Forward pricing in the shipping freight markets

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**Figure 1:** Daily spot freight rates for the Panamax Index (1/3/99–11/11/11).
Outline

1. Introduction

2. Stochastic dynamics of the spot freight rates
   - Geometric Brownian motion
   - A Lévy-based dynamics
   - Stochastic volatility model of BNS
   - Continuous autoregressive model CAR($p$)

3. Pricing of freight forward

4. Shape of the forward curves
More than 75% of the volume of the world trade in commodities and manufactured products are contributed from shipping industry.

Annualised volatility of shipping freight rates varies between 59% to 79% in the years 2008 to 2011.
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Seaborne trade: oil tanker, gas tanker, container, dry-bulk, other

Dry-bulk commodities:
- Major bulk: iron ore, coal, grain
- Minor bulk: steel products, fertilizer, sugar, cement, etc.

Major bulk are normally transported using Capesize and Panamax vessels.
Pricing forward using spot-forward relationship framework.

Stylized facts about the freight rate dynamics:
- Heavy-tailed logreturns
- Stochastic volatility
- Mean reversion
Pricing forward using spot-forward relationship framework.

Stylized facts about the freight rate dynamics:
- Heavy-tailed logreturns
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- Mean reversion

Stochastic models for spot freight rates (based on the findings in Benth, Koekebakker and Taib [3]):
- Geometric Brownian motion
- Normal inverse Gaussian (NIG) Lévy model
- Barndorff-Nielsen and Shephard (BNS) stochastic volatility
- Continuous-time autoregressive (CAR) driven by:
  - Brownian motion
  - NIG Lévy process
  - BNS stochastic volatility
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4 Shape of the forward curves
The explicit dynamics of GBM model

\[ S(t) = S(0) \exp(\mu t + \sigma B(t)) \, . \]

Define for a given discretization in time \( \Delta > 0 \),

\[ r_\Delta(t) := \ln S(t + \Delta) - \ln S(t) \, , \quad t = 0, \Delta, 2\Delta, \ldots. \]
Geometric Brownian motion

- The explicit dynamics of GBM model

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A Lévy-based dynamics

- Define the spot price $S(t)$, $t \geq 0$, as an exponential Lévy process,
  \[ S(t) = S(0) \exp(L(t)). \]

- The logreturns are defined as
  \[ r_\Delta(t) = L(t + \Delta) - L(t), \]
  for $t = 0, \Delta, 2\Delta, \ldots$.

- The logreturns are fitted using NIG($\delta, \alpha, \mu, \beta$) family of distribution.
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Stochastic volatility model of Barndorff-Nielsen and Shephard

- The BNS model is defined as follows,

\[ d \ln S(t) = \left( \mu + \beta \sigma^2(t) \right) dt + \sigma(t) dB(t), \quad (1) \]

where

\[ \sigma^2(t) = \sum_{i=1}^{n} \omega_i V_i(t), \]

for Ornstein-Uhlenbeck processes

\[ dV_i(t) = -\lambda_i V_i(t) dt + dZ_i(\lambda_i t), \quad i = 1, \ldots, n. \]

- \( r_\Delta(t) \) is a mean-variance mixture model.

- For \( n = 1 \) and the stationary distribution of \( \sigma^2(t) = V(t) \) is inverse Gaussian, then the logreturns will become approximately NIG distributed.
Continuous autoregressive model, CAR($p$)

Denote $\ln S(t) = Y(t)$ and for $p \geq 1$, the $p$-dimensional OU process $X(t)$ is defined as the solution of

$$dX(t) = AX(t) \, dt + e_p \sigma dB(t),$$

where $A$ is the $p \times p$ matrix given by

$$A = \begin{bmatrix} 0 & & I \\ -\alpha_p & \ldots & -\alpha_1 \end{bmatrix},$$

and $Y(t) = e'_1 X(t)$. 
Continuous autoregressive model, CAR\( (p) \)

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    A = \begin{bmatrix}
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        -\alpha_p & \cdots & I \\
        & -\alpha_1 & \\
    \end{bmatrix},
\]

and \( Y(t) = e_1' X(t) \).

- CAR model driven by NIG Lévy process

\[
    dX(t) = AX(t) \, dt + e_p \, dL(t).
\]
Continuous autoregressive model, CAR($p$)

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- CAR model driven by NIG Lévy process

$$dX(t) = AX(t) \, dt + e_p \, dL(t).$$

- CAR model driven by BNS-SV process

$$dX(t) = AX(t) \, dt + e_p \sigma(t) \, dB(t).$$
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The value of any derivative is defined as the **present value of its expected payoff** where the expectation is taken under risk neutral measure $Q$.

Pay nothing to enter the contract implies that,

$$e^{-r(T-t)}E_Q[S(T) - F(t, T) | \mathcal{F}_t] = 0.$$ 

The spot-forward relationship formula is

$$F(t, T) = E_Q[S(T) | \mathcal{F}_t].$$
The value of any derivative is defined as the present value of its expected payoff where the expectation is taken under risk neutral measure \( Q \).

Pay nothing to enter the contract implies that,

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e^{-r(T-t)} \mathbb{E}_Q [S(T) - F(t, T) \mid \mathcal{F}_t] = 0.
\]

The spot-forward relationship formula is

\[
F(t, T) = \mathbb{E}_Q [S(T) \mid \mathcal{F}_t].
\]

Esscher transform:
• The value of any derivative is defined as the present value of its expected payoff where the expectation is taken under risk neutral measure \( Q \).

• Pay nothing to enter the contract implies that,

\[
e^{-r(T-t)} E_Q [S(T) - F(t, T) | \mathcal{F}_t] = 0.
\]

• The spot-forward relationship formula is

\[
F(t, T) = E_Q [S(T)|\mathcal{F}_t].
\]

• **Esscher transform**: Let \( \theta_L \) be the market price of risk. For \( 0 \leq t \leq T \), we define a process \( \pi_L(t) \) as

\[
\pi_L(t) = \exp (\theta_L L(t) - \phi_L(\theta_L) t).
\]

Thus, the Radon-Nikodym derivative

\[
\frac{dQ}{dP} \bigg|_{\mathcal{F}_t} = \pi_L(t),
\]

such that \( \pi_L \) is the density process of a measure \( Q \sim P \).
Forward pricing: GBM model

- Introduce a parametric class of measure change of \textit{Girsanov transform} for the case of Gaussian model using

\[
B_\theta(t) = B(t) - \theta t,
\]

with \( \theta \) as a constant describing the \textit{market price of risk}.

- The \( Q \)-dynamics of GBM is now taking the form

\[
dS(t) = \kappa S(t) dt + \sigma S(t) dB_\theta(t),
\]

where \( \kappa = \mu + \sigma \theta \) and \( B_\theta \) is a \( Q \)-Brownian motion.

- The explicit solution of (3) for \( t \leq T \) is

\[
S(T) = S(t) \exp \left( \left( \kappa - \frac{\sigma^2}{2} \right) (T - t) + \int_t^T \sigma dB_\theta(u) \right). \tag{4}
\]
Forward pricing: GBM model

**Proposition**

The price at time $t$ for a forward contract with delivery at time $T \geq t \geq 0$ under geometric Brownian motion model is given as

$$F(t, T) = S(t) \exp(\kappa (T - t)),$$

where $\kappa = \mu + \sigma \theta$. 
Forward pricing: NIG Lévy model

Lemma

If \( g : [0, t] \mapsto \mathbb{R} \) is a bounded and measurable function and Condition ?? holds for \( k := \sup_{s \in [0, t]} |g(s)| \), then

\[
\mathbb{E} \left[ \exp \left( \int_0^t g(u) \, dL(u) \right) \right] = \exp \left( \int_0^t \phi(g(u)) \, du \right),
\]

where \( \phi(\lambda) = \psi(-i\lambda) \).

Proposition

The price at time \( t \) for a forward contract with delivery at time \( T \geq t \geq 0 \) under NIG Lévy model is given as

\[
F(t, T) = S(t) \exp \{ \phi_L(\theta_L + 1) - \phi_L(\theta_L) \} (T - t).
\]
Forward pricing: BNS stochastic volatility model

**Proposition**

The price at time $t$ for a forward contract with delivery at time $T \geq t \geq 0$ under **BNS stochastic volatility** model is given as

$$F(t, T) = S(t) \exp \left( (\mu + \theta)(T - t) + \sum_{j=1}^{n} \Theta(T - t) V_j(t) \right) \times \exp \left( \sum_{j=1}^{n} \int_{t}^{T} \{ \phi_{Z} (\Theta(T - \nu) + \theta_{V}) - \phi_{Z}(\theta_{V}) \} \, d\nu \right),$$

where $\Theta(\xi) = \frac{\omega_j}{\lambda_j} (\beta + 0.5) (1 - e^{-\lambda_j \xi})$. 

(5)
Forward pricing: CAR model

Proposition

The price of a forward contract at time $t$ for delivery at time $T \geq t \geq 0$ under CAR($p$) model driven by Brownian motion is given as

$$F(t, T) = \exp \left( e'_1 e^{A(T-t)} X(t) + \int_t^T \Sigma(T-u) \theta \sigma \, du \right) \times \exp \left( \frac{1}{2} \int_t^T \Sigma^2(T-u) \sigma^2 \, du \right),$$

where $\Sigma(T-u) = e'_1 e^{A(T-u)} e_p$. 
Proposition

The price of a forward contract at time $t$ for delivery at time $T \geq t \geq 0$ under $\text{CAR}(p)$ model driven by normal inverse Gaussian process is given as

$$F(t, T) = \exp \left( e_1' e^{A(T-t)} X(t) + \int_t^T \{ \phi_L(\Sigma(T - u) + \theta_L) - \phi_L(\theta_L) \} \, du \right).$$
Proposition

The price of a forward contract at time $t$ for delivery at time $T \geq t \geq 0$ under $\text{CAR}(p)$ driven by $\text{BNS}$ stochastic volatility process is given as

$$F(t, T) = \exp \left( e^t A(T-t) X(t) + \int_t^T \Sigma(T-u) \theta \, du \right)$$

$$\times \exp \left( \sum_{j=1}^n \frac{\omega_j}{2} \int_t^T \Sigma^2(T-u) e^{-\lambda_j(u-t)} \, du V_j(t) \right)$$

$$\times \exp \left( \sum_{j=1}^n \int_t^T \left\{ \phi_Z \left( \frac{\omega_j}{2} \int_\nu^T \Upsilon(u-\nu) \, du + \theta V \right) - \phi_Z(\theta V) \right\} \, d\nu \right)$$

where $\Upsilon(x) = \Sigma^2(T-u) e^{-\lambda_j x}$.
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Figure 2:  Slope of forward curves for Panamax vessels for several times to delivery.
Figure 3: The observed forward rate on 2 Jan 2009 together with the theoretical forward curves derived from GBM, NIG and BNS models.
Figure 4: Actual forward curve observed on 2 Jan 2009 together with the curve of forward from CAR(3) driven by Brownian motion for $\theta = 0.67$. 

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Introduction
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Shape of the forward curves


Terima kasih