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**Convenience yield and time adjusted  
basis stylized facts**

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## Motivation and objective of the paper

- **Goal of the paper: to revisit the empirical stylized facts of the convenience yield with a more precise definition than in previous empirical studies;**
- **We only present a preliminary draft for oil and copper commodities;**
- **Preliminary results: stylized facts studied under our definition are more robust than under the usual empirical measure;**

# The convenience yield (CY) is still subject to intense debates

- **Why studying the convenience yield: The convenience yield is a variable that is still subject to considerable debates (see e.g. Lautier, 2009):**
  - **Some authors believe in an economic rationale at the heart of the definition of this variable, e.g. Brennan, 1991:**
    - **The convenience yield is (the value of) the flow of services that accrue to the owner of commodity inventory as opposed to the owner of contracts for future delivery;**
  - **Other authors believe that it is just an ad-hoc variable to statistically reproduce backwardation, e.g. Hull (Options, futures and other derivatives);**

## Stylized facts involving the CY usually considered

- **Main stylized facts under consideration usually (e.g Routledge et al., 2000; Dincerler et al., 2005):**
  - **Link between the spot price and the convenience yield:**
    - **The correlation should be dynamic as a decreasing function of the level of inventory;**
  - **Link between the convenience yield and the level of inventory:**
    - **The convenience yield should be a convex decreasing function of inventory: the so called Kaldor-Working curve;**
  - **Link between the volatility of the convenience yield and the level of inventory:**
    - **The volatility should be dynamic as a decreasing function of the level of inventory;**

## The time adjusted is usually used as a proxy for the CY

- In empirical studies the convenience yield is usually proxied by the time adjusted basis,  $b(T)$ :

$$b(T) = -\frac{1}{T} [\ln[P(T)F(T)] - \ln[S]]$$

- P, F and S designates the zero coupon bond price, futures price and spot price, respectively; T is the (time to) maturity of the futures contract;
- This relation is justified by cash and carry arbitrage with an assumed deterministic convenience yield  $\delta$ :

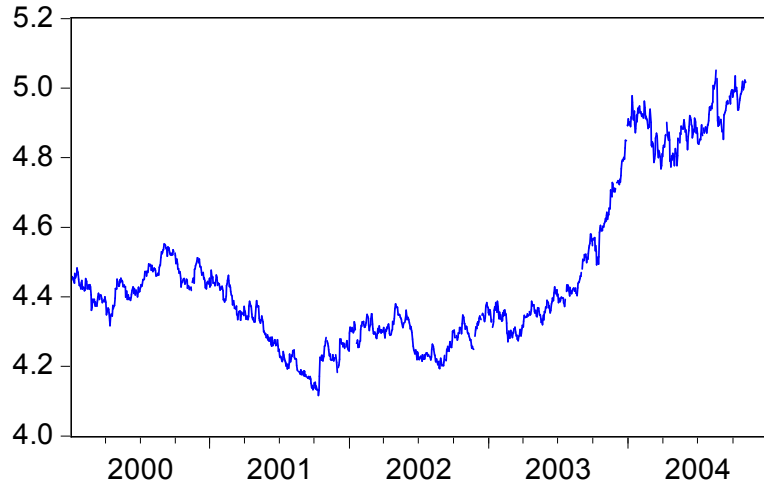
$$F(T) = \frac{S}{P(T)} \exp(-\delta * T)$$

## Preliminary Data Set

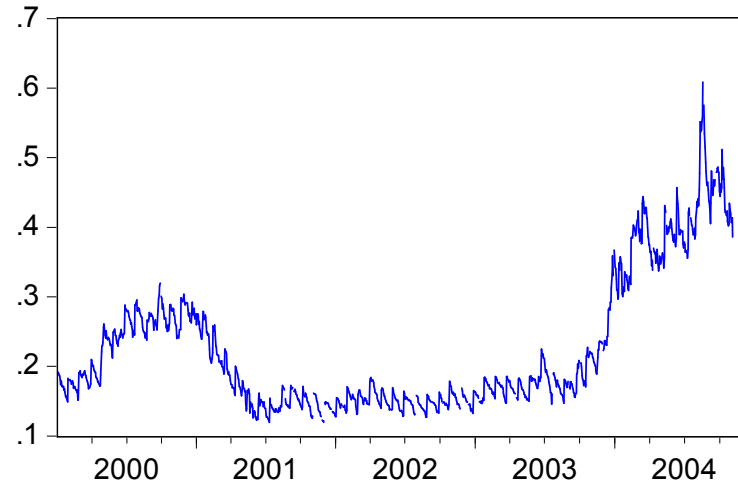
- **Stochastic Behavior of the CY**
- **The convenience yield and spot price state variables are obtained by the model of Casassus and Colin-Dufresne (2005):**
  - **This model is a conditionally Gaussian affine model with stochastic risk premia;**
  - **Data from 2000 to 2004;**

# Motivation and objective of the paper: Spot and CY from CCD model

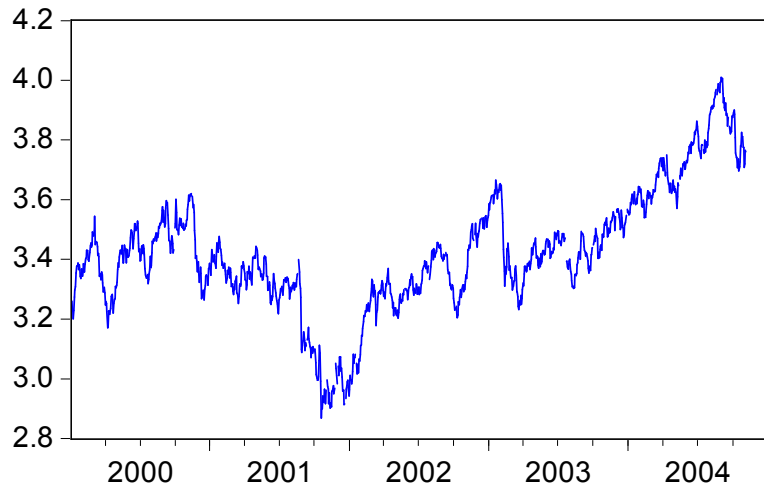
LOG\_SPOT\_COPPER



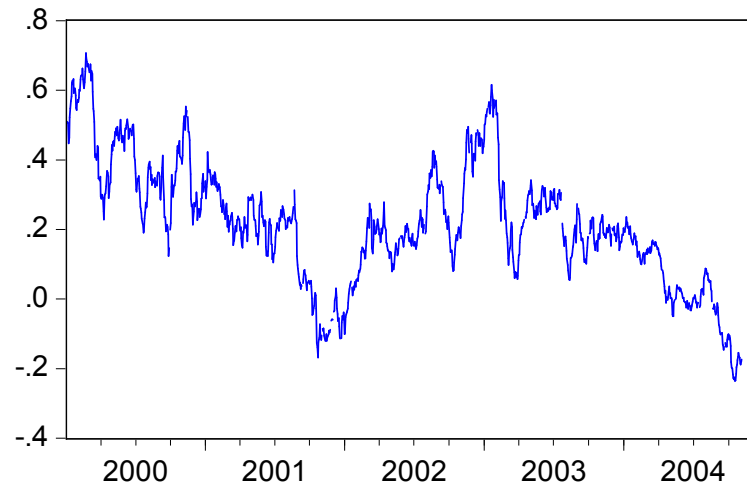
CY\_COPPER



LOG\_SPOT\_OIL



CY\_OIL



## A stochastic CY leads to a more complex relation with the basis

- **The assumption of a deterministic convenience yield can certainly not be made!**
- **Under a dynamic convenience yield, the following relation pertain between the time adjusted basis and the convenience yield:**

$$\exp(b_t(T)) = E_t^{P^{(S)}} \left[ \exp \left( \int_t^T \delta_u du \right) \right]$$

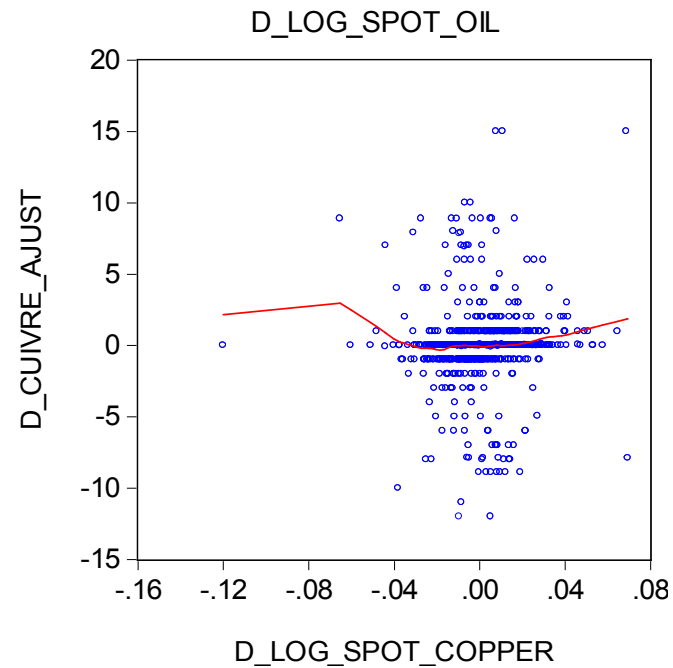
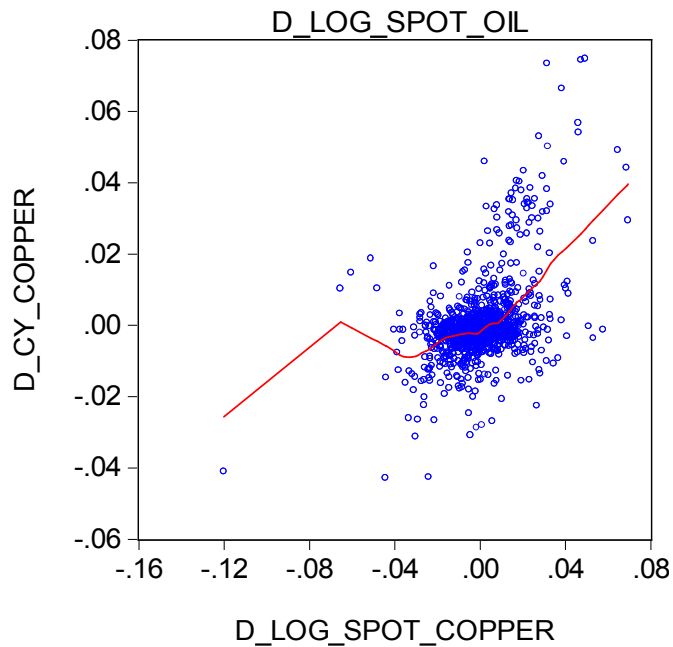
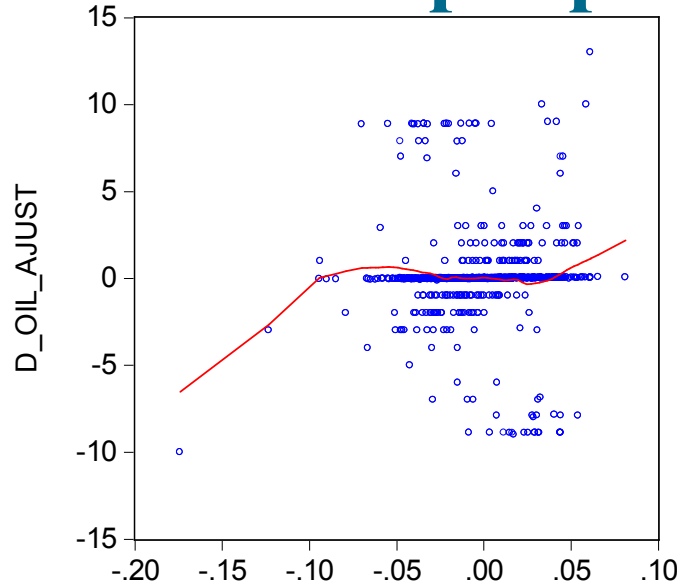
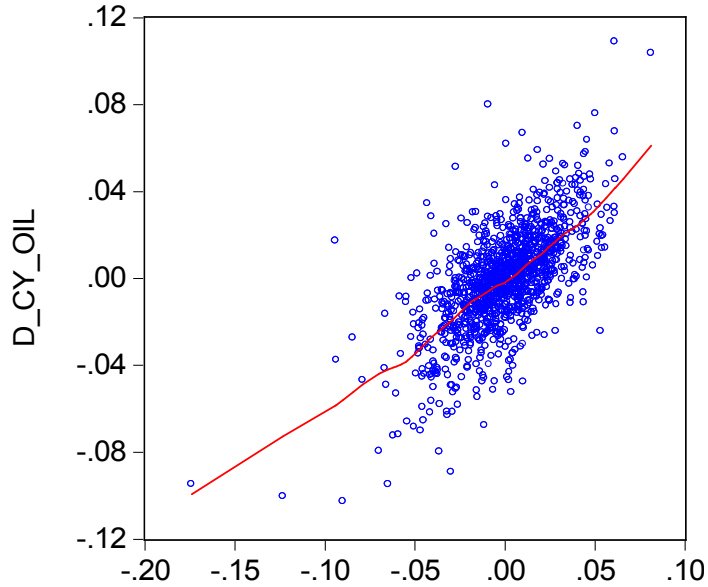
- **$P^{(S)}$  is a probability measure that takes into the convenience yield risk premium as well as the volatility of the spot price;**



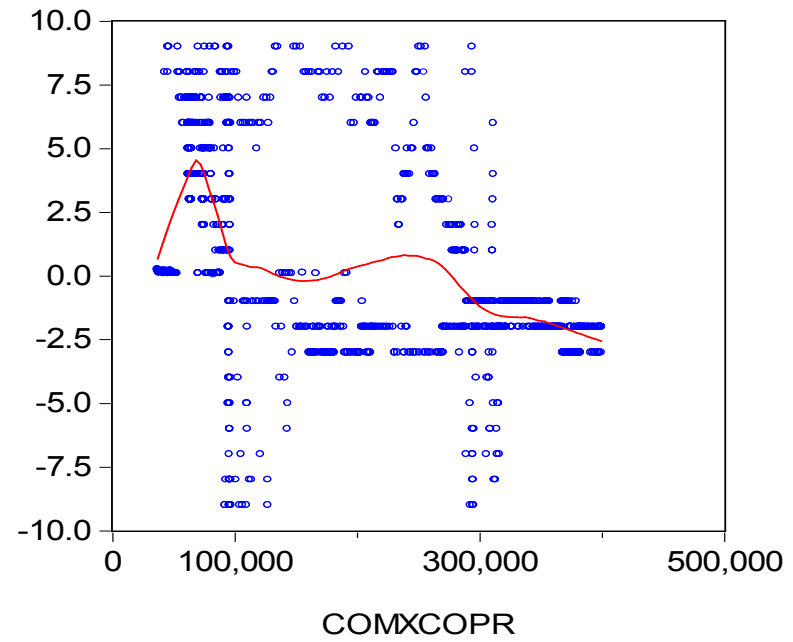
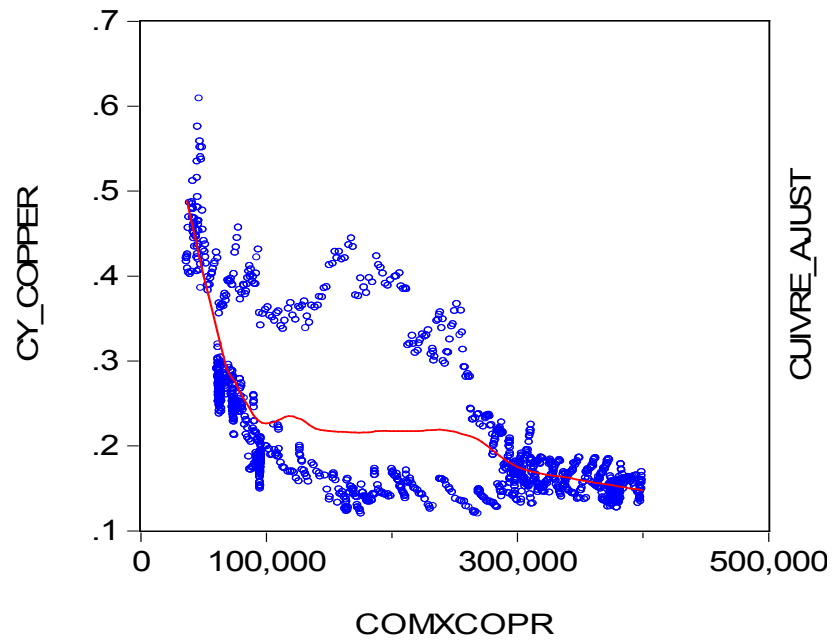
## **CY is model dependent but has an economic rational**

- **The time adjusted basis contains a risk premium in addition to the convenience yield (Considine and Donaldson, 2001);**
- **The convenience yield is not observable and depends on the model:**
  - **However, parameters are often consistent across models and commodities;**
  - **Schwartz and Smith (2000) find similar state variables implied by different but contemporaneous data set;**
  - **=> argue in favor of an economic interpretation of the convenience yield;**

# Link between the CY and the spot price



# Kaldor-Working curve for copper



## Conclusion

- **We justified both theoretically and empirically that the time adjusted basis could not be used as a proxy to study the empirical properties of the convenience yield;**
- **We have shown that (some of) the usual assumed properties of the convenience yield are more robust under the true definition of the convenience yield**
- **Some extensions need to be done:**
  - **To check other properties of the convenience yield, e.g.:**
    - **Spot price convenience yield correlation as a function of inventory;**
    - **Convenience yield volatility as a function of inventory;**
  - **To test robustness with other models of the convenience yield;**

## Conclusion con't

- **We are now considering recent time series:**
  - In light of the financialization of the commodity markets;
  - Does the theory of storage pertain?