcelona	19730101	20101231	20050401
Essen	19700101	20101231	20050401
London	19730101	20101231	20030401
Madrid	19730101	20101231	20050401
Paris	19730101	20101231	20030401
Rome	19730101	20101231	20050401
Stockholm	19730101	20101231	20030401

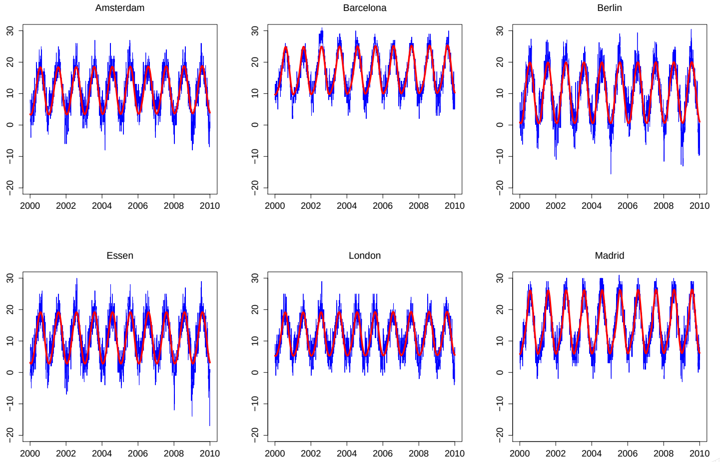
Table 1: Average Temperatures without 29th February. Source Bloomberg and DWD.

Volatility Functions $\sigma_i(t)$

Fit to data

- Seasonality $\Lambda(t)$,
- AR(p) process with seasonally heteroscedastic errors,
- impute RP_{ij}, $i = 1, \ldots, 9$ and $j = 1, \ldots, 7$,
- obtain $\sigma_i(t), t \in [1, 365]$ using residual standard deviation for each day of year smoothed by Fourier series.

Seasonality



Seasonality

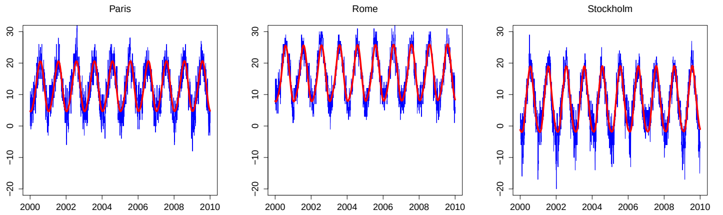


Figure 3: Daily average temperatures $T(t)$ (blue) and seasonality $\Lambda(t)$ (red).

	Essen		London		Madrid	
	estimate	t.stat	estimate	t.stat	estimate	t.stat
a	10.66	171.30	10.81	217.33	13.97	286.51
b	$0.1 \cdot 10^{-4}$	0.66	$0.7 \cdot 10^{-4}$	11.10	$0.9 \cdot 10^{-4}$	14.64
c_1	-2.32	-52.79	-2.48	-70.43	-3.30	-95.68
c_2	-7.82	-177.64	-6.42	-182.57	-8.93	-259.17
c_3	0.49	11.21	0.77	21.85	1.67	48.46
c_4	–	–	0.23	6.66	0.25	7.21
c_5	–	–	–	–	-0.19	-5.39
c_6	–	–	–	–	-0.34	-9.87

Table 2: Estimated Parameters of seasonality (1) for Essen, London, Paris

AR(p)

	Amsterdam		Barcelona		Berlin	
	estimate	t.stat	estimate	t.stat	estimate	t.stat
α_1	0.89	105.05	0.70	83.14	0.92	139.69
α_2	-0.19	-16.76	0.03	3.17	-0.20	-23.14
α_3	0.09	10.46	0.01	1.29	0.08	11.99
α_4	–	–	0.03	3.64	–	–

Table 3: Estimated Parameters of AR(p) for Amsterdam, Barcelona, Berlin

RP

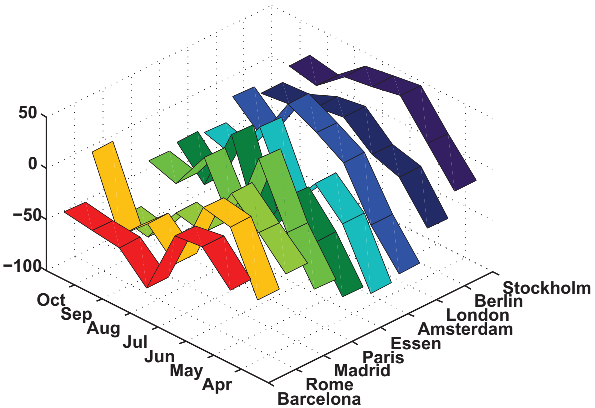
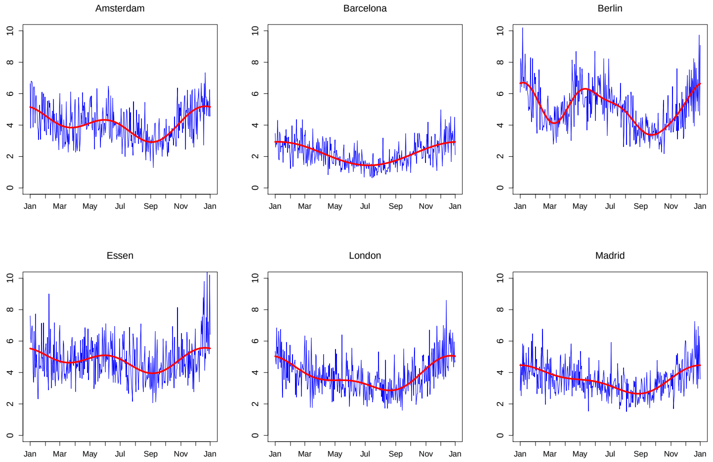


Figure 4: Average RP for traded locations computed according to (2)

Seasonal Variation



Seasonal Variation

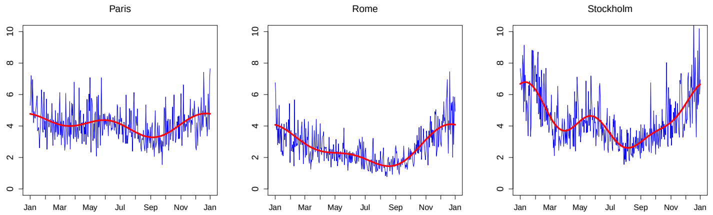
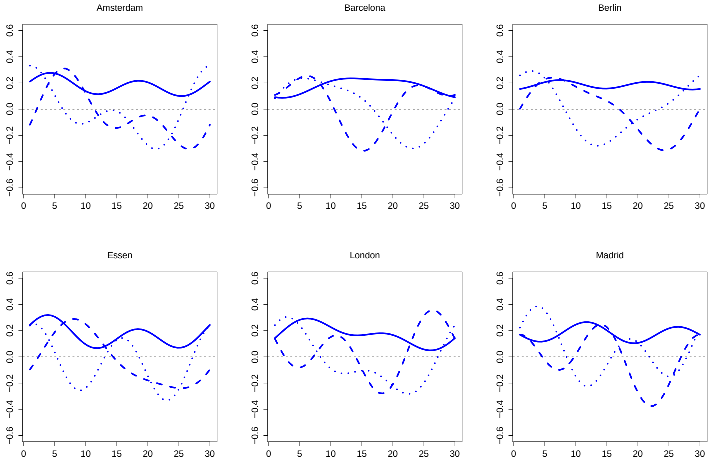


Figure 5: Estimated $\sigma(t)$ (blue) and smoothed by Fourier series (red).

Eigenfunctions



Eigenfunctions

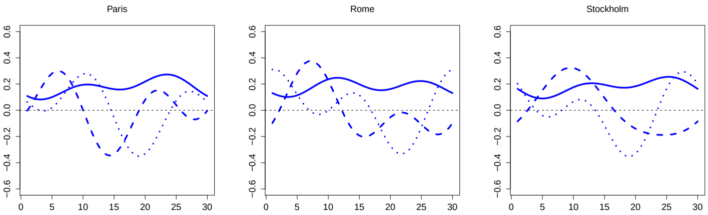
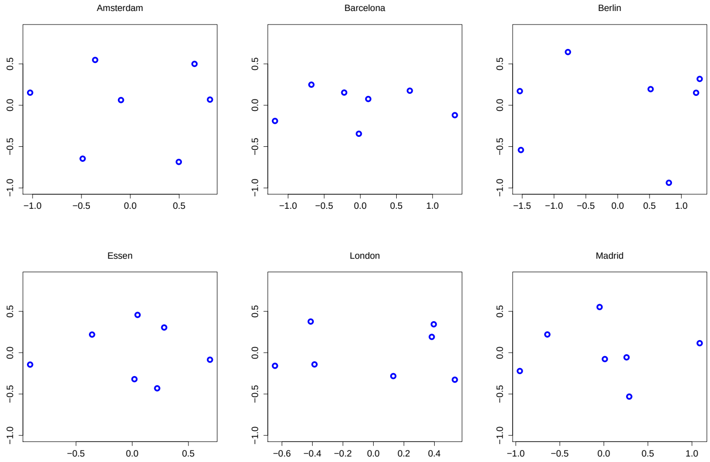


Figure 6: FPCA weight functions: eigenfunction ξ_1 (solid), ξ_2 (dashed), ξ_3 (dotted).

FPCA Scores


Amsterdam
Barcelona
Berlin
Essen
London
Madrid
−1.0 −0.5 0.0 0.5
−1.0 −0.5 0.0 0.5 1.0
−1.5 −1.0 −0.5 0.0 0.5 1.0
−0.5 0.0 0.5
−0.6 −0.4 −0.2 0.0 0.2 0.4
−1.0 −0.5 0.0 0.5 1.0

FPCA Scores

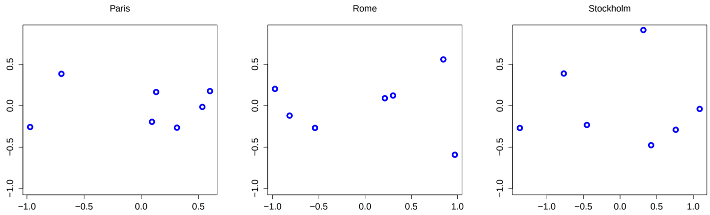
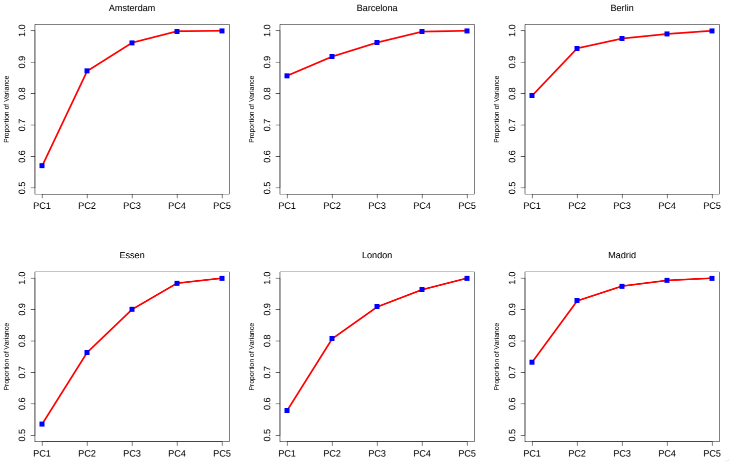


Figure 7: FPCA scores c_{ij1} and c_{ij2}

Explained Proportion of Variance



Explained Proportion of Variance

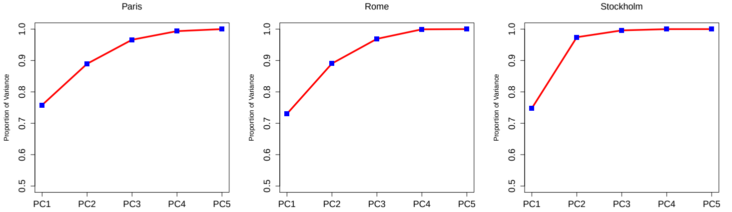
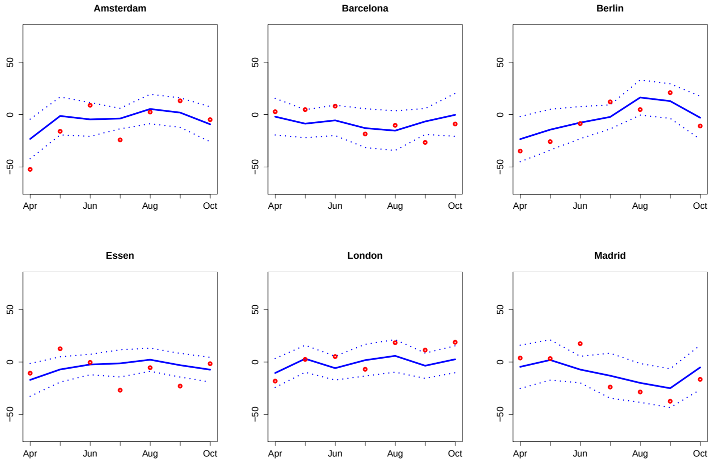


Figure 8: Proportion of Variance explained by the corresponding PC.

GWR Estimation Results



GWR Estimation Results

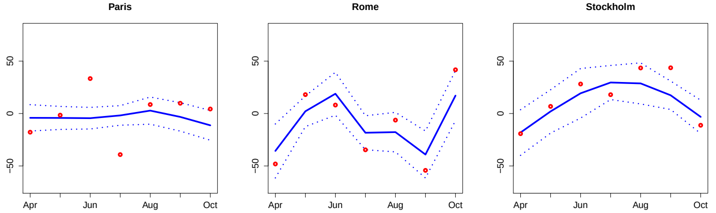


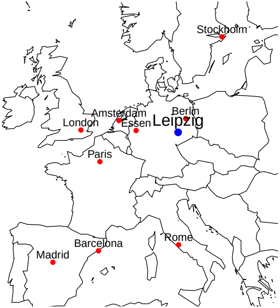
Figure 9: RP (red) and fitted values with 95% CI (blue) returned by GWR.

City	β_0	β_1	β_2	β_3	R^2_{loc}
Amsterdam	-4.68^{**}	-5.28	15.31^{**}	15.04^{**}	0.25
Barcelona	-7.35	6.17^{**}	-5.53	8.27	0.34
Berlin	-3.06	-8.07	17.58^{**}	-9.90^{*}	0.61
Essen	-5.10^{**}	-4.86	14.87^{**}	10.85^{*}	0.30
London	-3.27	-5.43	12.89^{**}	18.14^{**}	0.22
Madrid	-10.40	11.17^{**}	-7.98	30.55^{**}	0.42
Paris	-3.75^{**}	-2.85	10.98^{*}	13.38^{*}	0.21
Rome	-5.54°	12.00^{*}	-23.38^{**}	-77.01^{**}	0.73
Stockholm	10.82	-16.29	16.91^{**}	-41.96^{**}	0.72
Leipzig	-4.42	-6.64	16.07	-7.60	–

Table 4: Estimated Parameters of GWR (h^{*}=4.98). ** indicate significance on $\leq$ 1% level, * – on 5% and $^{\circ}$ – on 10%. Dummy variables for north and south sea coast cities omitted here. Weights for Leipzig $(0.13, 0, 0.42, 0.24, 0.02, 0, 0.05, 0.07, 0.07)^{\top}$.

Example: Hedging weather risk in electricity demand

- An electricity provider in Leipzig transfers risk via CAT futures.
- What RP one would pay for F_{CAT} in August 2010?



Out-of-Sample Forecast: Leipzig

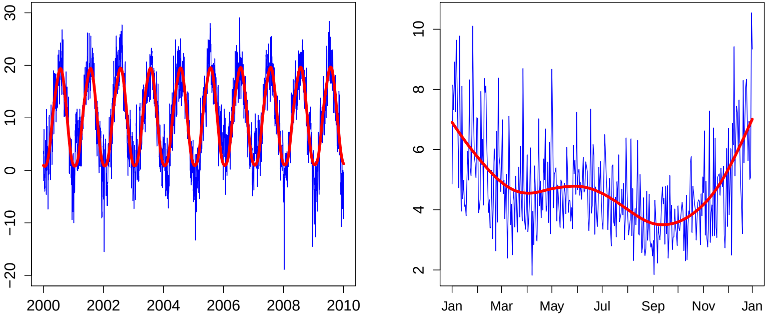


Figure 10: Λ_t, σ_t for Leipzig.

Out-of-Sample Forecast: Leipzig

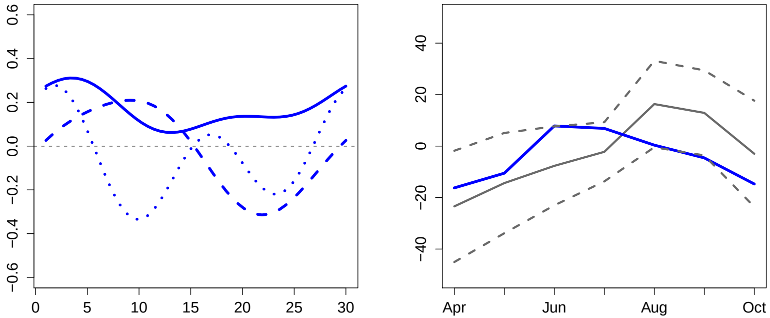
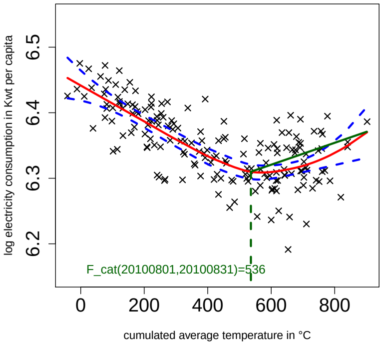


Figure 11: ξ and the resulting forecast for *RP* for Leipzig vs *RP* of Berlin.

$$F_{CAT}(\hat{\theta}, 20100801, 20100831) = 536$$



Example: Hedging weather risk in electricity demand

- c – marginal costs of meeting additional log demand of 1% per customer
- n – number of customers
- b – estimated marginal effects of 1°C CAT on log demand starting from threshold F_{CAT}
- α – number of WD hold, p – tick value of WD (for traded futures in Europe: 20EUR)

exposure	benefits
$cnb(CAT - F_{CAT})$	$\alpha p(CAT - F_{CAT})$

- hedging, s.t.

$$cnb = \alpha p$$

Example: Hedging weather risk in electricity demand

parameter	value	units
c	1	EUR per 1% log Kwt/pP ($\sim$ 0.25 EUR/Kwt)
n	$100,000$	customers
b	0.016	1% log Kwt/pP and 1°C CAT$\cdot$
p	20	EUR per 1°C CAT
α	80	contracts long

Table 5: An elementary example of a hedging strategy.

Appendix

$$
\begin{aligned}
RP_{ij} &= \underbrace{\int_{\tau_1}^{\tau_2} \theta_w^i(u)\bar{\sigma}_i(u)du}_{\beta_{i0}} + \int_{\tau_1}^{\tau_2} \underbrace{\theta_w^i(u)}_{\sum_m \beta_{im}\xi_{im}(u)} \underbrace{\{\sigma_{ij}(u) - \bar{\sigma}_i(u)\}}_{\sum_k c_{ijk}\xi_{ik}(u)} du \\
&= \beta_{i0} + \sum_m \sum_k \beta_{im} c_{ijk} \underbrace{\int_{\tau_1}^{\tau_2} \xi_{ik}(u)\xi_{im}(u)du}_{=\begin{cases} 0, & k \neq m, \\ 1, & k = m \end{cases}} \\
&= \beta_{i0} + \sum_k \beta_{ik} c_{ijk}.
\end{aligned}
$$

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