Integrated Electricity Spot and Forward Model

Andreas Wagner

Energy Finance Conference
Essen 2013

research supported by

October 10, 2013
Content

1. Motivation
2. Framework
3. Model and Results
4. Conclusions
Motivation

Aim of this talk

An electricity price framework and model for the joint modeling of electricity spot and forward prices.

- Information from the futures market is transferred to the spot market
- Combination of the structural and financial modeling approach
- Fundamental market information as well as information provided by the electricity futures and options market is used
- Some ideas similar to Burger et al. [2004] (SMaPS-Model)
- Model is a forward-dynamic extension to W. [2014] (Residual Demand Modeling, see EnergyFinance 2012) and Barlow [2002]
Remainder: Structural Models

- Structural models follow a basic economic concept: market price is the intersection of supply and demand.
- Structural models are well suited to model electricity prices and have become increasingly popular.
- Simple stochastic processes are enough to model the complicated electricity spot dynamics.
- Fundamental market information can be included.
Structural (fundamental, hybrid, supply/demand) models find strong attention in recent literature

- **Good summary and general structural framework** Carmona and Coulon [2012]
- **Barlow Model** Barlow [2002]
- **Stochastic Bid Stack Model** Coulon and Howison [2009], Coulon and Howison [2011], Carmona et al. [2011]
- **Fundamental Multi-Fuel Model** Carmona et al. [2011]
- **Structural Risk-Neutral Model** Aïd et al. [2009], Aïd et al. [2012]
- **and more** Burger et al. [2004], Cartea and Villaplana [2008], Pirrong and Jermakyan [2008], Lyle and Elliott [2009], de Maire D’Aertrycke and Smeers [2010], ...
Simple example

A simple structural model is (similar to Barlow [2002])

\[ S_t = f(D_t) \]

with spot price \( S_t \), supply function \( f \), and some stochastic process \( D_t \).

The framework we propose is basically as simple as above, but we allow for a forward dynamic of \( f \).
The electricity spot (=day ahead) price is often seen as a simple time series.

However, the spot price moves along two time dimensions.

Electricity for delivery at different times is not the same financial product (there may be some correlation though).

This is due to the non-storeability of electricity.
The abstract price

- Distinguish **delivery time** \( t \) and **observation time** \( \tau \)
- The **abstract (electricity) price** is the core of the framework

\[
S_{\tau}(t) = F(\tau)(t, D(t)), \quad \tau \leq t
\]

- \( F(\tau)(t, \cdot) \) is a **stochastic supply-function** for delivery at time \( t \)
- \( D(t) \) is the **driving factors process** (e.g. residual demand)
- The spot price is \( S_t = S_t(t, D(t)) \), i.e. the abstract price with \( \tau = t \)

The abstract price is a non-traded product depending on two different risk factors, namely the forward dynamic and the driving factors.
Driving factors

Abstract price

\[ S_\tau(t) = F(\tau)(t, D(t)), \quad \tau \leq t \]

- The abstract price evolves as a function of the driving factors process \( D(t) \)
- \( D(t) \) is assumed to be a time-\( \tau \)-stationary stochastic process, i.e. its distribution does not change in observation time
- \( D(t) \) represents unhedgeable risks like demand, renewable infeed, interconnectors, etc., i.e. factors where no knowledge is obtained in observation time (at least not until short before)
Abstract price

\[ S_\tau(t) = F(\tau)(t, D(t)), \quad \tau \leq t \]

delivery time \( t \), observation time \( \tau \)

- Stochastic supply-functional \( F(\cdot) \) represents hedgeable risks in the abstract price, e.g. changing fuel prices
- Multi-factor standard model (Black-model) is suitable

\[ dF(\tau)(t, x) = F(\tau)(t, x) \Sigma(\tau, t) \, dW_\tau, \quad F(0)(t, x) = f_0(t, x) \]

- \( W_\tau \): \( d \)-dimensional Brownian motion independent of \( F^D \)
- \( \Sigma(\tau, t) \): \( 1 \times d \) (deterministic) volatility vector

- Supply-functional is assumed independent of \( D(t) \), i.e. the structure of the generation stack is independent of the driving factors
In contrast to storeable commodities, there is no clear relationship between electricity spot and forward prices in the electricity market:

\[ F_\tau(t) \neq e^{r(t-\tau)}S_\tau \]

- This is (again) due to the non-storeability of electricity.
- Therefore we define the forward price in a suitable way.
- Traded forwards give an indication of the spot price level at delivery (remember also that a forward always covers a delivery period).
Definition (Forward price)

The time-$\tau$-price $F_\tau(t)$ of a forward contract with delivery at time $t$ is a random variable defined as the conditional expectation of $S_\tau(t)$, i.e.

$$F_\tau(t) := \mathbb{E} \left[ S_\tau(t) | \mathcal{F}_\tau^W \right],$$

where $\mathcal{F}_\tau^W$ is the filtration generated by $W_\tau$.

- Heuristically, the forward price is the expectation over the possible realisations of the driving factors process (which cannot be hedged)
- For each supply-dynamic (generated by the $W_\tau$), a different forward price path is realized
Properties

- The forward price follows a geometric Brownian motion (sensible, as the forward can be bought and stored without costs)
- The log-normal distribution makes the pricing of European options an easy exercise (Black-formula)
- In the market, only forwards with delivery period and European options on those are traded
- Using the common “the sum of lognormal random variables is itself lognormal“ assumption and a moment matching, we can handle those products
An explicit model

We define an explicit model within the framework for the German/Austria market.

Initial supply function

\[ F(0)(t, x) = a(t) + g(x) \]

- This additive structure of \( F \) allows for a quick calibration.
- For the driving factors process \( D(t) \), we use a stochastic model for residual demand (= total demand – wind infeed – solar infeed).
- For the volatility, we use the two-factor approach from Boerger et al. [2009] for the modeling of electricity futures (see next slides).
Calibration of $F(0)$

- Initial calibration of $F(0)$ on historical spot prices and residual demand data
- Bootstrapped term-structure of EEX futures is then used to shift $a(t)$
Empirical option-implied volatilities

The implied volatility of electricity futures depends on

- time-to-maturity (usually this equals time-to-delivery)
- the length of the delivery period
Volatility of supply-functional

This observation motivates the following volatility structure (as in Boerger et al. [2009])

Volatility structure

\[ \Sigma(\tau, t) = \left( e^{-\kappa (t - \tau)} \sigma_1, \sigma_2(t) \right), \]

where

- \( \sigma_1 \) is the (additional) short-term volatility,
- \( \kappa \) is a positive constant controlling the influence of the short-term volatility, and
- \( \sigma_2 : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) is a positive-valued function representing the long-term volatility depending on the delivery date.

\( \Sigma \) is calibrated on option volatilities and we assume \( \sigma_2(t) \) constant within the delivery periods of the futures.
Some results

Forward curve accounting for increasing capacities of wind and solar
Some results II

Boxplots of hourly spot prices

Spot Price Simulations for 11/12/2013

Andreas Wagner – Integrated Electricity Spot and Forward Model
Motivation
Framework
Model and Results
Conclusions

Some results III

Simulation of a forward with delivery period

Forward Price Simulation for Q1/2013

Time
Aug Sep Oct Nov Dec Jan

Price [EUR/MWh]
50
55
60
65

Forward Dynamic Realizations
- X1
- X2
- X3
- X4
- X5

Andreas Wagner – Integrated Electricity Spot and Forward Model
Conclusions

- Derivatives are priced market-consistent, but fundamental market information can also be included
- Spot price dynamics build up on prices for futures and options on futures
- Forward prices follow standard approach
- No detailed knowledge about the merit order required
- Model not suitable for fundamental analysis, no fuel prices included
Thank you for your attention

Contact

Andreas Wagner
+43 50607 21872
andreas.wagner@tiwag.at


