Model Risk and Power Plant Valuation

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Motivation and Introduction

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Motivation and Introduction: Questions

- How significant is the impact of the model’s choice on the value of a given instrument?
- How to assess the value of the parameters’ uncertainty?
- What is the main driver of the model risk in the energy markets?
General Approach

- We consider the model risk inherent in the valuation procedure of fossil power plants.

- We focus on a gas-fired power plant as flexible and low-carbon source of electricity which is an important building block in terms of the switch to a low-carbon energy generation.

- We model the generated financial streams as the spread option and investigate the reinvestment problem.

- To capture model risk we use a methodology recently established in a series of papers: [Cont, 2006, Bannör and Scherer, 2013].
Risk-Captured Price

Having

- a contingent claim $X$,
- a parameter space $\Theta$,
- a distribution $R$ on the parameters,
- a parameterised family of valuation measures $(Q_\theta)_{\theta \in \Theta}$,
- a law-invariant, normalised convex risk measure $\rho$

results in a risk-captured price of a contingent claim $X$ by

$$\Gamma(X) := \rho(\theta \mapsto \mathbb{E}_\theta[X]).$$
### Visualisation of the Steps of Parameter Risk-Capturing Valuation

<table>
<thead>
<tr>
<th>Model: complex financial market</th>
<th>Discounted derivative payout $X$</th>
</tr>
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<tbody>
<tr>
<td>Parameter space $\Theta$</td>
<td>Derivative price $E_\theta[X]$</td>
</tr>
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</table>

- Probability measure $R$ on $\Theta$
- Pricing function $\theta \to E_\theta[X]$
- Derivative price distribution induced by $R$ and $\theta \to E_\theta[X]$
- Risk measure $\rho$

**Ask price:** $\Gamma(X) = \rho(\theta \to E_\theta[X])$

**Bid price:** $-\Gamma(-X)$

Quantifies parameter risk of derivative price
Risk-Capturing Functional: AVaR Example

- Define AVaR w.r.t. the significance level $\alpha \in (0, 1)$ of some random variable $X$ as
  
  $\text{AVaR}_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha q_X(1 - \beta) d\beta,$

- where $q_X(\gamma)$ is $\gamma$ quantile of the random variable $X$.
- The AVaR measures the risk which may occur according to the previously specified model $Q_\theta$.
- When calculating the parameter risk-captured price of $X$ being induced by the AVaR, risk-neutral prices $(\mathbb{E}_\theta[X])_{\theta \in \Theta}$ w.r.t. different models $(Q_\theta)_{\theta \in \Theta}$ are compared and subsumed by the AVaR risk measure. Hence, the AVaR is used to quantify the parameter risk we are exposed to when pricing $X$. 
Clean Spark Spread Option and Virtual Power Plant

We model the daily profit (or loss) of the virtual power plant position as

\[ V_t = \max\{P_t - hG_t - \eta E_t, 0\}, \]

- \( P_t \) - is the power price;
- \( G_t \) - is the gas price;
- \( E_t \) - is the carbon certificate price;
- \( h \) - is the heat rate of the power plant;
- \( \eta \) - \( CO_2 \) emission rate of the power plant.
Energy Price Models

Let

- $\left( \Omega, \mathbb{P}, \mathcal{F}, \mathbb{F}_t, t \in [0, T] \right)$ be a complete filtered probability space;
- **carbon price**
  
  \[ dE_t = \alpha^E E_t \, dt + \sigma^E E_t \, dW_t^E; \]
- **gas price**
  
  \[ G_t = e^{g(t)+Z_t}, \]
  
  \[ dZ_t = -\alpha^G Z_t \, dt + \sigma^G dW_t^G; \]
- **power price**
  
  \[ P_t = e^{f(t)+X_t+Y_t}, \]
  
  base signal: \[ dX_t = -\alpha^P X_t \, dt + \sigma^P dW_t^P, \]
  
  spike signal: \[ dY_t = -\beta Y_t \, dt + J_t \, dN_t. \]
- **dependence structure**
  
  $W^E$, $W^G$ and $N$ are mutually independent processes,
  
  \[ dW_t^P \, dW_t^G = \rho \, dt. \]
Data

- **Phelix day base**: It is the average price of the hours 1 to 24 for electricity traded on the spot market. It is calculated for all calendar days of the year as the simple average of the auction prices for the hours 1 to 24 in the market area Germany/Austria, EUR/MWh.

- **NCG daily price**: Delivery is possible at the virtual trading hub in the market areas of NetConnect Germany GmbH & Co KG, EUR/MWh.

- **Emission certificate daily price**: One EU emission allowance confers the right to emit 1 tonne of carbon dioxide or 1 tonne of carbon dioxide equivalent, EUR/EUA.

- **Observation period**: 25.09.2009 - 08.06.2012.
Power, Gas, and Carbon Prices, 25.09.2009 - 08.06.2012
Clean Spark Spread, 25.09.2009 - 08.06.2012
Estimating the Model Parameters

- Estimation of the seasonal trends and deseasonalisation of power and gas.
- Separation of the power base and spike signals.
- Estimation of the mean-reverting rates.
- Estimation of the power base signal $X_t$.
- Estimation of the spike signal $Y_t$.
- Estimation of the correlation.

Following the above steps, we estimate the set of parameters with mainly an MLE approach

$$\{\alpha^E, \sigma^E, g(t), \alpha^G, \sigma^G, f(t), \alpha^P, \beta, \sigma^P, \lambda, \mu_s, \sigma_s, \rho\}.$$
General Procedure: Step 1

- After estimating all the parameters of our prices, we simulate them for the future time period and compute for every day $t$ the spark spread value $V_t$ given as

$$V_t = \max\{P_t - hG_t - \eta E_t, 0\}.$$

- Then, by fixing all the parameters except for the chosen one and setting the shift value $\xi$ (e.g. 1%), we compute shifted up and down spark spread values as

$$V_{t}^{up}(\theta + \xi),$$

$$V_{t}^{down}(\theta - \xi).$$
General Procedure: Step 2

► We compute the value of the power plant (VPP) by means of Monte Carlo simulations. For fixed large $N$ and $T = 3$ years we have

$$ VPP(t, T) = \frac{1}{N} \sum_{i=1}^{N} VPP_i(t, T), $$

$$ VPP_i(t, T) = \int_{t}^{T} e^{-r(s-t)} V_i(s) \, ds. $$

► For the chosen shift $\xi$ we also compute

$$ VPP^{up}(t, T; \theta) = VPP(t, T; \theta + \xi), $$

$$ VPP^{down}(t, T; \theta) = VPP(t, T; \theta - \xi). $$
General Procedure: Step 3

▶ We continue with sensitivity measuring of the VPP w.r.t. the parameter $\theta$ with the central finite difference [Glasserman, 2004]

$$
\nabla_\theta VPP(t, T) := \frac{\partial VPP}{\partial \theta} = \frac{VPP^{up}(t, T; \theta) - VPP^{down}(t, T; \theta)}{2\xi}
$$

▶ Finally, we compute the bid and ask prices by using a closed-form approximation formula for the AVaR to get the risk-captured prices by subtracting and adding risk-adjustment value to $VPP(t, T)$ respectively. This risk-adjustment value is computed as

$$
\varphi(\Phi^{-1}(1 - \alpha)) \frac{\sqrt{(\nabla_\theta VPP)' \cdot \Sigma_\theta \cdot \nabla_\theta VPP}}{\alpha} N,
$$

denoting by $\Sigma_\theta$ the asymptotic covariance matrix of the estimator for the parameter $\theta$ [McNeil et al., 2005].
Risk Values Results

- parameter risk in spike size: Laplace and Gaussian distributions;
- parameter risk in correlation;
- parameter risk in gas signal;
- joint parameter risk in gas and base power signal;
- joint parameter risk in gas, power and emissions (all processes except of jump size parameter).
Resulting values for the relative width of the bid-ask spread for various model risk sources. $\alpha_1 = 0.01$ (the highest risk-aversion), $\alpha_2 = 0.1$, $\alpha_3 = 0.5$

<table>
<thead>
<tr>
<th>Jumps size distribution</th>
<th>Gaussian</th>
<th>Laplace</th>
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<tbody>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
</tr>
<tr>
<td>Jumps</td>
<td>111.9%</td>
<td>73.71%</td>
</tr>
<tr>
<td>Correlation</td>
<td>6.95%</td>
<td>4.58%</td>
</tr>
<tr>
<td>Gas and power base</td>
<td>6.48%</td>
<td>4.27%</td>
</tr>
<tr>
<td>Gas</td>
<td>6.11%</td>
<td>4.03%</td>
</tr>
<tr>
<td>Gas, power and carbon</td>
<td>8.21%</td>
<td>5.41%</td>
</tr>
</tbody>
</table>
Parameter-risk implied bid-ask spread w.r.t. jump size distribution: Gaussian

![Bid and ask prices accounting for the parameter risk in jump distribution with normal jumps](image1)

![Relative bid-ask spread width accounting for the parameter risk in jump distribution with normal jumps](image2)
Parameter-risk implied bid-ask spread w.r.t. jump size distribution: Laplace
Parameter-risk implied bid-ask spread w.r.t. correlation parameter, Gaussian jumps
Parameter-risk implied bid-ask spread w.r.t. correlation parameter, Laplace jumps
Conclusive Remarks

► We suggested a methodology to quantify model risk in power plant valuation approaches (spread options).

► We studied various sources of risks and found out that the correlation and spike risks are dominating in the energy sector.

► We managed to estimate the lower boundary for the total model risk in terms of the chosen model.

► Future possible application in the energy markets could be a generation of an hourly power forward curve and valuation procedures for storages.
References


Thank you for your attention!
Parameter-risk implied bid-ask spread w.r.t. the gas and power base processes, Gaussian jumps
Parameter-risk implied bid-ask spread w.r.t. the gas and power base processes, Laplace jumps
Parameter-risk implied bid-ask spread w.r.t. the gas price process, Gaussian jumps

Simulations

Bid−Ask Delta Value

Relative bid−ask spread width accounting for the parameter risk in gas signals with normal jumps

Bid and ask prices accounting for the parameter risk in gas signals with normal jumps
Parameter-risk implied bid-ask spread w.r.t. the gas price process, Laplace jumps
Parameter-risk implied bid-ask spread w.r.t. all the parameters, except of the Gaussian jump size
Parameter-risk implied bid-ask spread w.r.t. all the parameters, except of the Laplace jump size