A Cointegrated Spot-Swap Model for Power Prices: A regime Switching Approach with Stochastic Volatility and Jumps

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Outline of the Presentation

- Introduction of the model
- Estimation and Data
- An application of the model: Option pricing
- Summary
Introduction of the model

- The spot price and the swap prices are cointegrated.
- The spot price follows a regime switching model with jumps.
- The spot price dynamics consists of three regimes:
  1. A normal regime with no jumps in the spot price
  2. A regime with up jumps
  3. A regime with down jumps
- The swap dynamics has stochastic volatility to capture smiles/smirks in the swaption market.
Introduction of the model: The spot dynamics in the normal regime

The spot price dynamics in the normal regime is given by:

\[ dS_N(t) = S_N(t) \left\{ -\kappa Z(t) dt + \sigma dB_1(t) \right\} \]

\[ Z(t) \equiv \xi(t) + \ln(S_N(t)) - \ln(F(t, t + \tau)) \]

where \( B \) is an independent Brownian motion and \( \xi \) is a deterministic function. \( F \) is the price of a monthly swap with delivery starts at time \( t + \tau \).
In the up-spike regime, $U$, the spot dynamics is given by

$$\ln S_U(t) = \ln F(t, t + \tau) + X_U(t)$$

and in the down-spike regime, $D$, the dynamics is

$$\ln S_D(t) = \ln F(t, t + \tau) + X_D(t)$$

Jumps are induced via the compound Poisson processes

$$X_U(t) = \sum_{i=1}^{1+P_U(t)} J_{i,U} \quad \text{and} \quad X_D(t) = \sum_{i=1}^{1+P_D(t)} J_{i,D}$$

where

$$P_k(t) \sim Po(\lambda_k) \quad \text{and} \quad J_{i,k} \sim N(\mu_k, \sigma_k) \quad \text{for} \quad k \in \{U, D\}$$
The swap price is defined by

\[ d \ln F(t, T) = \left( \mu(t, T) - \frac{1}{2} [\sigma(t, T)]^2 V(t) \right) dt + \sigma(t, T) \sqrt{V(t)} dB_2(t) \]

with the squared volatility process given by

\[ dV(t) = \kappa_V (\theta_V - V(t)) dt + \sigma_V \sqrt{V(t)} dB_3(t) \]

The two Browian motions are allowed to be correlated and \( \sigma \) is the deterministic function

\[ \sigma(t, T) = \gamma + \frac{\alpha}{\beta\tau_M} \left( e^{-\beta(T-t)} - e^{-\beta(T+\tau_M-t)} \right) \]

\( T \) is the start of the delivery period and \( \tau \) is the length of the swapping period.
The model is estimated using Nordic electricity spot, swap and swaption prices.

The spot mean reversion level (syntethic rolling swap price) is approximated with the front month swap price.

The time-continous spot price process is discretized to daily time steps.

The spot dynamics:
- Method: Maximum Likelihood
- Data: Spot and swap prices
- Sample period: 2006 – 2011

The swap dynamics:
- Method: Non linear least squares
- Data: Swap and swaption prices
- Sample period: 2010
The risk neutral dynamics

- The option prices are priced under the risk neutral measure.
- We need the risk neutral dynamics of the spot price.
- The swap price dynamics under the risk neutral measure were calibrated with the swaption prices.
- The spot price is unhedgeable and has no unique risk neutral spot price dynamics.
- The spot price dynamics was estimated under the historical measure.

  - We need to make assumptions about the risk neutral dynamics of the spot price.
    - We assume that the only risk which is priced is the continuous shocks in the normal regime i.e. the Brownian motion.
      \[
      dS_N(t) = S_N(t) \left\{ (-\kappa Z(t) - \sigma \zeta_1(t)) dt + \sigma dB^Q_1(t) \right\}
      \]
    - The price of risk $\zeta$ is assumed to be both time varying and stochastic such that
      \[
      E_t^Q [S^*(u) du] = F(t, T, T)
      \]
      for all $T$. 


Pricing European styled spot options

The log spot price is not of affine form and hence no Fourier based pricing methods can be used without any approximations (see Duffie et al 2001)

- Reason: The rolling swap price in combination with stochastic volatility

- Options are priced with Monte Carlo simulations

- Options can be priced with Fourier based methods by approximating the rolling swap contract with a fixed contract.

- The approximation error is very small because the mean reversion speed of the spot price is high.

- Choose the fixed contract which corresponds to the rolling contract at the “delivery” date of spot.
Pricing European styled spot options (cont’d)

- Pricing method: Monte Carlo simulations
  - The risk premium $\zeta$ needs to be defined which is difficult.
  - We approximate $\zeta$ by scaling the simulated prices such that the average simulated spot price is equal its forward price.

- Pricing method: Fourier based methods (approximation of the mean reversion level)
  - The risk premium can be backed out analytically.
The impact of the swaption smile on spot options:

- The impact of the swaption smile on spot options is relatively small.
- The impact of the swaption smile is increasing by time to maturity.
- For strips of independent daily call options (virtual power plants (VPPs)) the impact is even smaller as the VPPs consist of many in-, at- and out-of-the money options.
The implied volatility of swaptions
The implied volatility of spot options: 1 month to maturity

Maturity date: 2013-01-01

- SV
- No SV
The implied volatility of spot options: 7 months to maturity

Maturity date: 2013-06-30

- SV
- No SV
The implied volatility of spot options: 13 month to maturity

Maturity date: 2013-12-31

- SV
- No SV

Moneyness

Implied volatility
Valuation of a VPP

Daily option price

Daily forward curve

EUR/MWh

Monte Carlo
Fourier

Fwd curve
Strike level
The model features several important characteristics:
- The volatility smile in the swaption market
- Regime switching and jumps in the spot market
- Cointegration between the spot and the swap prices

The model parameters are easy to estimate as you can estimate the spot and swap parameters separately.

European styled options on the spot price can successfully be approximated and efficiently priced with Fourier based methods.

Stochastic volatility in the swaption market has no significant effect on the valuation of VPPs.
Estimation results of the spot parameters

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<th>Parameter</th>
<th>Estimated value</th>
<th>Standard error</th>
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<tr>
<td>$\xi$</td>
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<td>$\kappa$</td>
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