Spatial Risk Premium on Weather and Hedging Weather Exposure in Electricity

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"Everybody talks about the weather but nobody does anything about it." Mark Twain
Motivation
Figure 1: Estimated dependency of log electricity consumption in Germany on cumulative average temperature: 19960101-20100930, effects of prices and income removed (from Akdeniz Duran et al., 2011).
Future Contracts on Temperature

- transfer weather related risks,
- number of agents limited,
- weather non-tradable, insurance nature,
- geographically separated markets, spatial relationship.
Figure 2: Contracts on 11 cities in Europe are traded on the CME.
Pricing Model

- Econometrics of temperature (Benth et al., 2007)
  - seasonal function
    \[
    \Lambda(t) = a + bt + \sum_{j=1}^{k} \left\{ c_{2j-1} \sin \left( \frac{2j\pi t}{365} \right) + c_{2j} \cos \left( \frac{2j\pi t}{365} \right) \right\},
    \]
    (1)
  - \( p \)-dimensional Ornstein-Uhlenbeck process \( X(t) \):
    \[
    dX(t) = AX(t)dt + e_p \sigma(t)dB(t),
    \]
  - \( B(t) \) is Brownian motion, \( e_p \) \( p \)th column of \( I_p \), \( \sigma(t) \) seasonal variation.
Pricing Model

Price of a future on Cumulative Average Temperature index (CAT): (Benth et al., 2007)

\[ F_{\text{CAT}}(t, \tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \Lambda(u) du + a(t)X(t) \]
\[ + \int_t^{\tau_1} \theta(u)\sigma(u)a(u)e_p du \]
\[ + \int_{\tau_1}^{\tau_2} \theta(u)\sigma(u)e_1^\top A^{-1} \left[ \exp \{ A(\tau_2 - u) \} - l_p \right] e_p du. \]

with \( a(t) = e_1^\top A^{-1} \left[ \exp \{ A(\tau_2 - t) \} - \exp \{ A(\tau_1 - t) \} \right] \)

Market price of risk (MPR) \( \theta(t) \) unknown for specific locations.
Pricing Temperature around the Globe

- $\theta(t)$ varies across locations.
- Connect $\theta(t)$ to a known risk factor!
- Spatial model to interpolate for other locations.

How can FPCA help pricing an arbitrary location?
Outline

1. Motivation ✓
2. Spatial Model for Risk Premium
   - Functional Principal Components (FPCA)
   - Geographically Weighted Regression (GWR)
3. Empirical Risk Premia and Hedging Weather Exposure
4. Outlook
Risk Premium

Given (2) the RP at $t = \tau_1$ for $i$th location and $j$th contract ($RP_{ij}$):

$$RP_{ij}(t = \tau_1, \tau_2) = F_{CAT,ij}(\theta^i, t = \tau_1, \tau_2) - \hat{F}_{CAT,ij}(0, t = \tau_1, \tau_2) + \varepsilon_{ij},$$

$$= \int_{\tau_1}^{\tau_2} \theta^i(u)\sigma_{ij}(u)e_1^T A_i^{-1}$$
$$\times [\exp \{A_i(\tau_2 - u)\} - l_p] e_p du + \varepsilon_{ij}.$$ 

$\hat{F}_{CAT,ij}(0, t = \tau_1, \tau_2)$ estimated price for $j$th contract in $i$th location, zero MPR, $A_i$ is the matrix of O-U-Process coefficients for the $i$th location and $\varepsilon_{ij}$ noise.
a functional regression set up with scalar response:

\[ w^i(t) \overset{\text{def}}{=} e_1^\top A^{-1} \left[ \exp \left\{ A(\tau_2 - t) \right\} - l_p \right] e_p \] and

\[ \theta^i_{w}(t) = \theta(t)w^i(t): \]

\[
RP_{ij}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \theta^i_{w}(u)\sigma_{ij}(u)du + \varepsilon_{ij}.
\]

- Regress RP on PC scores of \( \sigma(t) \) for dimension reduction.
- Note \( \theta^i_{w}(t) \) contains location dependent parameters, need spatial setting.
Functional Principal Components

1. Decompose

\[ \sigma_{ij}(t) = \{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} + \bar{\sigma}_i(t), \]

\( \sigma_{ij}(t) \) variation curve for \( i \)th location and \( j \)th month, 
\( \bar{\sigma}_i(t) \) average curve for \( i \)th location.

\[ \text{RP}_{ij} = \int_{\tau_1}^{\tau_2} \theta^i_w(u)\bar{\sigma}_i(u)du + \int_{\tau_1}^{\tau_2} \theta^i_w(u)\{\sigma_{ij}(u) - \bar{\sigma}_i(u)\}du. \]

2. Perform FPCA: derive scores for temperature variation

Spatial Risk Premium on Weather
PC Scores

- PC scores for functions $\sigma_{ij}(t)$ $i = 1, \ldots, 9$ (9 cities), $j = 1, \ldots, 7$ (7 traded months):

$$c_{ijk} = \int_{\tau_1}^{\tau_2} \xi_{ik}(t) \{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} \, dt,$$

$c_{ijk}$ scores for $K$ largest eigenvalues, $\xi_{ik}(t)$ orthonormal eigenfunctions of $\text{Cov}\{\sigma(\cdot)\}$ operator.

- Collect scores capturing the variance in the data in matrix $C$.
- Parametrize the relationship to RP by geographically weighted regression.
3. Regress the response $RP_{ij}$ at $t = \tau_1$ on the PC scores (Ramsay & Silverman, 2008):

$$RP_{ij} = \beta_{i0} + \sum_{k=1}^{K} \beta_{ik} \int_{\tau_1}^{\tau_2} \xi_{ik}(t)\{\sigma_{ij}(t) - \bar{\sigma}_i(t)\} dt + \tilde{\varepsilon}_{ij}$$

for $i$th location and $j$th month, with $\tilde{\varepsilon}_{ij}$ containing $\varepsilon_{ij}$ and the truncation error resulting from taking first $K$ PC scores.

- Functional form of MPR: $\theta^i_w(t) = \sum_k \beta_{ik}\xi_{ik}(t)$
- Need spatial model for regression on PC scores.
Spatial Specification: GWR

Why GWR (Fotheringham et al., 2002)?

- distance based weights,
- local nature of spatial dependence,
- regression coefficients are functions of the geo locations,
- forecast to non-sampled locations.
GWR: the Model

\[ W_i^{1/2} RP = W_i^{1/2} C \beta_i + e_i, \ e_i \text{ vector of iid errors}, \]
\[ RP = (RP_{1,1}, RP_{2,1}, \ldots, RP_{n,1}, RP_{1,2}, \ldots, \ldots, RP_{n,7})^T, \]
\[ C = \begin{pmatrix} c_{1,1,1} & \cdots & c_{1,1,K} \\ c_{2,1,1} & \cdots & c_{1,1,K} \\ \vdots & \ddots & \vdots \\ c_{n,7,1} & \cdots & c_{n,7,K} \end{pmatrix} \]
\[ n \quad \text{total number of locations} \]
\[ K \quad \text{number of PC scores} \]
GWR: the Model

\[ W_i = \text{diag}(w_i), \quad i = 1, \ldots, n \]
\[ w_i = \text{diag} \left[ \exp \left\{ -\frac{1}{2} \left( \frac{d_{i1}}{h^*} \right)^2 \right\}, \ldots, \exp \left\{ -\frac{1}{2} \left( \frac{d_{in}}{h^*} \right)^2 \right\} \right], \]
\[ h^* = \arg \min_{h \in H} \sum_{m=1}^{7n} \left\{ RP_m - \hat{RP}_{\neq m}(h) \right\}^2, \]

with \( d_{il}, \ l = 1, \ldots, n \) euclidean distances to \( i \)th city, \( \hat{RP}_{\neq m}(h) \) estimated RP without the \( m \)th value using \( h \).
## Temperature Data

<table>
<thead>
<tr>
<th>City</th>
<th>First Date</th>
<th>Last Date</th>
<th>First $F_{\text{CAT}}$</th>
<th>Trade</th>
</tr>
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<tbody>
<tr>
<td>Amsterdam</td>
<td>19730101</td>
<td>20101231</td>
<td>20030401</td>
<td></td>
</tr>
<tr>
<td>Berlin</td>
<td>19480101</td>
<td>20101231</td>
<td>20030401</td>
<td></td>
</tr>
<tr>
<td>Barcelona</td>
<td>19730101</td>
<td>20101231</td>
<td>20050401</td>
<td></td>
</tr>
<tr>
<td>Essen</td>
<td>19700101</td>
<td>20101231</td>
<td>20050401</td>
<td></td>
</tr>
<tr>
<td>London</td>
<td>19730101</td>
<td>20101231</td>
<td>20030401</td>
<td></td>
</tr>
<tr>
<td>Madrid</td>
<td>19730101</td>
<td>20101231</td>
<td>20050401</td>
<td></td>
</tr>
<tr>
<td>Paris</td>
<td>19730101</td>
<td>20101231</td>
<td>20030401</td>
<td></td>
</tr>
<tr>
<td>Rome</td>
<td>19730101</td>
<td>20101231</td>
<td>20050401</td>
<td></td>
</tr>
<tr>
<td>Stockholm</td>
<td>19730101</td>
<td>20101231</td>
<td>20030401</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Average Temperatures without 29th February. Source Bloomberg and DWD.
Volatility Functions $\sigma_i(t)$

Fit to data

- Seasonality $\Lambda(t)$,
- AR($p$) process with seasonally heteroscedastic errors,
- impute $RP_{ij}, i = 1, \ldots, 9$ and $j = 1, \ldots, 7$,
- obtain $\sigma_i(t), t \in [1, 365]$ using residual standard deviation for each day of year smoothed by Fourier series.
Seasonality

Spatial Risk Premium on Weather
Seasonality

Figure 3: Daily average temperatures $T(t)$ (blue) and seasonality $\Lambda(t)$ (red).
### Estimated Parameters of seasonality (1) for Essen, London, Paris

<table>
<thead>
<tr>
<th></th>
<th>Essen</th>
<th>London</th>
<th>Madrid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>estimate</td>
<td>t.stat</td>
<td>estimate</td>
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<tr>
<td>$a$</td>
<td>10.66</td>
<td>171.30</td>
<td>10.81</td>
</tr>
<tr>
<td>$b$</td>
<td>$0.1 \cdot 10^{-4}$</td>
<td>0.66</td>
<td>$0.7 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$c_1$</td>
<td>-2.32</td>
<td>-52.79</td>
<td>-2.48</td>
</tr>
<tr>
<td>$c_2$</td>
<td>-7.82</td>
<td>-177.64</td>
<td>-6.42</td>
</tr>
<tr>
<td>$c_3$</td>
<td>0.49</td>
<td>11.21</td>
<td>0.77</td>
</tr>
<tr>
<td>$c_4$</td>
<td>$-$</td>
<td>$-$</td>
<td>0.23</td>
</tr>
<tr>
<td>$c_5$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>$c_6$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 2: Estimated Parameters of seasonality (1) for Essen, London, Paris
### Table 3: Estimated Parameters of AR($p$) for Amsterdam, Barcelona, Berlin

<table>
<thead>
<tr>
<th></th>
<th>Amsterdam</th>
<th></th>
<th>Barcelona</th>
<th></th>
<th>Berlin</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>estimate</td>
<td>t.stat</td>
<td>estimate</td>
<td>t.stat</td>
<td>estimate</td>
<td>t.stat</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.89</td>
<td>105.05</td>
<td>0.70</td>
<td>83.14</td>
<td>0.92</td>
<td>139.69</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.19</td>
<td>-16.76</td>
<td>0.03</td>
<td>3.17</td>
<td>-0.20</td>
<td>-23.14</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.09</td>
<td>10.46</td>
<td>0.01</td>
<td>1.29</td>
<td>0.08</td>
<td>11.99</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>–</td>
<td>–</td>
<td>0.03</td>
<td>3.64</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
Figure 4: Average RP for traded locations computed according to (2)

Spatial Risk Premium on Weather
Seasonal Variation

Spatial Risk Premium on Weather
Seasonal Variation

Figure 5: Estimated $\sigma(t)$ (blue) and smoothed by Fourier series (red).
Empirical Risk Premia and Hedging Weather Exposure

Eigenfunctions

Spatial Risk Premium on Weather
Eigenfunctions

Figure 6: FPCA weight functions: eigenfunction $\xi_1$ (solid), $\xi_2$ (dashed), $\xi_3$ (dotted).
FPCA Scores

Spatial Risk Premium on Weather
FPCA Scores

Figure 7: FPCA scores $c_{ij1}$ and $c_{ij2}$
Explained Proportion of Variance

Spatial Risk Premium on Weather
Empirical Risk Premia and Hedging Weather Exposure

Explained Proportion of Variance

Figure 8: Proportion of Variance explained by the corresponding PC.

Spatial Risk Premium on Weather
GWR Estimation Results
GWR Estimation Results

Figure 9: RP (red) and fitted values with 95% CI (blue) returned by GWR.
<table>
<thead>
<tr>
<th>City</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$R^2_{loc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amsterdam</td>
<td>$-4.68^{**}$</td>
<td>$-5.28$</td>
<td>$15.31^{**}$</td>
<td>$15.04^{**}$</td>
<td>0.25</td>
</tr>
<tr>
<td>Barcelona</td>
<td>$-7.35$</td>
<td>$6.17^{**}$</td>
<td>$-5.53$</td>
<td>$8.27$</td>
<td>0.34</td>
</tr>
<tr>
<td>Berlin</td>
<td>$-3.06$</td>
<td>$-8.07$</td>
<td>$17.58^{**}$</td>
<td>$-9.90^*$</td>
<td>0.61</td>
</tr>
<tr>
<td>Essen</td>
<td>$-5.10^{**}$</td>
<td>$-4.86$</td>
<td>$14.87^{**}$</td>
<td>$10.85^*$</td>
<td>0.30</td>
</tr>
<tr>
<td>London</td>
<td>$-3.27$</td>
<td>$-5.43$</td>
<td>$12.89^{**}$</td>
<td>$18.14^{**}$</td>
<td>0.22</td>
</tr>
<tr>
<td>Madrid</td>
<td>$-10.40$</td>
<td>$11.17^{**}$</td>
<td>$-7.98$</td>
<td>$30.55^{**}$</td>
<td>0.42</td>
</tr>
<tr>
<td>Paris</td>
<td>$-3.75^{**}$</td>
<td>$-2.85$</td>
<td>$10.98^*$</td>
<td>$13.38^*$</td>
<td>0.21</td>
</tr>
<tr>
<td>Rome</td>
<td>$-5.54^\circ$</td>
<td>$12.00^*$</td>
<td>$-23.38^{**}$</td>
<td>$-77.01^{**}$</td>
<td>0.73</td>
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<tr>
<td>Stockholm</td>
<td>$10.82$</td>
<td>$-16.29$</td>
<td>$16.91^{**}$</td>
<td>$-41.96^{**}$</td>
<td>0.72</td>
</tr>
<tr>
<td>Leipzig</td>
<td>$-4.42$</td>
<td>$-6.64$</td>
<td>$16.07$</td>
<td>$-7.60$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 4: Estimated Parameters of GWR ($h^* = 4.98$). ** indicate significance on $\leq 1\%$ level, * – on 5% and $^\circ$ – on 10%. Dummy variables for north and south sea coast cities omitted here. Weights for Leipzig $(0.13, 0, 0.42, 0.24, 0.02, 0, 0.05, 0.07, 0.07)^\top$. Spatial Risk Premium on Weather
Example: Hedging weather risk in electricity demand

- An electricity provider in Leipzig transfers risk via CAT futures.
- What RP one would pay for $F_{CAT}$ in August 2010?
Out-of-Sample Forecast: Leipzig

Figure 10: $\Lambda_t, \sigma_t$ for Leipzig.
Out-of-Sample Forecast: Leipzig

Figure 11: $\xi$ and the resulting forecast for $RP$ for Leipzig vs $RP$ of Berlin.
\[ F_{CAT}(\hat{\theta}, 20100801, 20100831) = 536 \]
Example: Hedging weather risk in electricity demand

- $c$ – marginal costs of meeting additional log demand of 1% per customer
- $n$ – number of customers
- $b$ – estimated marginal effects of $1^\circ$C CAT on log demand starting from threshold $F_{CAT}$
- $\alpha$ – number of WD hold, $p$ – tick value of WD (for traded futures in Europe: 20EUR)

<table>
<thead>
<tr>
<th>exposure</th>
<th>benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cnb(CAT - F_{CAT})$</td>
<td>$\alpha p(CAT - F_{CAT})$</td>
</tr>
</tbody>
</table>

- hedging, s.t.

$$cnb = \alpha p$$
Example: Hedging weather risk in electricity demand

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>1</td>
<td>EUR per 1% log Kwt/pP ($\sim 0.25$ EUR/Kwt)</td>
</tr>
<tr>
<td>n</td>
<td>100,000</td>
<td>customers</td>
</tr>
<tr>
<td>b</td>
<td>0.016</td>
<td>1% log Kwt/pP and 1°C CAT·</td>
</tr>
<tr>
<td>p</td>
<td>20</td>
<td>EUR per 1°C CAT</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>80</td>
<td>contracts long</td>
</tr>
</tbody>
</table>

Table 5: An elementary example of a hedging strategy.
Appendix

\[ RP_{ij} = \left[ \int_{\tau_1}^{\tau_2} \theta_{w}^i(u) \sigma_i(u) \, du + \int_{\tau_1}^{\tau_2} \theta_{w}^i(u) \left\{ \sigma_{ij}(u) - \bar{\sigma}_i(u) \right\} \, du \right] + \beta_i \sum_k c_{ik} \xi_{ik}(u) \]

\[ = \beta_i \sum_k c_{ik} \xi_{ik}(u) \]

\[ = \beta_i + \sum_k c_{ik} \xi_{ik}(u) \]

\[ = \begin{cases} 0, & k \neq m, \\ 1, & k = m \end{cases} \]

\[ = \beta_i + \sum_k c_{ik} \xi_{ik} \]

Spatial Risk Premium on Weather
Literature

E. Akdeniz Duran and W.K. Härdle and M. Osipenko
*Difference based ridge and Liu type estimators in semiparametric regression models*

F.E. Benth and J.S. Benth and S. Koekebakker
*Putting a Price on Temperature*

A. Fotheringham and C. Brudson and M. Charlton
*Geographically Weighted Regression: the Analysis of Spatially Varying Relationships*
Literature

- W.K. Härdle and B.López Cabrera
  *Implied Market Price of Weather Risk*

- W.K. Härdle and M. Osipenko
  *Spatial Risk Premium on Weather Derivatives and Hedging Weather Exposure in Electricity*

- B. Øksendal
  *Stochastic Differential Equations*
Literature

F. Perez-Gonzalez and H. Yun
*Risk Management and Firm Value: Evidence from Weather Derivatives*

J.O. Ramsay, B.W. Silverman
*Functional Data Analysis*
Springer Verlag, Heidelberg, 2008
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