Karlsruhe Institute of Technology

Empirical Performance of Reduced-Form Models for Emission Permit Prices

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SSRN: http://ssrn.com/abstract=2297121

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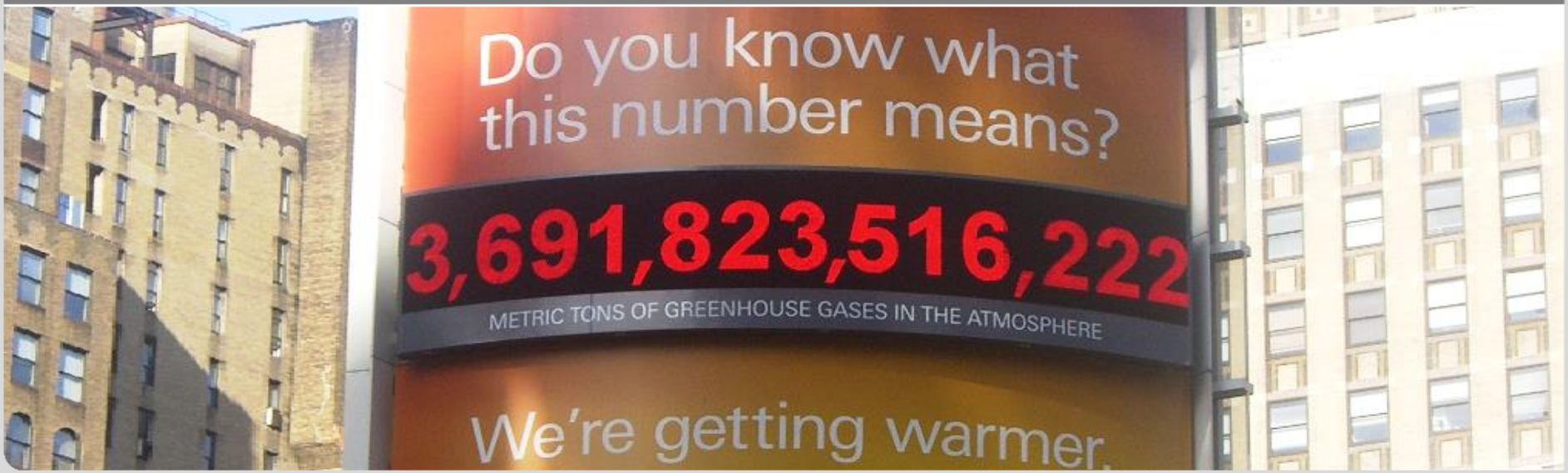


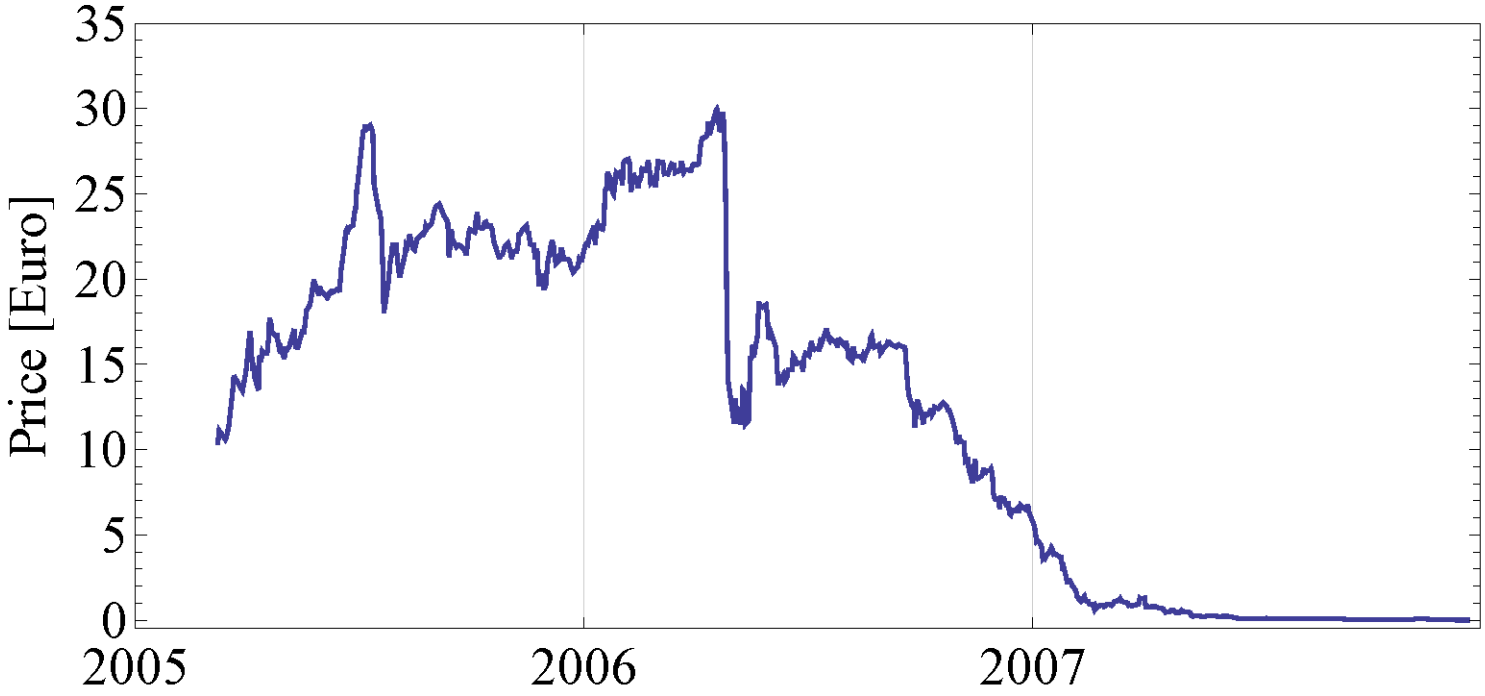
Motivation

- Design of emission trading systems induces specific features of permit price dynamics
- Direct implications for hedging strategies, derivatives pricing, investment decisions

- Do reduced-form models for CO_2 prices outperform standard models with respect to
  (1) historical model fit and
  (2) option pricing performance?

Example: EU ETS Phase I

- Design: 1 period, unused permits become worthless



- Grüll and Taschini (2011): Reduced-form CO_2 models outperform standard models with respect to historical fit

Design of Today's Emission Trading Systems

- n consecutive compliance periods $[0,T_1]$, $[T_1,T_2]$, ..., $[T_{n-1},T_n]$
- Banking between compliance periods, but no borrowing
- Non-compliance penalties, later delivery of lacking permits

- Resulting structure of futures prices

$$F(t,\bar{t}) = \sum_{j=1}^{n} e^{-r(T_j-\bar{t})} \cdot \underbrace{Q\left\{x_{T_j} > q_j\right\}}_{\text{probability of penalties}} \cdot \underbrace{p_j}_{\text{penalty}}$$

x_{T_j}: economy-wide cumulative emissions

q_j: economy-wide cumulative allocation

Example: EU ETS Phase II

Model Specification

- Reduced-form approach (see Carmona and Hinz 2011): specify $x_{t,T_k} = E_t\{x_{T_k}\}$ directly under the risk-neutral measure
- For a large class of processes for x_{t,T_k}, the dynamics of $A_{k,t} = Q\{x_{T_k,T_k} > q_k\} = Q\{x_{T_k} > q_k\}$ follows as

$$dA_{k,t} = \phi'\left(\phi^{-1}\left(A_{k,t}\right)\right)\sqrt{\frac{\beta_k}{T_k - t}}\,dW_t{}^k$$

- Correlations of Wiener processes: ρ_{jk}

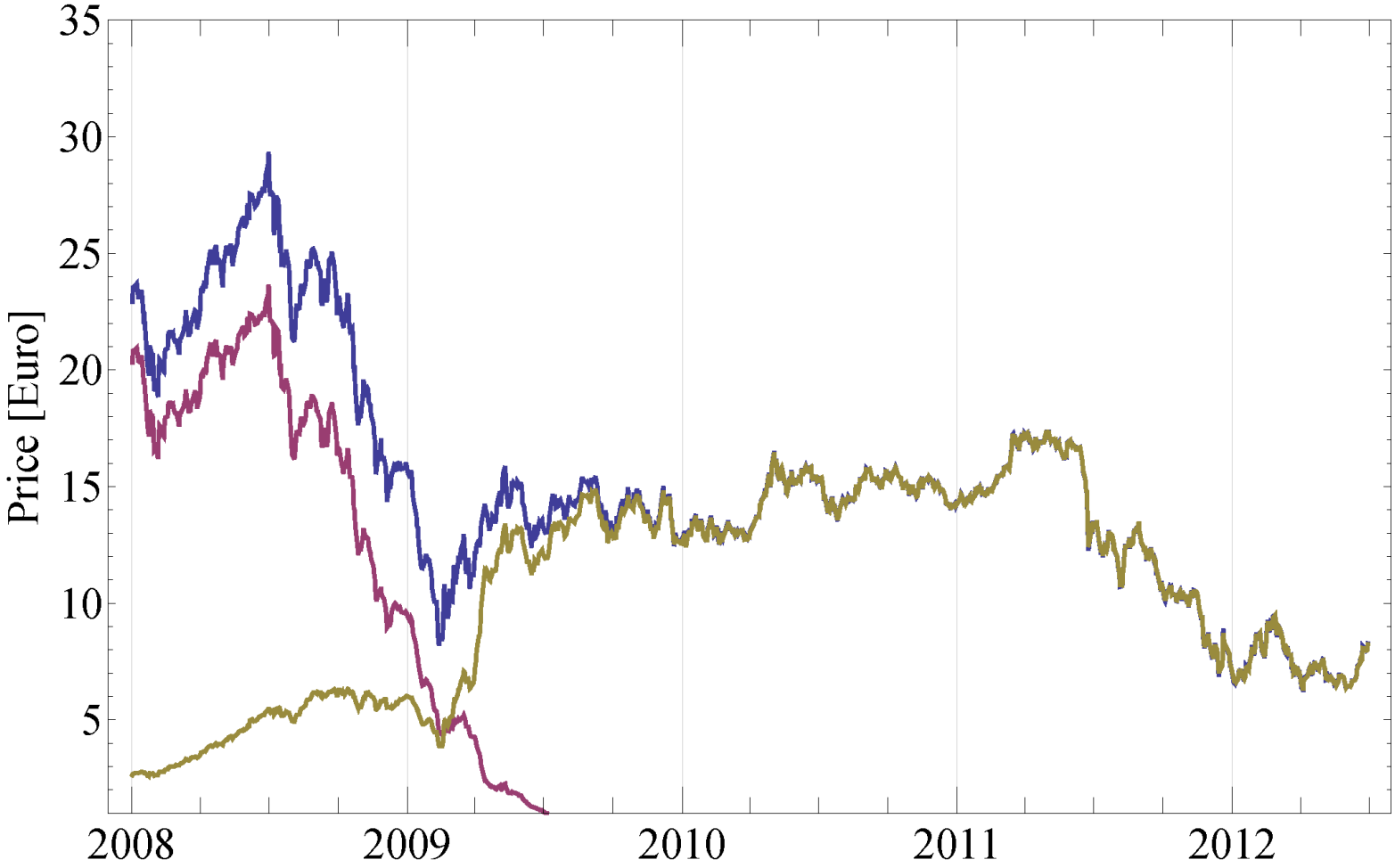
Historical Estimation Approach

- Consider modified specification

$$F(t,\bar{t}) = \sum_{j=1}^{m} e^{-r(T_j-\bar{t})} \cdot A_{j,t} \cdot p_j + R_t$$

- Model variants
  - GBM: m = 0, R_t follows GBM
  - 1PCH: m = 1, R_t = 0
  - 1PCHR: m = 1, R_t follows GBM
  - 2PCH: m = 2, R_t = 0
- Estimation based on the Unscented Kalman Filter

Filtered Price Components: 2PCH

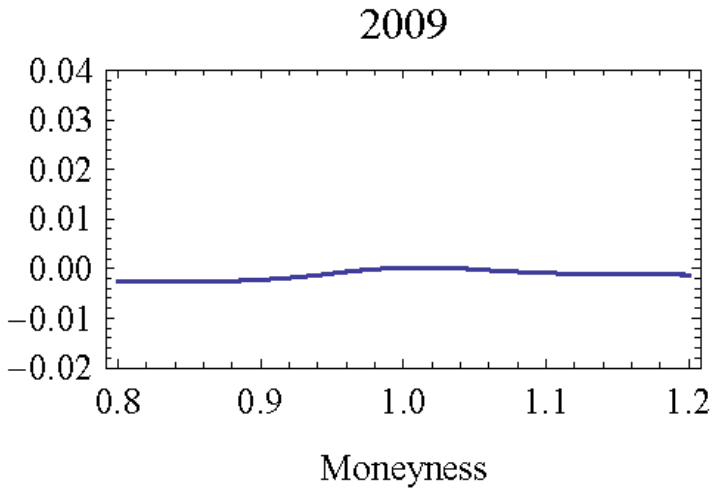


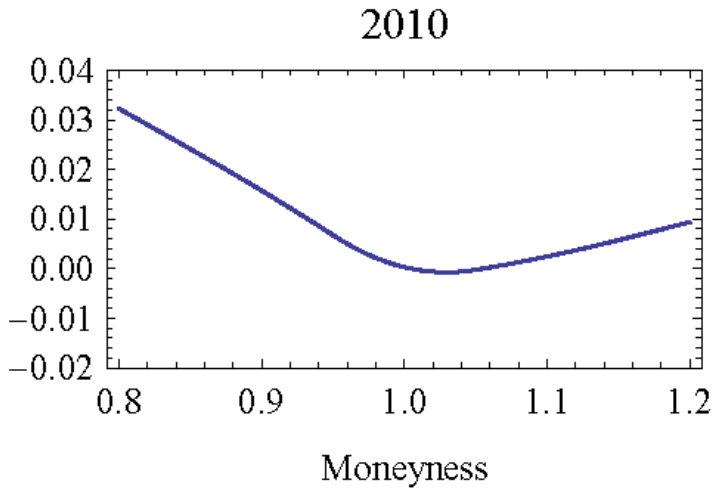
Historical Estimation Results

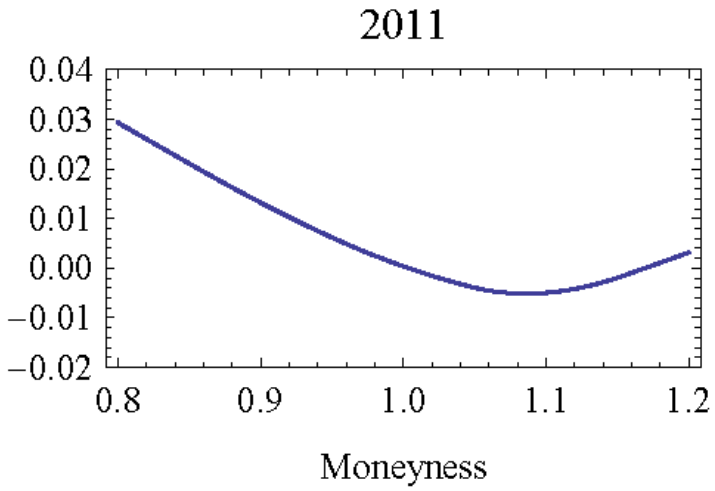
	h_1	β_1	h_2	β_2	h_R	σ_R	ρ		LLF	AIC
GBM					-0.458	0.376			568.27	**-1130.53**
					(0.465)	(0.009)				
1PCH	-0.603	0.182							469.68	**-933.35**
	(0.468)	(0.009)								
1PCHR	-5.610	0.401			0.216	0.327	0.518		574.44	**-1134.87**
	(0.582)	(0.044)			(0.498)	(0.011)	(0.061)			
2PCH	-5.587	0.394	0.191	0.444			0.419		575.88	**-1137.76**
	(0.555)	(0.043)	(0.498)	(0.025)			(0.081)			

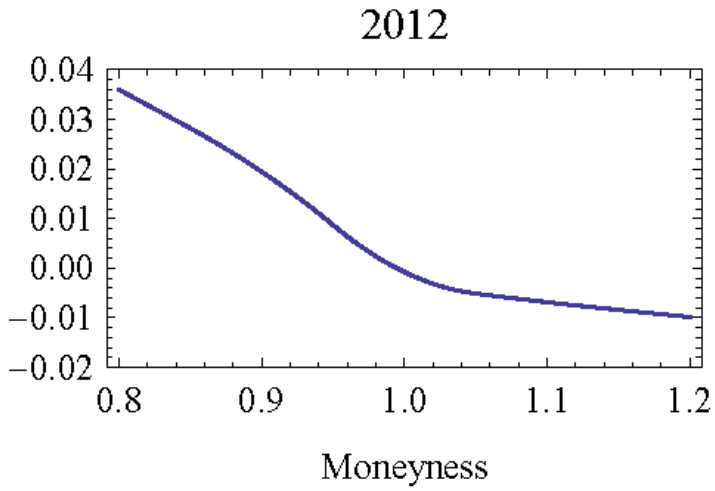
GBM: n=0, R=GBM
1PCHR: n=1, R=GBM
1PCH: n=1, R=0
2PCH: n=2, R=0

Option Pricing









Analysis of Option Pricing Performance

- First observations
  - Carmona and Hinz (2011): until the end of September 2009, “traders [...] priced EUA options using Black 76 formula”
  - Development to strong downward-sloping smile from 2010 to 2012

- Design of analysis
  - In-Sample: Monthly recalibration of each model variant, evaluation of option pricing errors
  - Out-of-Sample: Evaluation of option pricing errors using parameter estimates for previous month

In-Sample Analysis

	Min	Max	PPPE	MAE	MAPE	RMSE	RRMSE
2011							
GBM	-0.46	0.30	53.49%	0.09	9.69%	0.13	**13.44%**
1PCH	-0.38	0.29	53.68%	0.08	8.45%	0.12	**11.70%**
1PCHR	-0.47	0.30	53.29%	0.09	9.77%	0.13	**13.54%**
2PCH	-0.42	0.29	52.52%	0.09	8.83%	0.12	**12.14%**
2012							
GBM	-0.18	0.16	50.00%	0.05	4.54%	0.07	**5.80%**
1PCH	-0.12	0.15	49.38%	0.04	3.22%	0.05	**4.41%**
1PCHR	-0.18	0.17	51.25%	0.06	4.70%	0.07	**6.00%**
2PCH	-0.12	0.17	50.00%	0.04	3.41%	0.05	**4.54%**

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Out-of-Sample Analysis

	Min	Max	PPPE	MAE	MAPE	RMSE	RRMSE
2011							
GBM	-0.73	0.35	39.53%	0.14	15.19%	0.17	**18.71%**
1PCH	-0.49	1.87	44.57%	0.20	18.57%	0.39	**32.58%**
1PCHR	-0.77	0.35	39.53%	0.14	15.25%	0.17	**18.79%**
2PCH	-0.66	0.41	38.37%	0.13	13.34%	0.16	**16.28%**
2012							
GBM	-0.28	0.21	48.12%	0.12	9.48%	0.14	**11.12%**
1PCH	-0.18	0.28	59.38%	0.11	9.33%	0.13	**11.09%**
1PCHR	-0.28	0.22	47.50%	0.12	9.50%	0.14	**11.22%**
2PCH	-0.22	0.24	49.38%	0.10	8.04%	0.11	**9.37%**

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1PCH	-0.49	1.87	44.57%	0.20	18.57%	0.39	**32.58%**
1PCHR	-0.77	0.35	39.53%	0.14	15.25%	0.17	**18.79%**
2PCH	-0.66	0.41	38.37%	0.13	13.34%	0.16	**16.28%**
2012							
GBM	-0.28	0.21	48.12%	0.12	9.48%	0.14	**11.12%**
1PCH	-0.18	0.28	59.38%	0.11	9.33%	0.13	**11.09%**
1PCHR	-0.28	0.22	47.50%	0.12	9.50%	0.14	**11.22%**
2PCH	-0.22	0.24	49.38%	0.10	8.04%	0.11	**9.37%**

Conclusion and Outlook

- Reduced-form models for emission permit prices
  - Account for the specific payoff structure of emission permits induced by the design of the system
  - Outperform standard models (GBM) with respect to historical model fit and option pricing
- But: Appropriate specification is crucial (number of compliance periods)
- New emissions markets (California, China, Australia, ...) extend the scope of application
- How are optimal investment/hedging decisions affected by the special form of price dynamics?