Convenience yield and time adjusted basis stylized facts
Motivation and objective of the paper

• Goal of the paper: to revisit the empirical stylized facts of the convenience yield with a more precise definition than in previous empirical studies;

• We only present a preliminary draft for oil and copper commodities;

• Preliminary results: stylized facts studied under our definition are more robust than under the usual empirical measure;
The convenience yield (CY) is still subject to intense debates

- Why studying the convenience yield: The convenience yield is a variable that is still subject to considerable debates (see e.g. Lautier, 2009):
  - Some authors believe in an economic rational at the heart of the definition of this variable, e.g. Brennan, 1991:
    - The convenience yield is (the value of) the flow of services that accrue to the owner of commodity inventory as opposed to the owner of contracts for future delivery;
  - Other authors believe that it is just an ad-hoc variables to statistically reproduce backwardation, e.g. Hull (Options, futures and other derivatives);
Stylized facts involving the CY usually considered

- Main stylized facts under consideration usually (e.g. Routledge et al., 2000; Dincerler et al., 2005):
  - Link between the spot price and the convenience yield:
    - The correlation should be dynamic as a decreasing function of the level of inventory;
  - Link between the convenience yield and the level of inventory:
    - The convenience yield should be a convex decreasing function of inventory: the so-called Kaldor-Working curve;
  - Link between the volatility of the convenience yield and the level of inventory:
    - The volatility should be dynamic as a decreasing function of the level of inventory;
The time adjusted is usually used as a proxy for the CY

- In empirical studies the convenience yield is usually proxied by the time adjusted basis, $b(T)$:
  \[
  b(T) = -\frac{1}{T} \left[ \ln[P(T)F(T)] - \ln[S] \right]
  \]
  - $P$, $F$ and $S$ designates the zero coupon bond price, futures price and spot price, respectively; $T$ is the (time to) maturity of the futures contract;

- This relation is justified by cash and carry arbitrage with an assumed deterministic convenience yield $\delta$:
  \[
  F(T) = \frac{S}{P(T)} \exp(-\delta*T)
  \]
Preliminary Data Set

- Stochastic Behavior of the CY

- The convenience yield and spot price state variables are obtained by the model of Casassus and Colin-Dufresne (2005):
  - This model is a conditionally Gaussian affine model with stochastic risk premia;
  - Data from 2000 to 2004;
Motivation and objective of the paper:
Spot and CY from CCD model
A stochastic CY leads to a more complex relation with the basis

- The assumption of a deterministic convenience yield can certainly not be made!

- Under a dynamic convenience yield, the following relation pertains between the time adjusted basis and the convenience yield:

\[
\exp(b_t(T)) = E_t^{P(S)} \left[ \exp \left( \int_t^T \delta_u du \right) \right]
\]

- \(P^{(S)}\) is a probability measure that takes into the convenience yield risk premium as well as the volatility of the spot price;
CY is model dependent but has an economic rational

- The time adjusted basis contains a risk premium in addition to the convenience yield (Considine and Donaldson, 2001);

- The convenience yield is not observable and depends on the model:
  - However, parameters are often consistent across models and commodities;
  - Schwartz and Smith (2000) find similar state variables implied by different but contemporaneous data set;
  - => argue in favor of an economic interpretation of the convenience yield;
Link between the CY and the spot price
Kaldor-Working curve for copper
Conclusion

• We justified both theoretically and empirically that the time adjusted basis could not be used as a proxy to study the empirical properties of the convenience yield;

• We have shown that (some of) the usual assumed properties of the convenience yield are more robust under the true definition of the convenience yield

• Some extensions need to be done:
  – To check other properties of the convenience yield, e.g.:
    • Spot price convenience yield correlation as a function of inventory;
    • Convenience yield volatility as a function of inventory;
  – To test robustness with other models of the convenience yield;
Conclusion con’t

• We are now considering recent time series:
  – In light of the financialization of the commodity markets;
  – Does the theory of storage pertain?