Volatility modelling of log returns of EU CO_2 emission allowances with regime switching GARCH models

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Motivation (1)

- EU emission allowances (EUA) are a new class of assets with their own characteristics deserving their own approach
- market for EUAs is constantly growing
- risk in energy sector is mainly linked with high volatility of prices
- short-term modeling important for risk management and hedging strategies
- both traders and emitting companies need reliable price forecasts

Motivation (2)

- few publications on econometric modeling of logreturns and volatility of EUA prices due to lack of historical data
- existing studies focus on short time series with known break points and on long-term relationships (e.g. Chevallier, 2009; Hintermann, 2010)
- focus on first trading period (2005-2007)
- studies apply either GARCH or regime switching models

Motivation (3)

- Paolella and Taschini (2008) use mixed GARCH models
- mixed GARCH models do not fit the recent data
- Benz and Trück (2009) apply both GARCH models and regime switching models
- GARCH models have high volatility persistence
- regime switching models do not capture the conditional heteroskedastidity of the series
- several authors suggest the use of regime switching GARCH models (e.g. Paolella and Taschini, 2008)

Contribution

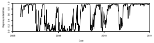
- analysis of short-term spot price behaviour
- MS-ARCH models introduced by Hamilton and Susmel (1994)
- problem of path-dependence with MS-GARCH solved by Klaassen (2002)
- MS-GARCH captures both shifts in volatility and volatility clustering, as observed in data
- apply Benz and Trück's approach (2009) and extend by
  1. using MS-GARCH models
  2. using data from phase II instead of phase I
  3. using spot market prices instead of OTC data

Outline

1. Motivation ✓
2. Contribution ✓
3. EU ETS and CO_2 emission trading
4. Methodology
5. Empirical analysis
6. Estimation results
7. Forecasting
8. Conclusion

EU Emission Trading System

- cap-and-trade system to reduce greenhouse gas emissions and meet Kyoto Protocol emission targets
- EU Emissions Trading System (ETS) entered into force in January 2005
- created a new market for CO_2 allowances
- three trading periods
  - (i) EU-ETS I, 2005-2007, trial period
  - (ii) EU-ETS II, 2008-2012, period under consideration
  - (iii) EU-ETS III, 2013-2020, auctioning replaces free allocation

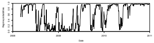


EU carbon market

- EU ETS is the world's largest carbon market
- EU Emission Allowances (EUAs) are traded on several exchanges, amongst others on Bluenext, Climex, European Energy Exchange, Green Exchange, Intercontinental Exchange and Nord Pool

Year	Number of EUA (in bn.)	Traded value (in USD bn.)	Year	Number of EUA (in bn.)	Traded value (in USD bn.)
2005	0.3	7.9	2009	6.3	118.5
2006	1.1	24.4	2010	6.8	133.6
2007	2.1	49.1	2011	7.9	147.8
2008	3.1	100.5			

Table 1: Total trade volumes of EUAs on the aforementioned exchanges
Source: World Bank, 2012

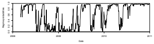


Characteristics of EU carbon market in Phase II

- EUAs are allocated to installations free of charge
- allowances can only be used during the commitment period
- prices are determined by expected market supply and demand
- firms can influence their demand by abatement
- changes in policies influence short-term supply and demand
- CO_2 production depends on the weather, fuel prices and economic growth
- EAU can be considered a commodity

Models

- several models considered in order to provide benchmarks for comparing performance of the regime switching GARCH models
- estimated models: Normal distribution, AR, GARCH, AR-GARCH with and without regime switching
- GARCH allows for conditional variance
- regime switching models allow for periods with different stochastic processes



Regime switching model (1)

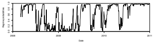
Markov regime switching model (Hamilton, 1990)

- modeling breaks in time series (e.g. policy changes)
- different model specifications for each regime or state
- current regime determined by latent variable
- we consider models with 2 states with state space

$$\mathcal{S} = \{1, 2\} \tag{1}$$

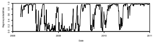
- s_t, the state at time t, is a realization of two-state Markov chain with transition probability

$$p_{ij} = \mathsf{P}(s_t = j | s_{t-1} = i) \tag{2}$$



Regime switching model (2)

- current state depends only on most recent state due to Markov property
- inference on s_t can only be made through the observations of y_t, as s_t is not observable
- two sources of uncertainty: the latent state and the model specification in each state
- estimation of Markov Switching model as in Hamilton (1990)
  ▸ Appendix I - Estimation of MS Model
- use previous models for model specifications in regimes



MS-GARCH model

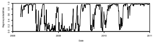
- several specifications of MS-GARCH models in the literature
- MS-GARCH models solve problem of volatility persistence
- most specifications show the problem of path dependence in the variance equation, which makes estimation intractable
- we apply the model according to Klaassen (2002), which has several advantages:
  - (i) conditional variance specification is not path dependent
  - (ii) allows for recursive estimation alghorithm using maximum likelihood estimation ▸ Appendix II - Estimation of MS-GARCH
  - (iii) allows for recursive forecasting

Data

- data from Bluenext from EU ETS II, as this is the exchange with the highest trading volume
- retrieved from Bloomberg, ticker PNXCSPT2
- time series from February 26, 2008 until November 28, 2012
- 2008 - 2010 for parameter estimation
- 2011 - 2012 for out-of-sample forecasting
- analysis performed on the log returns

$$y_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \tag{3}$$

where p_t is the spot price of EUA at time t

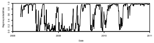


Summary statistics

- prices are skewed, log returns less skewed
- both prices and log returns show excess kurtosis

period	N	Mean	Median	Min	Max	Std Dev	Skew	Kurt
				Prices				
2008-2012	1182	14.016	13.940	6.040	28.730	5.071	0.76	3.32
2008-2010	724	16.273	14.660	7.960	28.730	4.581	1.09	2.99
2011-2012	458	10.433	8.565	6.040	16.930	3.505	0.61	8.44
				Log returns				
2008-2012	1182	-0.0009	0	-0.1081	0.2038	0.0276	0.03	8.03
2008-2010	724	-0.0006	0	-0.1029	0.1055	0.0244	-0.20	5.02
2011-2012	458	-0.0015	-0.0011	-0.1081	0.2038	0.0320	0.61	8.84

Table 2: Summary statistics for daily prices and daily log returns



Prices and log returns

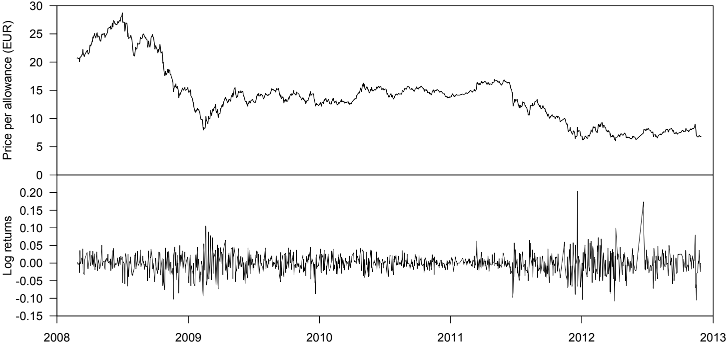
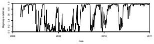


Figure 1: EUA spot prices (upper panel) and log returns (lower panel) from February 26, 2008 until November 28, 2012

Stationarity tests

period	test statistic	p-value	lags
Augmented Dickey-Fuller test			
2008-2012	-7.437	$<$0.01	22
2008-2010	-5.321	$<$0.01	20
2011-2012	-4.879	$<$0.01	17
KPSS			
2008-2012	0.069	$>$0.1	7
2008-2010	0.108	$>$0.1	6
2011-2012	0.071	$>$0.1	4

Table 3: Results of the Augmented Dickey-Fuller and KPSS tests for stationarity

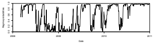


i.i.d. Normal and AR

AR(4), optimal lag order according to AIC criteria
(G)ARCH effects in residuals of AR model significant by LM test

Parameter	Coefficient i.i.d. Normal	Coefficient AR(4)
μ	-0.0006	–
c	–	-0.0006
ϕ_1	–	0.0988
ϕ_2	–	-0.1391
ϕ_3	–	0.0795
ϕ_4	–	0.0609
$\mathrm{E}[y_t]$	-0.0006	-0.0006
σ	0.0244	0.0240

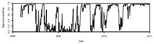
Table 4: Parameter estimates of i.i.d. Normal and AR models



GARCH and AR-GARCH

Parameter	Coefficient	Coefficient
	GARCH(1,1)	AR(4)-GARCH(1,1)
	Mean equation	
c	-0.0003	-0.0002
ϕ_1	–	0.0031
ϕ_2	–	-0.0696
ϕ_3	–	0.0550
ϕ_4	–	0.0199
$E[y_t]$	-0.0003	-0.0002
	Variance equation	
α_0	0.0000	0.0000
α_1	0.0726	0.0697
β_1	0.9199	0.9214
$E[\sigma_t]$	0.0239	0.0247

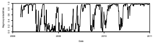
Table 5: Parameter estimates of GARCH and AR-GARCH models



MS-Normal and MS-AR (1)

	MS-Gaussian		MS-AR(4)	
Regime (i)	1 (low)	2 (high)	1 (low)	2 (high)
μ_1	0.0014	-0.0037	–	–
σ_i	0.0161	0.0336	0.0159	0.0324
c	–	–	0.0017	-0.0033
ϕ_1	–	–	-0.0597	0.1647
ϕ_2	–	–	-0.0662	-0.1947
ϕ_3	–	–	0.0086	0.1116
ϕ_4	–	–	-0.0870	0.1078
Markov estimates				
p_{ii}	0.9864	0.9749	0.9818	0.9698

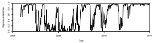
Table 6: Parameter estimates of Markov switching i.i.d. and AR models



MS-Normal and MS-AR (2)

	MS-Gaussian		MS-AR(4)	
Regime (i)	1 (low)	2 (high)	1 (low)	2 (high)
Unconditional expectations				
$E[y_{t,i}]$	0.0014	-0.0037	0.0014	-0.0041
$E[\sigma_{t,i}]$	0.0161	0.0336	0.0159	0.0324
Markov estimates				
$P(s_t = i)$	0.6486	0.3514	0.6240	0.3760

Table 7: Unconditional expectations of mean, standard deviation and state probabilities for Markov switching i.i.d. and AR models



MS-AR

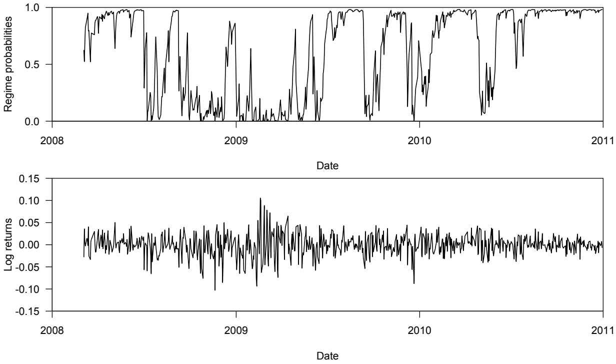
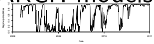


Figure 2: Estimated probabilities to be in the 'low' state for MS-AR(4) (upper panel) model and log returns (lower panel)

MS-GARCH and MS-AR-GARCH(1)

	MS-GARCH(1,1)		MS-AR(4)-GARCH(1,1)	
Regime (i)	1 (low)	2 (high)	1 (low)	2 (high)
		Mean equation		
c	0.0009	-0.0042	0.0011	-0.0090
ϕ_1	–	–	-0.0339	0.3013
ϕ_2	–	–	-0.0637	-0.2108
ϕ_3	–	–	0.0261	0.1965
ϕ_4	–	–	-0.0315	0.2512
		Variance equation		
α_0	0.0001	0.0003	0.0000	0.0002
α_1	0.0013	0.1038	0.0078	0.1952
β	0.7166	0.7233	0.8645	0.7510
		Markov estimates		
p_{ii}	0.9923	0.9821	0.9740	0.8818

Table 8: Estimates of Markov switching GARCH and AR-GARCH models



MS-GARCH and MS-AR-GARCH(2)

	MS-GARCH(1,1)		MS-AR(4)-GARCH(1,1)	
Regime (i)	1 (low)	2 (high)	1 (low)	2 (high)
Unconditional expectations				
$\mathrm{E}[y_{t,i}]$	0.0009	-0.0042	0.0010	-0.0218
$\mathrm{E}[\sigma_{t,i}]$	0.0136	0.0409	0.0101	0.0707
Markov estimates				
$\mathrm{P}(s_t = i)$	0.6988	0.3012	0.8198	0.1802

Table 9: Unconditional expectations of mean, standard deviation and state probabilities for Markov switching GARCH(1,1) and AR(4)-GARCH(1,1) model

MS-GARCH

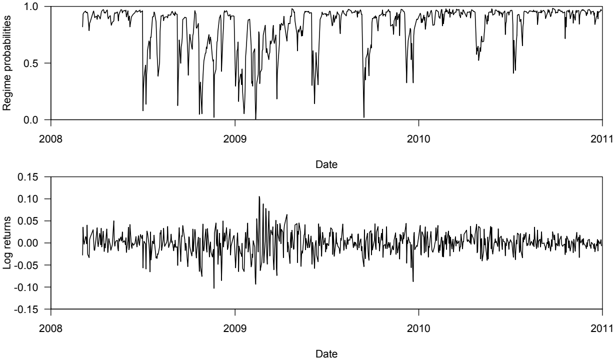
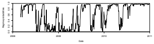


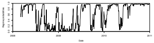
Figure 3: Estimated probabilities to be in the 'low' state for MS-AR(4)-GARCH(1,1) model (upper panel) and log returns (lower panel)



Comparison of in-sample estimation results

model	number of parameters	log likelihood	AIC
i.i.d. Normal	2	1651.06	-3298.11
AR(4)	6	1673.85	-3335.69
GARCH(1,1)	4	1732.45	-3456.89
AR(4)-GARCH (1,1)	8	1735.33	-3454.67
MS i.i.d.	6	1720.00	-3408.99
MS-AR(4)	14	1732.92	-3437.84
MS-GARCH(1,1)	10	1739.21	-3458.43
MS-AR(4)-GARCH(1,1)	18	1750.94	-3465.87

Table 10: Number of parameters, maximum log likelihood value and Akaike Information Criteria (AIC) for the estimated models



Forecasting log returns and volatility

Point forecasts

- out-of-sample 1-day-ahead forecast with recursive window estimation
- comparison of performance by mean absolute error (MAE) and mean squared error (MSE)

Density forecasts

- out-of-sample 1-day-ahead forecast with recursive window estimation
- allows to construct forecasted confidence intervals
- comparison of performance by performing a distributional test (Diebold et al., 1998) Appendix III - Distributional test

Comparison of out-of-sample results

Small differences in MAE and MSE

model	MAE	MSE	KS	p-value KS
i.i.d. Normal	0.02226	0.0010263	0.4737	<2.2e-16
AR(4)	0.02244	0.0010583	0.0469	0.2657
GARCH(1,1)	0.02230	0.0010282	0.0536	0.1446
AR(4)-GARCH (1,1)	0.02231	0.0010391	0.0501	0.2005
MS i.i.d.	0.02234	0.0010266	0.0367	0.5695
MS-AR(4)	0.02260	0.0010407	0.0346	0.6419
MS-GARCH(1,1)	0.02232	0.0010254	0.0321	0.7314
MS-AR(4)-GARCH(1,1)	0.02229	0.0010268	0.0370	0.5592

Table 11: Mean absolute error (MAE) and mean squared error (MSE) for point forecasts and Kolmogorov-Smirnov (KS) test for density forecasts

Density forecasts (1)

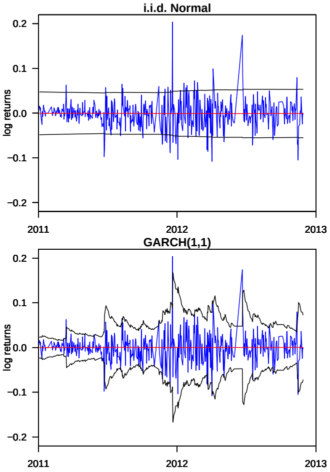


Figure 4: Forecasted confidence intervals, point forecasts and true values

Density forecasts (2)

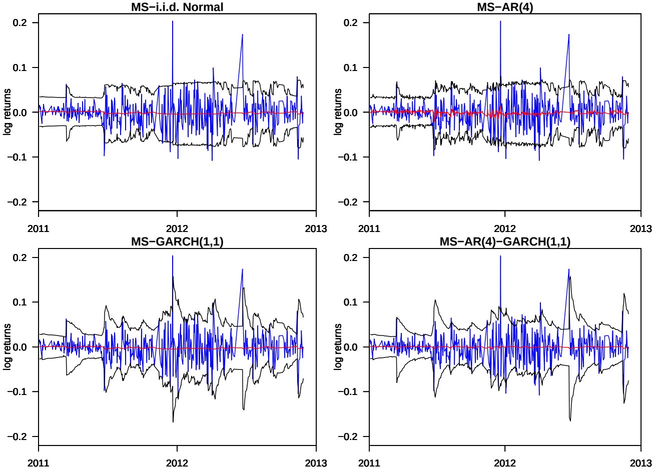


Figure 5: Forecasted confidence intervals, point forecasts and true values

Kernel density plots (1)

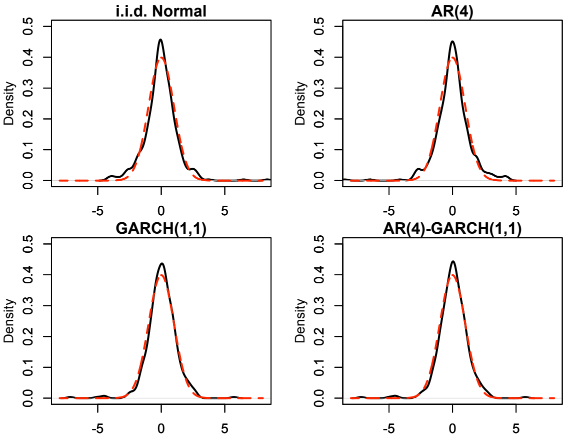


Figure 6: Kernel density plots of standardised forecast errors and Normal densities

Kernel density plots (2)

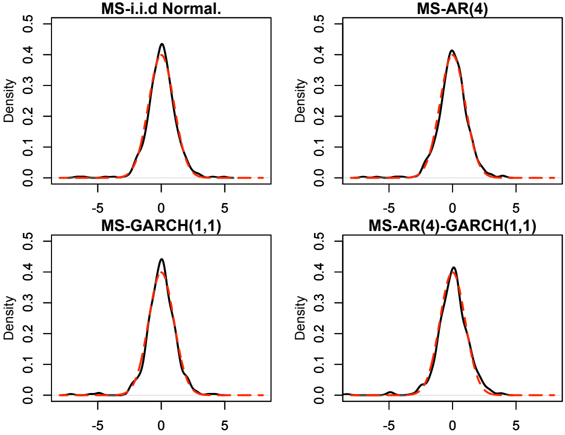


Figure 7: Kernel density plots of standardised forecast errors and Normal densities

Conclusion

- data justify use of MS-GARCH models
- best in-sample fit by MS-AR(4)-GARCH(1,1) model
- MS-GARCH models have best out-of-sample density forecasts
- MS models distinguish well between states
- changes in regime and volatility structure capture series well
- MS-GARCH models solve the problem of variance persistence faced by the GARCH models
- MS-GARCH performs best for volatility forecasting and risk management

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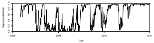

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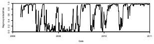

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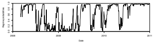


Appendix I - Estimation of MS Model (1)

- probability of being in state j at time t is

$$\xi_{jt} = \mathsf{P}(s_t = j|\Omega_t;\theta) \tag{4}$$

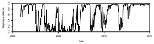
where $\Omega_t = \{y_t, y_{t-1}, \dots, y_1\}$ and θ is the parameter vector

- inference on the state probailites ξ_{jt} is performed iteratively by evaluating the density η_{jt} under both regimes

$$\eta_{jt} = g_j(y_t|s_t = j, \Omega_{t-1};\theta) \tag{5}$$

where g_j is the density function of the process in state j

◂ Markov Switching Model



Appendix I - Estimation of MS Model (2)

Knowing $\xi_{i,t-1}$ the conditional density of the observation y_t is

$$f(y_t|\Omega_{t-1};\theta) = \sum_{i=1}^{2}\sum_{j=1}^{2} p_{ij}\xi_{i,t-1}\eta_{jt} \tag{6}$$

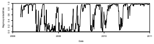
and the probability to be in state j at time t is

$$\xi_{jt} = \frac{\sum_{i=1}^{2} p_{ij}\xi_{i,t-1}\eta_{jt}}{f(y_t|\Omega_{t-1};\theta)} \tag{7}$$

This yields the conditonal log likelihood of the observed data

$$\ell(y_1, y_2, \ldots, y_T|y_0;\theta) = \sum_{t=1}^{T} \ln f(y_t|\Omega_{t-1};\theta) \tag{8}$$

◂ Markov Switching Model



Appendix II - Estimation of MS-GARCH model

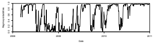
The variance specification for the MS-GARCH model according to Klaassen (2002) integrates out the path dependence by using the law of iterated expectations.

The variance of y_t evaluated at time $t-1$ is described by

$$\begin{aligned} \mathrm{Var}_{t-1}(y_t|s_t = j) &= \qquad Var_{t-1}(\varepsilon_t|s_t = j) \\ &= \alpha_{0j} + \alpha_{1j}\varepsilon_{t-1} + \beta_{1j}\, \mathrm{E}_{t-1}\left[\mathrm{Var}_{t-2}(\varepsilon_{t-1}|s_{t-1})\right] \end{aligned}$$

The model is estimated by a differential evolution algorithm.

◂ MS-GARCH model



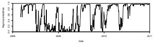
Appendix III - Distributional test to evaluate density forecasts

- forecast of the distribution of y_{t+1} is

$$y_{t+1} \sim \mathrm{N}\left(\widehat{\mu}, \widehat{\sigma^2}\right) \tag{9}$$

where $\widehat{\mu}$ is the point forecast and $\widehat{\sigma^2}$ the forecasted variance.
- if this is the correct distribution with forecasted density function $\hat{f}(y_{t-1})$ and distribution function $\hat{F}(y_{t-1})$, then $\hat{F}(y_{t-1})$ is normally distributed (Diebold et al., 1998)
- the density forecast can be evaluated by testing u_{t+1} for uniformity by using for example the Kolmogorov-Smirnov test

◂ Forecasting log returns and volatility



Appendix VI - Normal distribution, AR

Normal distribution

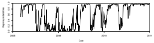
$$y_t = \mu + \varepsilon_t \tag{10}$$

where $\varepsilon_t \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma^2\right)$

AR(k)

$$y_t = c + \sum_{h=1}^{k} \phi_h y_{t-k} + \varepsilon_t \tag{11}$$

where $\varepsilon_t \overset{\text{iid}}{\sim} \mathrm{N}\left(0, \sigma^2\right)$



Appendix V - GARCH and AR-GARCH

GARCH(p,q) (Bollerslev, 1986)

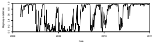
$$y_t = c + \varepsilon_t \sigma_t \tag{12}$$

where $\varepsilon_t \overset{\text{iid}}{\sim} \mathrm{N}(0,1)$ and $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$

AR(k)-GARCH(p,q)

$$y_t = c + \sum_{h=1}^{k} \phi_h y_{t-k} + \varepsilon_t \sigma_t \tag{13}$$

where $\varepsilon_t \overset{\text{iid}}{\sim} \mathrm{N}(0,1)$ and $\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$



Appendix VI - Q-Q plots of residuals (1)

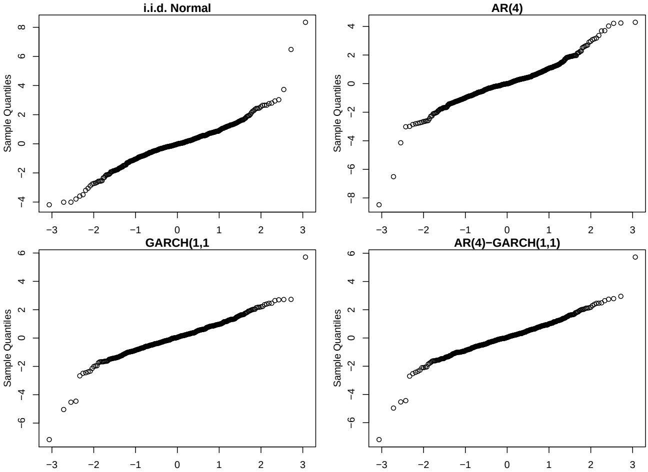


Figure 8: Q-Q plots of the standardised forecast errors

Appendix VII - Q-Q plots of residuals (2)

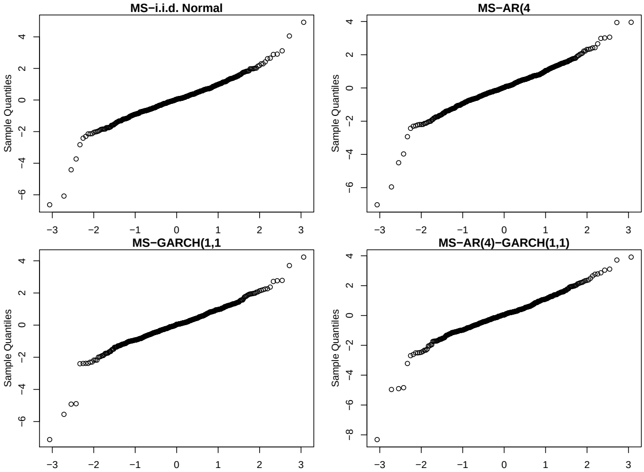


Figure 9: Q-Q plots of the standardised forecast errors