Forward pricing in the shipping freight markets

Che Mohd Imran Che Taib

Department of Mathematics, Faculty of Science and Technology,
Universiti Malaysia Terengganu, 21030 Kuala Terengganu, Terengganu,
MALAYSIA

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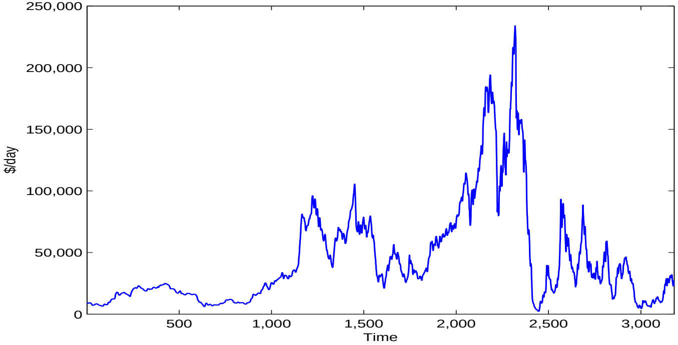

Figure 1: Daily spot freight rates for the Panamax Index (1/3/99–11/11/11).

Outline

1. Introduction
2. Stochastic dynamics of the spot freight rates
   - Geometric Brownian motion
   - A Lévy-based dynamics
   - Stochastic volatility model of BNS
   - Continuous autoregressive model CAR(p)
3. Pricing of freight forward
4. Shape of the forward curves

- More than 75% of the volume of the world trade in commodities and manufactured products are contributed from shipping industry.
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- Seaborne trade: oil tanker, gas tanker, container, dry-bulk, other
- Dry-bulk commodities:
  - Major bulk: iron ore, coal, grain
  - Minor bulk: steel products, fertilizer, sugar, cement, etc.
- Major bulk are normally transported using Capesize and Panamax vessels.

- Pricing forward using spot-forward relationship framework.
- Stylized facts about the freight rate dynamics:
  - Heavy-tailed logreturns
  - Stochastic volatility
  - Mean reversion

- Pricing forward using spot-forward relationship framework.
- Stylized facts about the freight rate dynamics:
  - Heavy-tailed logreturns
  - Stochastic volatility
  - Mean reversion
- Stochastic models for spot freight rates (based on the findings in Benth, Koekebakker and Taib [3]):
  - Geometric Brownian motion
  - Normal inverse Gaussian (NIG) Lévy model
  - Barndorff-Nielsen and Shephard (BNS) stochastic volatility
  - Continuous-time autoregressive (CAR) driven by:
    - Brownian motion
    - NIG Lévy process
    - BNS stochastic volatility

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Geometric Brownian motion

- The explicit dynamics of GBM model

$$S(t) = S(0)\exp(\mu t + \sigma B(t)) \, .$$

- Define for a given discretization in time $\Delta > 0$,

$$r_\Delta(t) := \ln S(t+\Delta) - \ln S(t) \, , t = 0, \Delta, 2\Delta \dots .$$

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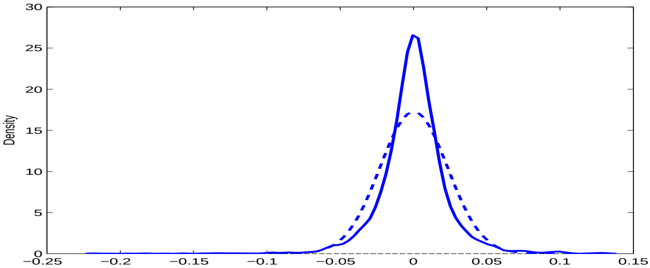

A Lévy-based dynamics

- Define the spot price $S(t)$, $t \geq 0$, as an exponential Lévy process,

$$S(t) = S(0)\exp(L(t)) .$$

- The logreturns are defined as

$$r_\Delta(t) = L(t+\Delta) - L(t) ,$$

  for $t = 0, \Delta, 2\Delta, \ldots$.
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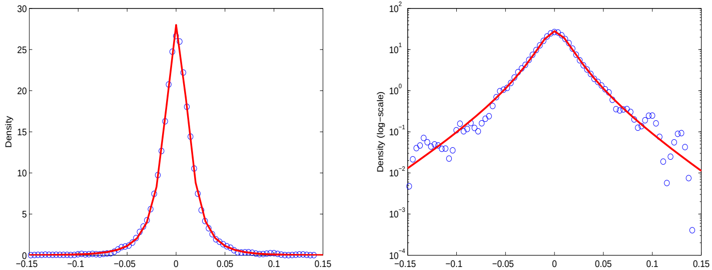

Stochastic volatility model of Barndorff-Nielsen and Shephard

- The BNS model is defined as follows,

$$d\ln S(t) = (\mu + \beta\sigma^2(t))\,dt + \sigma(t)\,dB(t)\,, \tag{1}$$

where

$$\sigma^2(t) = \sum_{i=1}^{n} \omega_i V_i(t)\,,$$

for Ornstein-Uhlenbeck processes

$$dV_i(t) = -\lambda_i V_i(t)\,dt + dZ_i(\lambda_i t)\,, i = 1, \ldots, n\,.$$

- $r_\triangle(t)$ is a mean-variance mixture model.
- For $n = 1$ and the stationary distribution of $\sigma^2(t) = V(t)$ is inverse Gaussian, then the logreturns will become approximately NIG distributed.

Continuous autoregressive model, CAR(p)

- Denote $\ln S(t) = Y(t)$ and for $p \geq 1$, the p-dimensional OU process $\mathbf{X}(t)$ is defined as the solution of

$$d\mathbf{X}(t) = A\mathbf{X}(t)\,dt + \mathbf{e}_p \sigma \, dB(t)\,,$$

where A is the $p \times p$ matrix given by

$$A = \begin{bmatrix} \mathbf{0} & & I \\ -\alpha_p & \dots & -\alpha_1 \end{bmatrix},$$

and $Y(t) = \mathbf{e}_1' \mathbf{X}(t)$.

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- CAR model driven by NIG Lévy process

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- CAR model driven by NIG Lévy process

$$d\mathbf{X}(t) = A\mathbf{X}(t)\,dt + \mathbf{e}_p\, dL(t)\,.$$

- CAR model driven by BNS-SV process

$$d\mathbf{X}(t) = A\mathbf{X}(t)\,dt + \mathbf{e}_p\, \sigma(t)\, dB(t)\,.$$

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- The value of any derivative is defined as the **present value of its expected payoff** where the expectation is taken under risk neutral measure Q.
- Pay nothing to enter the contract implies that,

$$e^{-r(T-t)}\mathbb{E}_Q\left[S(T) - F(t,T) \mid \mathcal{F}_t\right] = 0.$$

- The spot-forward relationship formula is

$$F(t,T) = \mathbb{E}_Q[S(T)|\mathcal{F}_t].$$

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- Esscher transform: Let θ_L be the *market price of risk*. For $0 \leq t \leq T$, we define a process $\pi_L(t)$ as

$$\pi_L(t) = \exp\left(\theta_L L(t) - \phi_L(\theta_L)t\right).$$

Thus, the Radon-Nikodym derivative

$$\left.\frac{dQ}{dP}\right|_{\mathcal{F}_t} = \pi_L(t),$$

such that π_L is the density process of a measure $Q \sim P$.

Forward pricing: GBM model

- Introduce a parametric class of measure change of Girsanov transform for the case of Gaussian model using

$$B_\theta(t) = B(t) - \theta t, \tag{2}$$

with θ as a constant describing the *market price of risk*.

- The Q-dynamics of GBM is now taking the form

$$dS(t) = \kappa S(t)dt + \sigma S(t)dB_\theta(t), \tag{3}$$

where $\kappa = \mu + \sigma\theta$ and B_θ is a Q-Brownian motion.

- The explicit solution of (3) for $t \leq T$ is

$$S(T) = S(t)\exp\left(\left(\kappa - \frac{\sigma^2}{2}\right)(T-t) + \int_t^T \sigma\, dB_\theta(u)\right). \tag{4}$$

Forward pricing: GBM model

Proposition

The price at time t for a forward contract with delivery at time $T \geq t \geq 0$ under geometric Brownian motion model is given as

$$F(t, T) = S(t) \exp\left(\kappa\left(T - t\right)\right),$$

where $\kappa = \mu + \sigma\theta$.

Forward pricing: NIG Lévy model

Lemma

If $g : [0, t] \mapsto R$ *is a bounded and measurable function and Condition ?? holds for* $k := \sup_{s \in [0,t]} |g(s)|$, *then*

$$\mathbb{E}\left[\exp\left(\int_0^t g(u)\, dL(u)\right)\right] = \exp\left(\int_0^t \phi(g(u))du\right),$$

where $\phi(\lambda) = \psi(-i\lambda)$.

Proposition

The price at time t *for a forward contract with delivery at time* $T \geq t \geq 0$ *under NIG Lévy model is given as*

$$F(t, T) = S(t) \exp\left(\{\phi_L(\theta_L + 1) - \phi_L(\theta_L)\}(T - t)\right).$$

Forward pricing: BNS stochastic volatility model

Proposition

The price at time t for a forward contract with delivery at time $T \geq t \geq 0$ under BNS stochastic volatility model is given as

$$
\begin{aligned}
F(t,T) = & S(t)\exp\left((\mu+\theta)(T-t)+\sum_{j=1}^{n}\Theta(T-t)V_j(t)\right) \\
& \times \exp\left(\sum_{j=1}^{n}\int_t^T\left\{\phi_Z\left(\Theta(T-\nu)+\theta_V\right)-\phi_Z(\theta_V)\right\}d\nu\right),
\end{aligned}
\tag{5}
$$

where $\Theta(\xi) = \frac{\omega_j}{\lambda_j}(\beta+0.5)\left(1-e^{-\lambda_j\xi}\right)$.

Forward pricing: CAR model

Proposition

The price of a forward contract at time t for delivery at time $T \geq t \geq 0$ under CAR(p) model driven by Brownian motion is given as

$$F(t,T) = \exp\left(\mathbf{e}_1' e^{A(T-t)}\mathbf{X}(t) + \int_t^T \Sigma(T-u)\theta\sigma\, du\right) \times \exp\left(\frac{1}{2}\int_t^T \Sigma^2(T-u)\sigma^2 du\right),$$

where $\Sigma(T-u) = \mathbf{e}_1' e^{A(T-u)}\mathbf{e}_p$.

Proposition

The price of a forward contract at time t for delivery at time $T \geq t \geq 0$ under CAR(p) model driven by normal inverse Gaussian process is given as

$$F(t,T) = \exp\left(\mathbf{e}_1' e^{A(T-t)}\mathbf{X}(t) + \int_t^T \{\phi_L(\Sigma(T-u)+\theta_L) - \phi_L(\theta_L)\}\,du\right).$$

Proposition

The price of a forward contract at time t for delivery at time $T \geq t \geq 0$ under CAR(p) driven by BNS stochastic volatility process is given as

$$F(t,T) = \exp\left(\mathbf{e}_1'^{A(T-t)}\mathbf{X}(t) + \int_t^T \Sigma(T-u)\theta du\right)$$
$$\times \exp\left(\sum_{j=1}^{n} \frac{\omega_j}{2} \int_t^T \Sigma^2(T-u)e^{-\lambda_j(u-t)}\, du V_j(t)\right)$$
$$\times \exp\left(\sum_{j=1}^{n} \int_t^T \left\{\phi_Z\left(\frac{\omega_j}{2}\int_\nu^T \Upsilon(u-v)\, du + \theta_V\right) - \phi_Z(\theta_V)\right\} d\nu\right.$$

where $\Upsilon(x) = \Sigma^2(T-u)e^{-\lambda_j x}$.

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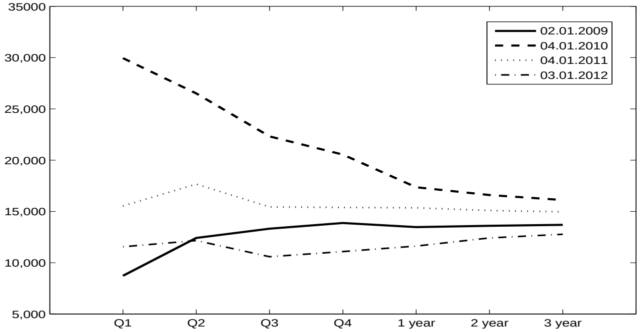

Figure 2: Slope of forward curves for Panamax vessels for several times to delivery.

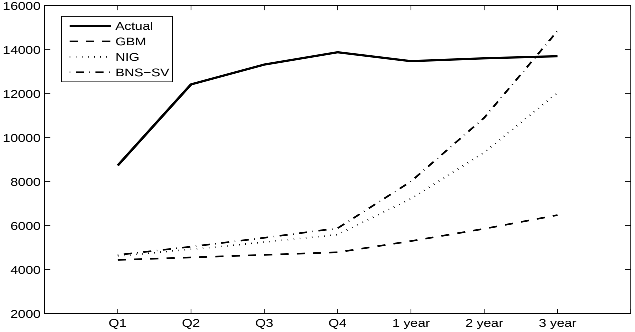

Figure 3: The observed forward rate on 2 Jan 2009 together with the theoretical forward curves derived from GBM, NIG and BNS models.

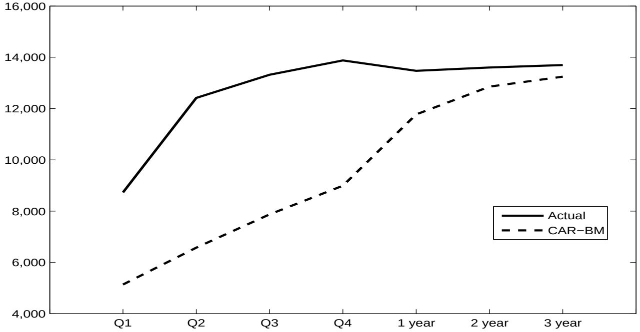

Figure 4: Actual forward curve observed on 2 Jan 2009 together with the curve of forward from CAR(3) driven by Brownian motion for $\theta = 0.67$.

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Terima kasih