A Cointegrated Spot-Swap Model for Power Prices:
A regime Switching Approach with Stochastic Volatility and Jumps

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Outline of the Presentation

- Introduction of the model
- Estimation and Data
- An application of the model: Option pricing
- Summary

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Introduction of the model

- The spot price and the swap prices are cointegrated.

- The spot price follows a regime switching model with jumps.

- The spot price dynamics consists of three regimes:
  1. A normal regime with no jumps in the spot price
  2. A regime with up jumps
  3. A regime with down jumps

- The swap dynamics has stochastic volatility to capture smiles/smirks in the swaption market.

Introduction of the model: The spot dynamics in the normal regime

The spot price dynamics in the normal regime is given by:

$$dS_N(t) = S_N(t)\{-\kappa Z(t)dt + \sigma dB_1(t)\}$$

$$Z(t) \equiv \xi(t) + \ln(S_N(t)) - \ln(F(t, t+\tau))$$

where B is an independent Brownian motion and ξ is a determinsic function. F is the price of a monthly swap with delivery starts at time t+τ.

Introduction of the model: The spot price dynamics in the jump regimes

In the up-spike regime, U, the spot dynamics is given by

$$\ln S_U(t) = \ln F(t, t+\tau) + X_U(t)$$

and in the down-spike regime, D, the dynamics is

$$\ln S_D(t) = \ln F(t, t+\tau) + X_D(t)$$

Jumps are induced via the compound Poission processes

$$X_U(t) = \sum_{i=1}^{1+P_U(t)} J_{i,U} \quad \text{and} \quad X_D(t) = \sum_{i=1}^{1+P_D(t)} J_{i,D}$$

where

$$P_k(t) \sim \mathrm{Po}(\lambda_k) \quad \text{and} \quad J_{i,k} \sim \mathrm{N}(\mu_k, \sigma_k) \quad \text{for} \quad k \in \{U, D\}$$

Introduction of the model: The swap dynamics

The swap price is defined by

$$d\ln F(t,T) = \left(\mu(t,T) - \frac{1}{2}\left[\sigma(t,T)\right]^2 V(t)\right) dt + \sigma(t,T)\sqrt{V(t)}dB_2(t)$$

with the squared volatility process given by

$$dV(t) = \kappa_V\left(\theta_V - V(t)\right) dt + \sigma_V\sqrt{V(t)}dB_3(t)$$

The two Browian motions are allowed to be correlated and and σ is the deterministic function

$$\sigma(t,T) = \gamma + \frac{\alpha}{\beta\tau_M}\left(e^{-\beta(T-t)} - e^{-\beta(T+\tau_M-t)}\right)$$

T is the start of the delivery period and τ is the length of the swapping period.

Estimation and Data

- The model is estimated using Nordic electricity spot, swap and swaption prices.
- The spot mean reversion level (syntethic rolling swap price) is approximated with the front month swap price.
- The time-continous spot price process is discretized to daily time steps.
- The spot dynamics:

Method:	Maximum Likelihood
Data:	Spot and swap prices
Sample period:	2006 – 2011

- The swap dynamics:

Method:	Non linear least squares
Data:	Swap and swaption prices
Sampe period:	2010

The risk neutral dynamics

- The option prices are priced under the risk neutral measure.
- We need the risk neutral dynamics of the spot price.
- The swap price dynamics under the risk neutral measure were calibrated with the swaption prices.
- The spot price is unhedgeable and has no unique risk neutral spot price dynamics.
- The spot price dynamics was estimated under the historical measure.
  - We need to make assumptions about the risk neutral dynamics of the spot price.
    - We assume that the only risk which is priced is the continous shocks in the normal regime i.e. the Brownian motion.

$$dS_N(t) = S_N(t)\left\{\left(-\kappa Z(t) - \sigma\zeta_1(t)\right)dt + \sigma dB_1^{\mathbb{Q}}(t)\right\}$$

    - The price of risk ς is assumed to be both time varying and stochastic such that

$$E_t^{\mathbb{Q}}\left[S^*(u)du\right] = F(t,T,T)$$

for all T.

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Pricing European styled spot options

The log spot price is not of affine form and hence no Fourier based pricing methods can be used without any approximations (see Duffie et al 2001)

- Reason: The rolling swap price in combination with stochastic volatility

- Options are priced with Monte Carlo simulations

- Options can be priced with Fourier based methods by approximating the rolling swap contract with a fixed contract.

- The approximation error is very small because the mean reversion speed of the spot price is high.

- Choose the fixed contract which corresponds to the rolling contract at the "delivery" date of spot.

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Pricing European styled spot options (cont'd)

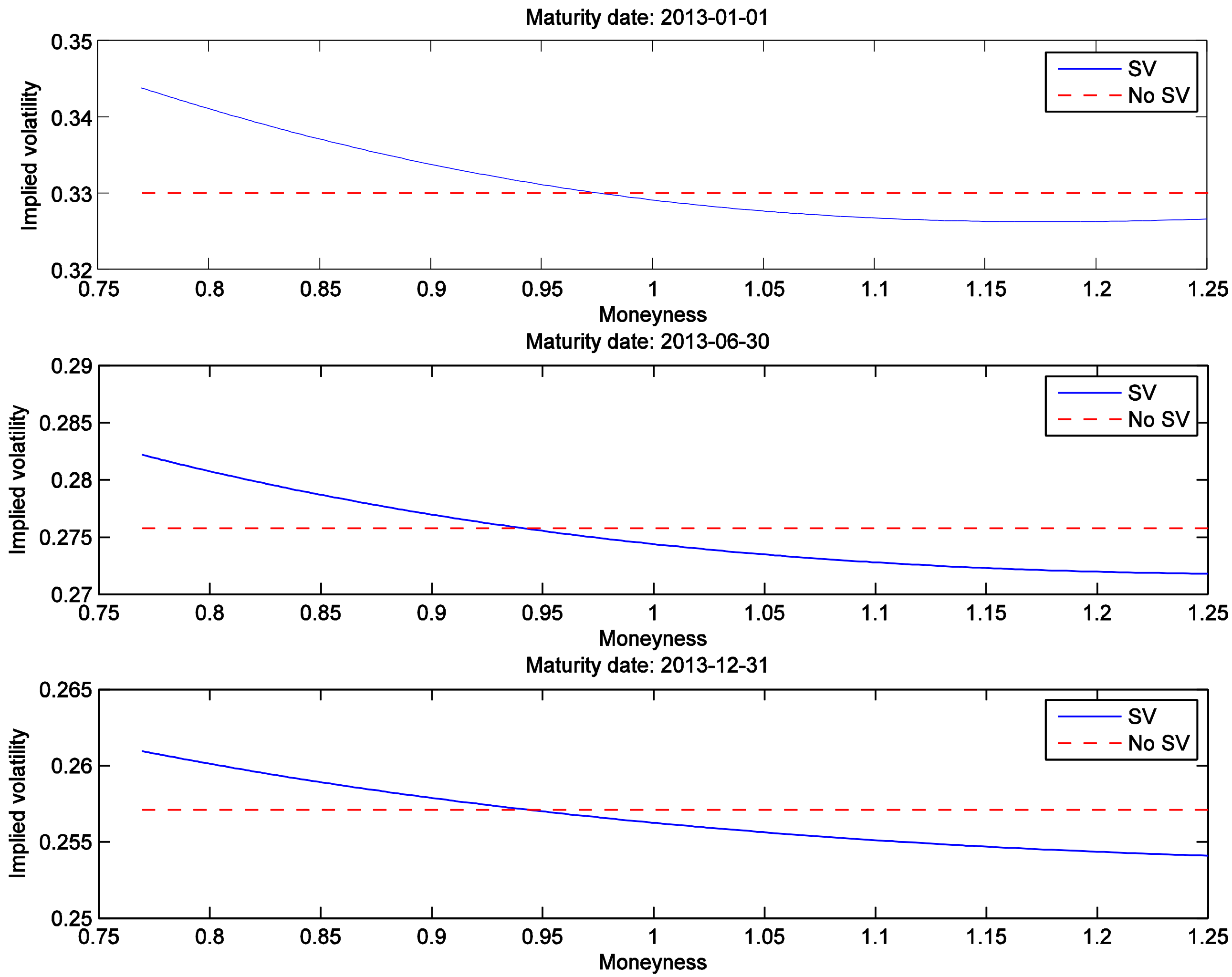
- Pricing method: Monte Carlo simulations
  - The risk premium ς needs to be defined which is difficult.
  - We approximate ς by scaling the simulated prices such that the average simulated spot price is equal its forward price

- Pricing method: Fourier based methods (approximation of the mean reversion level)
  - The risk premium can be backed out analytically.

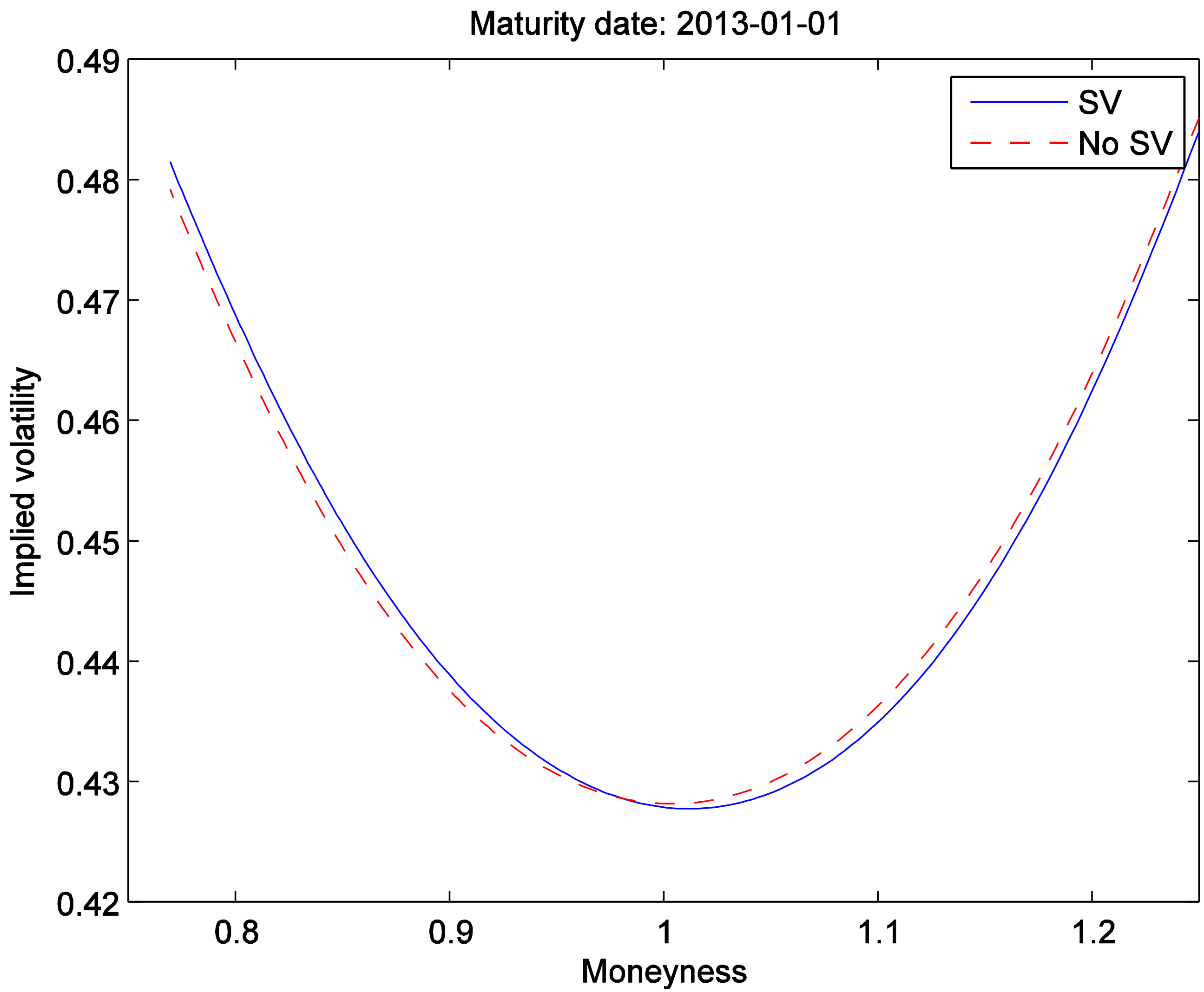
The impact of the swaption smile on spot options:

- The impact of the swaption smile on spot options is relative small.
- The impact of the swaption smile is increasing by time to maturity.
- For strips of independent daily call options (virtual power plants (VPPs)) the impact is even smaller as the VPPs consist of many in-, at- and out-of-the money options.

The implied volatility of swaptions



The implied volatility of spot options: 1 month to maturity



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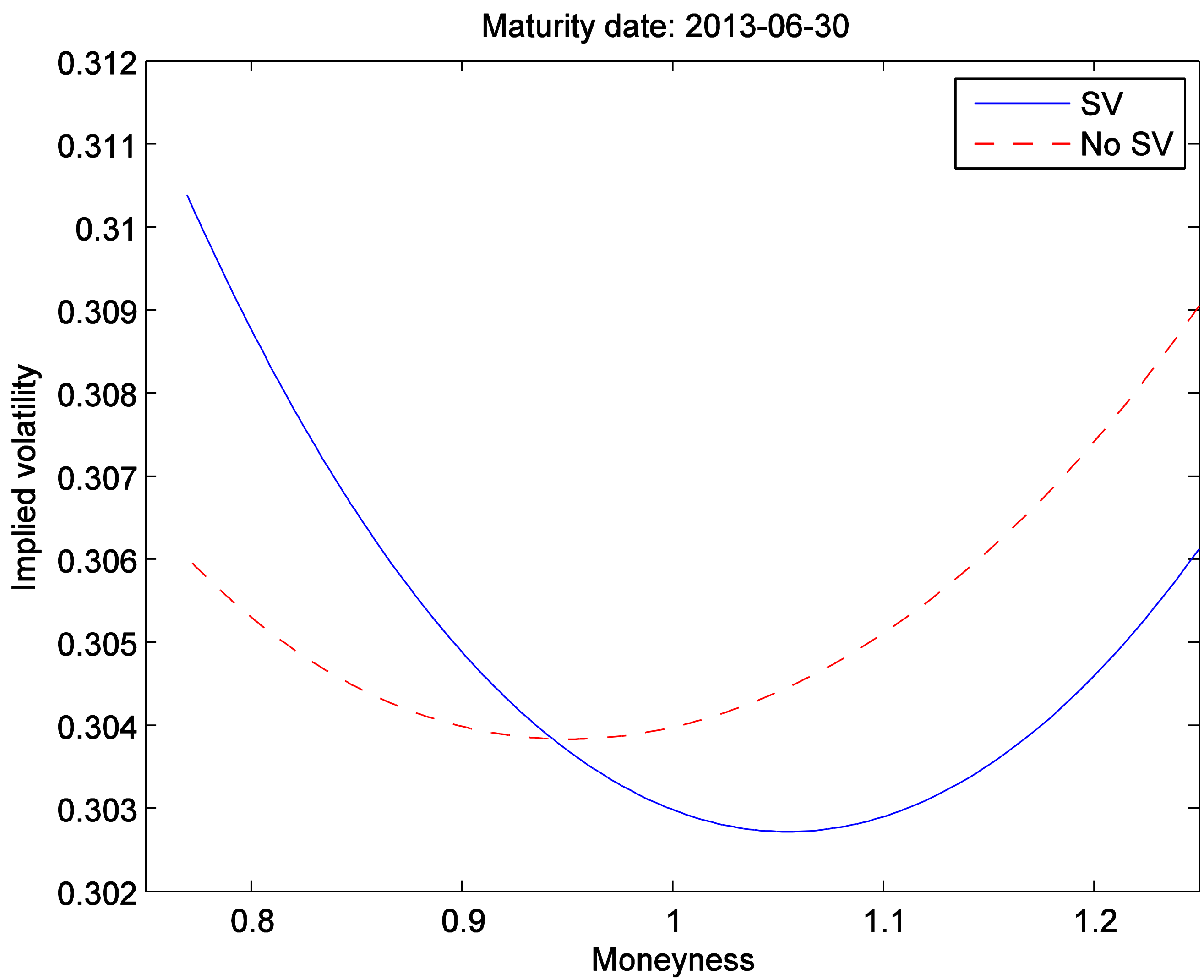
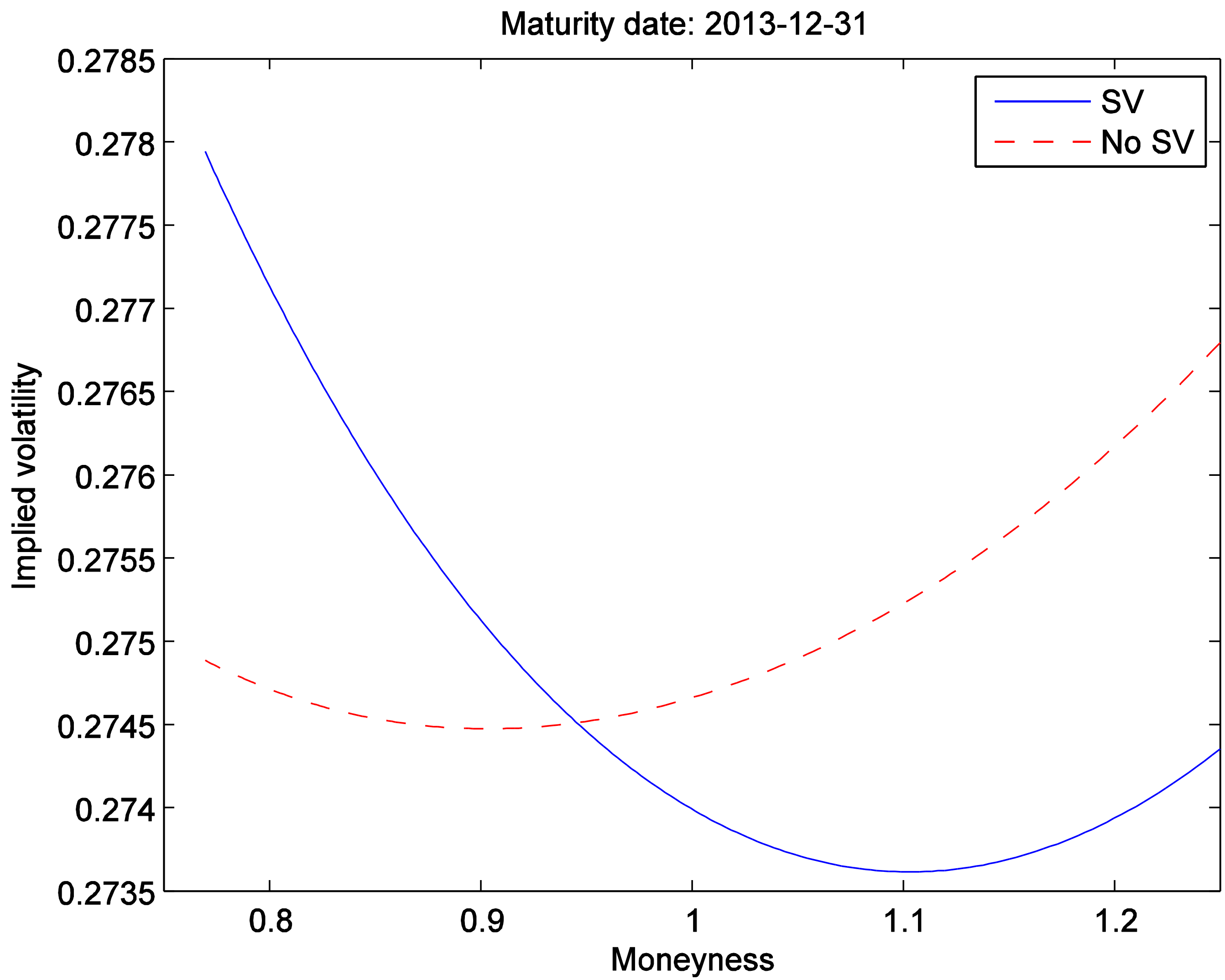
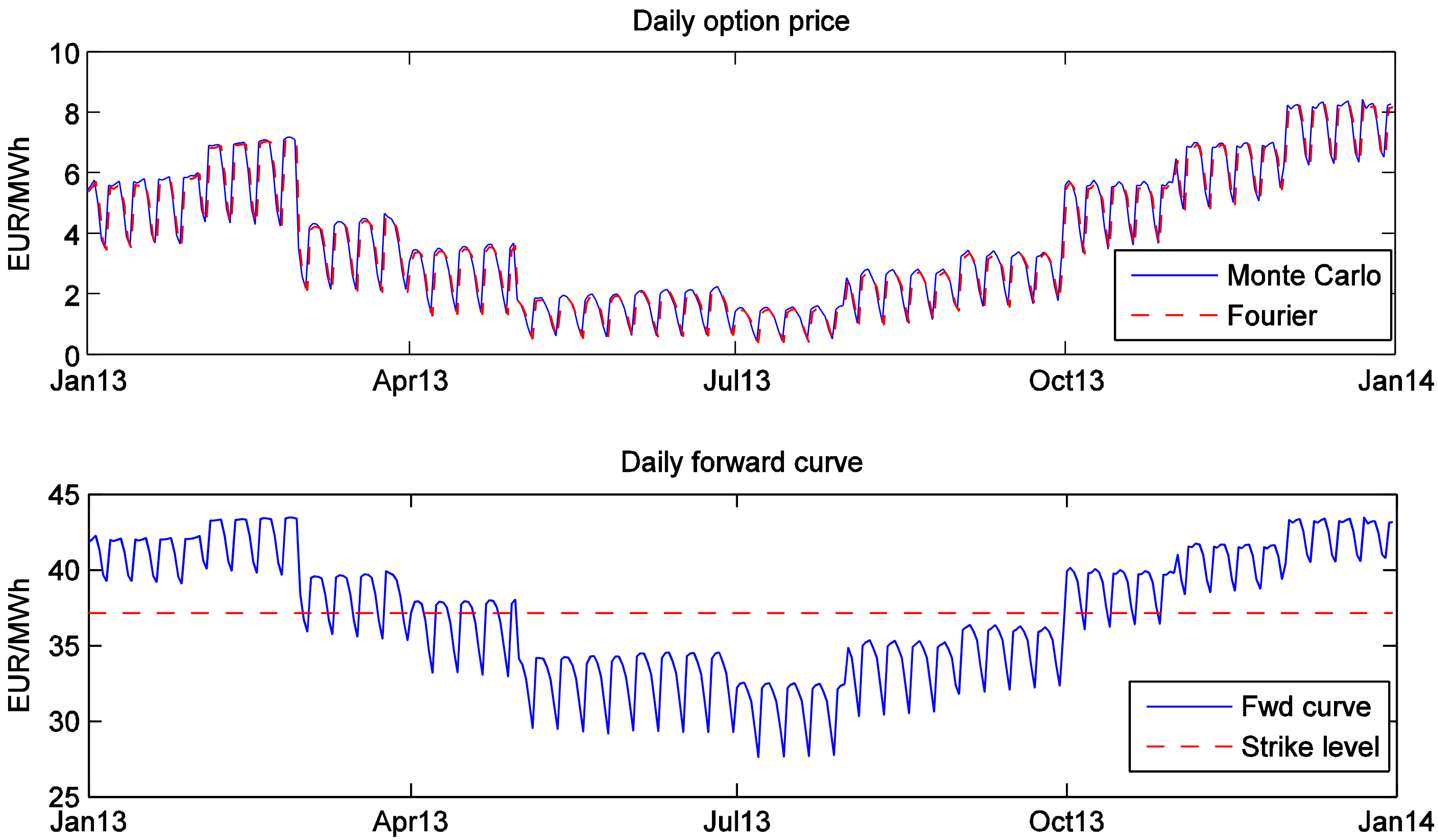
The implied volatility of spot options: 7 months to maturity



The implied volatility of spot options: 13 month to maturity



Valuation of a VPP



Summary

- The model features several important characteristics:
  - The volatility smile in the swaption market
  - Regime switching and jumps in the spot market
  - Cointegration between the spot and the swap prices
- The model parameters are easy to estimate as you can estimate the spot and swap parameters separately.
- European styled options on the spot price can succesfully be approximated and efficiently priced with Fourier based methods.
- Stochastic volatility in the swaption market has no significant effect on the valuation of VPPs.

Estimation results of the spot parameters

Parameter	Estimated value	Standard error
ξ	0.0366	0.0040
κ	67.1445	2.7850
σ	0.8052	0.0144
λ_U	0.2297	0.1077
λ_D	0.6790	0.5884
μ_U	0.1510	0.0122
μ_D	-0.1526	0.0596
σ_U	0.0574	0.0069
σ_D	0.1302	0.0374
π^{NU}	0.0369	0.0089
π^{ND}	0.0087	0.0065
π^{UN}	0.3407	0.0464
π^{DN}	0.1398	0.0366

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