Integrated Electricity Spot and Forward Model

Andreas Wagner

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Motivation

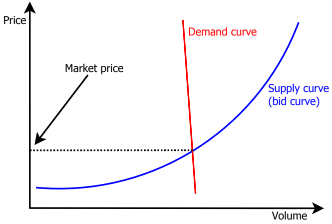
Aim of this talk

An electricity price framework and model for the joint modeling of electricity spot and forward prices.

- Information from the futures market is transferred to the spot market
- Combination of the structural and financial modeling approach
- Fundamental market information as well as information provided by the electricity futures and options market is used
- Some ideas similar to Burger et al. [2004] (SMaPS-Model)
- Model is a forward-dynamic extension to W. [2014] (Residual Demand Modeling, see EnergyFinance 2012) and Barlow [2002]

Remainder: Structural Models

- Structural models follow a basic economic concept: market price is the intersection of supply and demand
- Structural models are well suited to model electricity prices and have become increasingly popular
- Simple stochastic processes are enough to model the complicated electricity spot dynamics
- Fundamental market information can be included



Literature

Structural (fundamental, hybrid, supply/demand) models find strong attention in recent literature

- Good summary and general structural framework Carmona and Coulon [2012]
- Barlow Model Barlow [2002]
- Stochastic Bid Stack Model Coulon and Howison [2009], Fundamental Multi-Fuel Model Carmona et al. [2011]
- Structural Risk-Neutral Model Aïd et al. [2009], Aïd et al. [2012]
- and more Burger et al. [2004], Cartea and Villaplana [2008], Pirrong and Jermakyan [2008], Lyle and Elliott [2009], de Maire D'Aertrycke and Smeers [2010], ...

Simple example

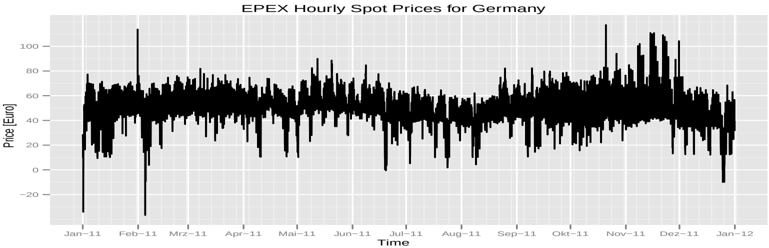
A simple structural model is (similar to Barlow [2002])

$$S_t = f(D_t)$$

with spot price S_t, supply function f, and some stochastic process D_t.

The framework we propose is basically as simple as above, but we allow for a forward dynamic of f

Basic idea



- The electricity spot (=day ahead) price is often seen as a simple time series
- However, the spot price moves along two time dimensions
- Electricity for delivery at different times is not the same financial product (their may be some correlation though)
- This is due to the non-storeability of electricity

The abstract price

- Distinguish delivery time t and observation time τ
- The abstract (electricity) price is the core of the framework

$$S_\tau(t) = \mathbf{F}(\tau)(t, D(t)), \quad \tau \leq t$$

- $\mathbf{F}(\tau)(t, \cdot)$ is a stochastic supply-function for delivery at time t
- $D(t)$ is the driving factors process (e.g. residual demand)
- The spot price is $S_t = S_t(t, D(t))$, i.e. the abstract price with $\tau = t$

The abstract price is a non-traded product depending on two different risk factors, namely the forward dynamic and the driving factors.

Driving factors

Abstract price

$$S_{\tau}(t) = \mathbf{F}(\tau)(t, D(t)), \quad \tau \leq t$$

delivery time t, observation time τ

- The abstract price evolves as a function of the driving factors process $D(t)$
- $D(t)$ is assumed to be a time-τ-stationary stochastic process, i.e. its distribution does not change in observation time
- $D(t)$ represents unhedgeable risks like demand, renewable infeed, interconnectors, etc., i.e. factors where no knowledge is obtained in observation time (at least not until short before)

Forward Dynamic

Abstract price

$$S_\tau(t) = \mathbf{F}(\tau)(t, D(t)), \quad \tau \leq t$$

delivery time t, observation time τ

- Stochastic supply-functional $\mathbf{F}(\cdot)$ represents hedgeable risks in the abstract price, e.g. changing fuel prices
- Multi-factor standard model (Black-model) is suitable

$$d\mathbf{F}(\tau)(t, x) = \mathbf{F}(\tau)(t, x)\ \Sigma(\tau, t)\ dW_\tau, \quad \mathbf{F}(0)(t, x) = f_0(t, x)$$

  - W_τ: d-dimensional Brownian motion independent of $\mathcal{F}^D$
  - $\Sigma(\tau, t)$: $1 \times d$ (deterministic) volatility vector

- Supply-functional is assumed independent of $D(t)$, i.e. the structure of the generation stack is independent of the driving factors

Forwards I

- In contrast to storeable commodities, there is no clear relationship between electricity spot and forward prices in the electricity market

$$F_\tau(t) \neq e^{r(t-\tau)} S_\tau$$

  $F_\tau(t)$: Forward-price at time τ for delivery at time t
  S_τ: Spot-price at time τ

- This is (again) due to the non-storeability of electricity
- Therefore we define the forward price in a suitable way
- Traded forwards give an indication of the spot price level at delivery (remember also that a forward always covers a delivery period)

Forwards II

Definition (Forward price)

The time-τ-price $F_\tau(t)$ of a forward contract with delivery at time t is a random variable defined as the conditional expectation of $S_\tau(t)$, i.e.

$$F_\tau(t) := \mathbb{E}\left[S_\tau(t)|\mathcal{F}_\tau^W\right],$$

where $\mathcal{F}_\tau^W$ is the filtration generated by W_τ.

- Heuristically, the forward price is the expectation over the possible realisations of the driving factors process (which cannot be hedged)
- For each supply-dynamic (generated by the W_τ), a different forward price path is realized

Properties

- The forward price follows a geometric Brownian motion (sensible, as the forward can be bought and stored without costs)
- The log-normal distribution makes the pricing of European options an easy exercise (Black-formula)
- In the market, only forwards with delivery period and European options on those are traded
- Using the common “the sum of lognormal random variables is itself lognormal“ assumption and a moment matching, we can handle those products

An explicit model

We define an explicit model within the framework for the German/Austria market

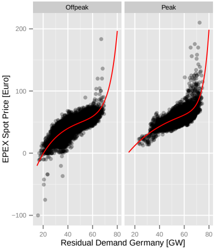
Initial supply function

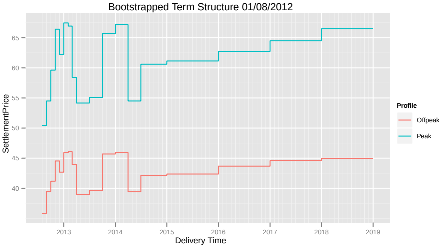
$$\mathbf{F}(0)(t, x) = a(t) + g(x)$$

- This additive structure of $\mathbf{F}$ allows for a quick calibration
- For the driving factors process $D(t)$, we use a stochastic model for residual demand (= total demand – wind infeed – solar infeed)
- For the volatility, we use the two-factor approach from Boerger et al. [2009] for the modeling of electricity futures (see next slides)

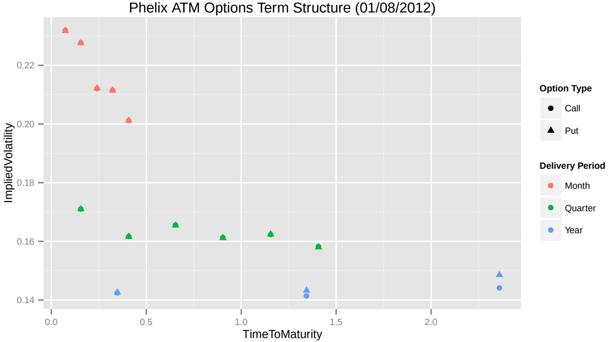
Calibration of $\mathbf{F}(0)$

- Initial calibration of $\mathbf{F}(0)$ on historical spot prices and residual demand data
- Bootstrapped term-structure of EEX futures is then used to shift $a(t)$





Empirical option-implied volatilities



The implied volatility of electricity futures depends on

- time-to-maturity (usually this equals time-to-delivery)
- the length of the delivery period

Volatility of supply-functional

This observation motivates the following volatility structure (as in Boerger et al. [2009])

Volatility structure

$$\Sigma(\tau, t) = \left(e^{-\kappa(t-\tau)}\sigma_1, \sigma_2(t)\right),$$

where

σ_1 is the (additional) short-term volatility,

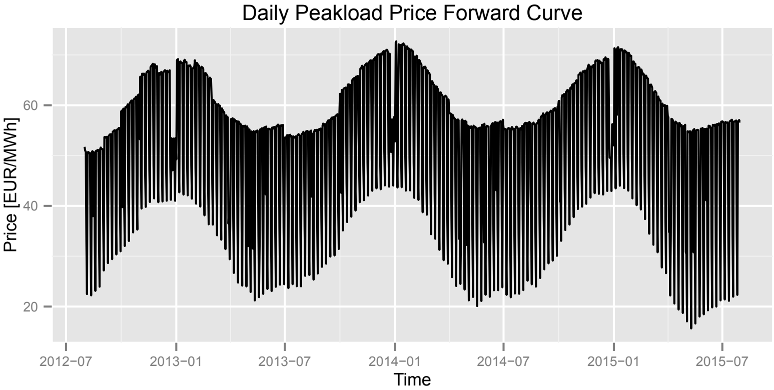
κ is a positive constant controlling the influence of the short-term volatility, and

$\sigma_2 : \mathbb{R}^+ \to \mathbb{R}^+$ is a positive-valued function representing the long-term volatility depending on the delivery date.

Σ is calibrated on option volatilities and we assume $\sigma_2(t)$ constant within the delivery periods of the futures

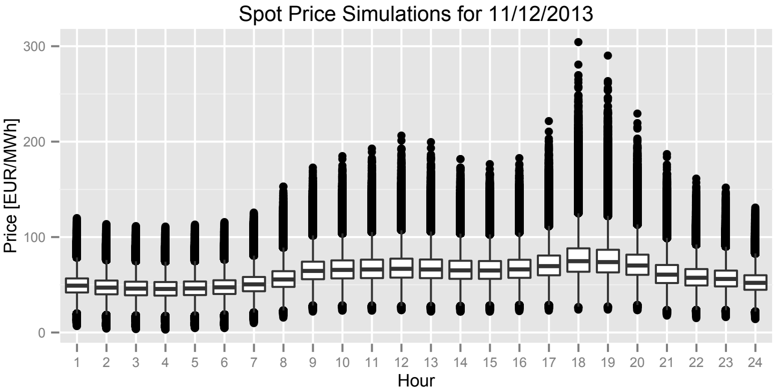
Some results I

Forward curve accounting for increasing capacities of wind and solar



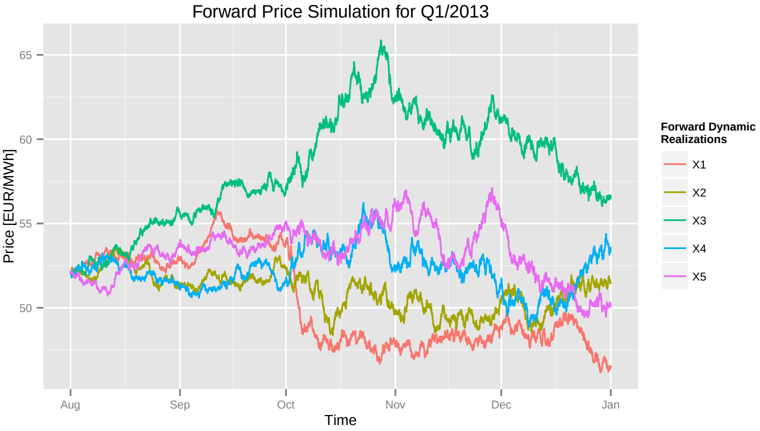
Some results II

Boxplots of hourly spot prices



Some results III

Simulation of a forward with delivery period



Conclusions

- Derivatives are priced market-consistent, but fundamental market information can also be included
- Spot price dynamics build up on prices for futures and options on futures
- Forward prices follow standard approach
- No detailed knowledge about the merit order required
- Model not suitable for fundamental analysis, no fuel prices included

Thank you for your attention

Contact

Andreas Wagner

+43 50607 21872

andreas.wagner@tiwag.at

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