A Commodity Spot Price Model with Short-, Medium- and Long-Term Components

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Outline of presentation

Model Specification

In the following let $S(t)$ denote the spot price and $s(t) = \ln S(t)$. We decompose $s(t)$ into three components;

$$s(t) = \lambda(t) + q(t) + \eta(t).$$

$\lambda(t)$ is a deterministic seasonal price component and $\eta(t)$ follows a Wiener process with drift,

$$d\eta(t) = \mu dt + \sigma_\eta dW_\eta.$$

$q(t)$ satisfies the following system of stochastic differential equations;

$$\begin{aligned} dq(t) =& [-\gamma q(t) + \mathbf{\Gamma X}(t)]dt + \sigma_q dW_q, \\ d\mathbf{X}(t) =& -\mathbf{\Delta X}(t)dt + \mathbf{B_X} d\mathbf{W_X}. \end{aligned}$$

Model Specification

$\mathbf{X}(t)$ is an m-dimensional stochastic process driven by the m-dimensional standard Wiener process $\mathbf{W_X}(t)$, where $\mathrm{E}[\, d\mathbf{W_X} d\mathbf{W_X}'] = \mathbf{I}_{m\times m}dt$. Any correlation structure is represented by $\mathbf{B_X}$.

$\gamma \geq 0$ and $\sigma_q \geq 0$ are scalar parameters and $\mathbf{\Gamma}, \mathbf{\Delta}$ and $\mathbf{B_X}$ are matrix parameters with $\dim(\mathbf{\Gamma}) = (1 \times m)$, $\mathbf{\Delta}$ is diagonal with $\dim(\mathbf{\Delta}) = (m \times m)$ and $\mathbf{B_X}$ is upper triangular with $\dim(\mathbf{B_X}) = (m \times m)$.

Model Specification

$\mathbf{X}(t)$ may, or may not, be observable.

When $\mathbf{X}(t)$ is observable, $\mathbf{\Gamma}$ represents a scaling of the effect of the observed process $\mathbf{X}(t)$ on $q(t)$. We treat $\mathbf{X}(t)$ as unobservable, and the numerical choice of the elements in $\mathbf{\Gamma} \neq \mathbf{0}$ is arbitrary.

Any scaling of the effect of $\mathbf{X}(t)$ on $q(t)$ is equally matched by a scaling of $\mathbf{\Delta}$ and $\mathbf{B_X}$.

Let $\sigma_q dW_q \mathbf{B_X} d\mathbf{W_X} = \mathbf{K}dt$, $dW_\eta dW_q = 0$ and $dW_\eta d\mathbf{W_X} = \mathbf{0}$.

Model Solution

Analytical expressions for the distribution of state variables and spot prices in our model is found as follows:

$$q(t) = e^{-\gamma t} q_0 + \mathbf{\Gamma}(\gamma \mathbf{I} - \mathbf{\Delta})^{-1}(e^{-\mathbf{\Delta} t} - e^{-\gamma \mathbf{I} t})\mathbf{X}_0 + \int_0^t e^{-\gamma(t-\tau)} \sigma_q dW_q(\tau) -$$

$$\mathbf{\Gamma}(\gamma \mathbf{I} - \mathbf{\Delta})^{-1} \int_0^t (e^{-\gamma \mathbf{I}(t-\tau)} - e^{-\mathbf{\Delta}(t-\tau)})\mathbf{B_X} d\mathbf{W_X}(\tau),$$

$$\mathbf{X}(t) = e^{-\mathbf{\Delta} t}\mathbf{X}_0 + \int_0^t e^{-\mathbf{\Delta}(t-\tau)} \mathbf{B_X} d\mathbf{W_X}(\tau),$$

since $\mathbf{\Delta}$ is diagonal $e^{-\mathbf{\Delta}(t-\tau)} = \text{diag}\left\{e^{-\delta_i(t-\tau)}\right\}$ and $\mathbf{I}$ is the $(m \times m)$ identity matrix. $\eta(t)$ has solution

$$\eta(t) = \eta_0 + \mu t + \int_0^t \sigma_\eta dW_\eta(\tau).$$

Model Properties

Assume $\mathbf{\Delta} \neq \gamma \mathbf{I}$, such that the expected value is given by

$$\mathrm{E}\begin{bmatrix} q(t) \\ \mathbf{X}(t) \\ \eta(t) \end{bmatrix} = \begin{bmatrix} e^{-\gamma t} q_0 + \mathbf{\Gamma}(\gamma \mathbf{I} - \Delta)^{-1}(e^{-\Delta t} - e^{-\gamma \mathbf{I} t})\mathbf{X}_0 \\ e^{-\Delta t}\mathbf{X}_0 \\ \eta_0 + \mu t \end{bmatrix}.$$

Define $\mathbf{\Sigma} = \{\sigma_{ij}\} = \mathbf{B}_x \mathbf{B}_x', \mathbf{\Omega}_t = \{\omega_{ij}(t)\}$ and $\mathbf{\Xi}_t = \{\xi_{ij}(t)\}$, where $\omega_{ij}(t) = \sigma_{ij}[1 - e^{-(\delta_i + \delta_j)t}]/(\delta_i + \delta_j)$, $\xi_{ij}(t) = \sigma_{ij}[1 - e^{-(\gamma + \delta_j)t}]/(\gamma + \delta_j)$.

Model Properties

The variance of the process is then

$$\text{Var}\begin{bmatrix} q(t) \\ \mathbf{X}(t) \\ \eta(t) \end{bmatrix} = \begin{bmatrix} V_q & \mathbf{V}_{\mathbf{q},\mathbf{X}} & 0 \\ \mathbf{V}_{\mathbf{q},\mathbf{X}} & \mathbf{V}_{\mathbf{X}} & \mathbf{0} \\ 0 & \mathbf{0} & V_\eta \end{bmatrix},$$

where

$$\begin{aligned} V_q(t) =& \boldsymbol{\Gamma}(\gamma \mathbf{I} - \boldsymbol{\Delta})^{-1}\left[\frac{1-e^{-2\gamma t}}{2\gamma}\boldsymbol{\Sigma} - \boldsymbol{\Xi}_t - \boldsymbol{\Xi}_t' + \boldsymbol{\Omega}_t\right](\gamma \mathbf{I} - \boldsymbol{\Delta})^{-1}\boldsymbol{\Gamma}' + \\ & \boldsymbol{\Gamma}(\gamma \mathbf{I} - \boldsymbol{\Delta})^{-1}\left[2(\gamma \mathbf{I} + \boldsymbol{\Delta})^{-1}[1 - e^{-(\gamma \mathbf{I} + \boldsymbol{\Delta})t}] - \frac{1-e^{-2\gamma t}}{\gamma}\mathbf{I}\right]\mathbf{K} + \\ & \frac{1-e^{-2\gamma t}}{2\gamma}\sigma_q^2, \\ \mathbf{V}_{\mathbf{X}}(t) =& \boldsymbol{\Omega}_t, \\ \mathbf{V}_{\mathbf{q},\mathbf{X}}(t) =& (\boldsymbol{\Omega}_t - \boldsymbol{\Xi}_t)(\gamma \mathbf{I} - \boldsymbol{\Delta})^{-1}\boldsymbol{\Gamma}' + (\gamma \mathbf{I} + \boldsymbol{\Delta})^{-1}[1 - e^{-(\gamma \mathbf{I} + \boldsymbol{\Delta})t}]\mathbf{K} \\ V_\eta(t) =& \sigma_\eta^2 t. \end{aligned}$$

Model Properties

The log spot price now has the following expression;

$$s(t) = \boldsymbol{\Gamma}(\gamma \mathbf{I} - \boldsymbol{\Delta})^{-1}(e^{-\boldsymbol{\Delta} t} - e^{-\gamma \mathbf{I} t})\mathbf{X}_0 + \int_0^t e^{-\gamma(t-\tau)} \sigma_q dW_q(\tau) -$$
$$\boldsymbol{\Gamma}(\gamma \mathbf{I} - \boldsymbol{\Delta})^{-1} \int_0^t (e^{-\gamma \mathbf{I}(t-\tau)} - e^{-\boldsymbol{\Delta}(t-\tau)}) \mathbf{B}_{\mathbf{X}} d\mathbf{W}_{\mathbf{X}}(\tau) +$$
$$\lambda(t) + e^{-\gamma t} q_0 + \eta_0 + \mu t + \int_0^t \sigma_\eta dW_\eta(\tau).$$

Model Properties

Taking the expectation we obtain

$$\mathrm{E}[s(t)] = \lambda(t) + e^{-\gamma t} q_0 + \mathbf{\Gamma}(\gamma \mathbf{I} - \mathbf{\Delta})^{-1}(e^{-\mathbf{\Delta} t} - e^{-\gamma \mathbf{I} t})\mathbf{X}_0 + \eta_0 + \mu t.$$

The variance of the log spot price is given by

$$\mathrm{Var}[s(t)] = V_q + V_\eta,$$

Financial Derivatives

The model can be used for pricing of financial derivatives.

Under the risk-neutral probability measure, the forward price is equal to the expected spot price, i.e.,

$$\begin{aligned}\ln(F_{t,T}) =& \ln(\mathrm{E}^Q[S_T|\mathcal{F}_t]),\\ =& \mathrm{E}^Q[\ln S_T|\mathcal{F}_t] + \frac{1}{2}\mathrm{Var}^Q[\ln S_T|\mathcal{F}_t],\\ =& \lambda(T) + e^{-\gamma(T-t)}q_t + \mathbf{\Gamma}(\gamma\mathbf{I} - \mathbf{\Delta})^{-1}(e^{-\mathbf{\Delta}(T-t)} - e^{-\gamma\mathbf{I}(T-t)})\mathbf{X}_t\\ & + \eta_t + A(T-t),\end{aligned}$$

where

$$A(T-t) = \mu(T-t) + \frac{V_q(T-t) + V_\eta(T-t)}{2},$$

Financial Derivatives

Given q_0, $\mathbf{X}_0$ and η_0, the log forward price is normally distributed with expectation given by

$$\begin{aligned}
\mathrm{E}[\ln F_{t,T}] =& \lambda(T) + e^{-\gamma(T-t)}[e^{-\gamma t}q_0 + \mathbf{\Gamma}(\gamma\mathbf{I} - \mathbf{\Delta})^{-1}(e^{-\mathbf{\Delta}t} - e^{-\gamma\mathbf{I}t})\mathbf{X}_0] + \\
& \mathbf{\Gamma}(\gamma\mathbf{I} - \mathbf{\Delta})^{-1}(e^{-\mathbf{\Delta}(T-t)} - e^{-\gamma\mathbf{I}(T-t)})e^{\mathbf{\Delta}t}\mathbf{X}_0 + \\
& \mu t + \eta_0 + A(T-t),
\end{aligned}$$

Financial Derivatives

The variance is given by

$$\begin{aligned}\mathrm{Var}[\ln F_{t,T}] =& e^{-2\gamma(T-t)} V_q(t) + \sigma_\eta^2 t + \\ & \mathbf{\Gamma}(\gamma \mathbf{I} - \mathbf{\Delta})^{-1}(e^{-\mathbf{\Delta}(T-t)} - e^{-\gamma \mathbf{I}(T-t)})\mathbf{V_X}(t) \times \\ & (e^{-\mathbf{\Delta}(T-t)} - e^{-\gamma \mathbf{I}(T-t)})(\gamma \mathbf{I} - \mathbf{\Delta})^{-1}\mathbf{\Gamma}' + \\ & 2\mathbf{\Gamma}(\gamma \mathbf{I} - \mathbf{\Delta})^{-1}(e^{-\mathbf{\Delta}(T-t)} - e^{-\gamma \mathbf{I}(T-t)})e^{-2\gamma(T-t)}\mathbf{V_{q,X}}.\end{aligned}$$

The instantaneous variance is found by differentiating $\mathrm{Var}[\ln F_{t,T}]$ with respect to t. The model can be used to price European style financial options by inserting the variance into the Black 76 formula.

State Space Form

Let $t_k, k = 1, \ldots, n$ denote the sampling dates, and assume that $t_k - t_{k-1} = \tau$ is a constant. The evolution of the state variables is described by the transition equation, which is given by

$$\boldsymbol{\alpha}_{t+1} = \mathbf{u} + \mathbf{T}\boldsymbol{\alpha}_t + \boldsymbol{\omega}_t, \tag{1}$$

where $\boldsymbol{\alpha}_t = \begin{bmatrix} q_t & \mathbf{X}_t & \eta_t \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 0 & \mathbf{0} & \mu \end{bmatrix}$ and

$$\mathbf{T} = \begin{bmatrix} e^{-\gamma\tau} & \boldsymbol{\Gamma}(\gamma\mathbf{I} - \boldsymbol{\Delta})^{-1}(e^{-\boldsymbol{\Delta}\tau} - e^{-\gamma\tau}\mathbf{I}) & 0 \\ \mathbf{0} & e^{-\boldsymbol{\Delta}\tau} & \mathbf{0} \\ 0 & \mathbf{0} & 1 \end{bmatrix}.$$

$\boldsymbol{\omega}_t$ is a vector of serially uncorrelated, Gaussian distributed disturbances with $\mathrm{E}[\boldsymbol{\omega}_t] = \mathbf{0}$ and $\mathrm{Var}[\boldsymbol{\omega}_\mathbf{t}] = \begin{bmatrix} V_q(\tau) & \mathbf{V}_{\mathbf{q},\mathbf{X}}(\tau) & 0 \\ \mathbf{V}_{\mathbf{q},\mathbf{X}}(\tau) & \mathbf{V}_{\mathbf{X}}(\tau) & \mathbf{0} \\ 0 & \mathbf{0} & V_\eta(\tau) \end{bmatrix}$.

State Space Form

Define $\mathbf{y}_t = [\ln F(t, T_1), \ldots, \ln F(t, T_n)]'$, where $\ln F(t, T_i)$, $i = 1, \ldots, n$ are observed log forward prices at time t. The measurement equation is obtained by adding measurement errors $\boldsymbol{\epsilon}_t$ to the forward valuation formula;

$$\mathbf{y}_t = \mathbf{d}_t + \mathbf{Z}_t \boldsymbol{\alpha}_t + \boldsymbol{\epsilon}_t.$$

Here $\mathbf{d}_t = [A(T_1 - t), \ldots, A(T_n - t)]'$, $\boldsymbol{\epsilon}_t$ is a vector of serially uncorrelated, normally distributed disturbances; $\epsilon_t \overset{IID}{\sim} N(\mathbf{0}, \mathbf{H})$, and

$$\mathbf{Z}_t = \begin{bmatrix} e^{-\gamma(T_1 - t)} & \Gamma(\gamma \mathbf{I} - \boldsymbol{\Delta})^{-1}\left(e^{-\boldsymbol{\Delta}(T_1 - t)} - e^{-\gamma \mathbf{I}(T_1 - t)}\right) & 1 \\ \vdots & \vdots & \vdots \\ e^{-\gamma(T_n - t)} & \Gamma(\gamma \mathbf{I} - \boldsymbol{\Delta})^{-1}\left(e^{-\boldsymbol{\Delta}(T_n - t)} - e^{-\gamma \mathbf{I}(T_n - t)}\right) & 1 \end{bmatrix}.$$

ICE Brent Oil Futures

- The ICE Brent crude data is a collection of daily future prices, Pos 1 – Pos 30.
- The data set spans 07.02.2005 – 06.09.2010, 43170 observations denominated in US Dollars.
- No distinct seasonality in the forward prices
- Fitting the model to the ICE Brent data is fairly straight forward

ICE Forward Price Surface

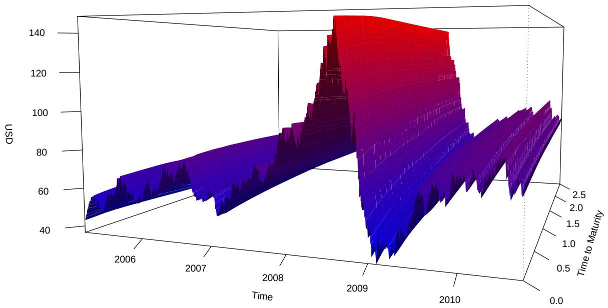

Figure: Plot of the ICE Brent futures contracts surface.

Nord Pool Electricity Swaps

- Nord Pool Electricity data is a panel of daily observations of traded forward and future contracts.
- The observation period spans 01.01.2004 to 30.04.2010, counting 26878 observations denominated in Euro.
- The dataset consists of weekly forward contracts with Pos 1 – 3, monthly contracts with Pos 1 – 6, quarterly contracts with Pos 3 – 4, and yearly contracts with Pos 2 – 5.
- The observed prices are the price of swaps, not forwards.

Approximating the Swap contracts with a single forward contract

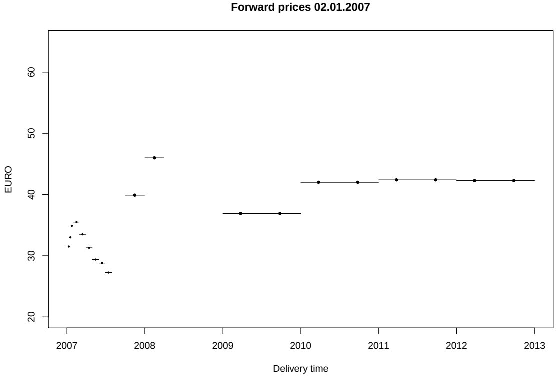

Figure: Approximating the Nord Pool swap contracts with a forward contract.

Seasonality

- Due to seasonal patterns in demand of electricity, it is to be expected that the spot price has a seasonal component.
- There is apparently no seasonal variation in the spot price in our sample period.
- Market participants incorporate a seasonal effect in their derivative valuation process. We therefore calibrate the seasonal terms to expected seasonality as observed in the forward market.

Nord Pool Spot Price

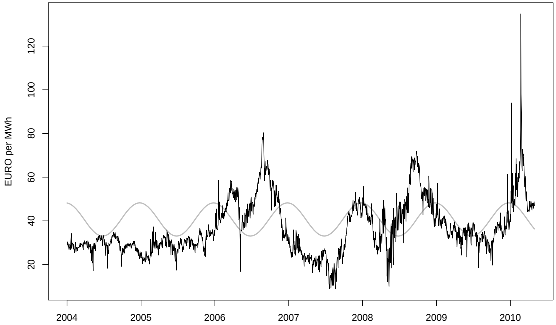

Figure: Nord Pool spot price and seasonality estimated from monthly forward contracts. There is not much seasonality or price spikes observed in Nord Pool spot prices in our sample period.

Seasonality

The seasonal term $\lambda(t)$ is assumed to follow a sinusoidal function on the form

$$\lambda(t) = a_1 + a_2 \cos\left((t + a_3)\frac{2\pi}{365}\right).$$

We use the monthly contracts to extract the seasonality. Parameter estimates are given by

$$\begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \hat{a}_3 \end{bmatrix} = \underset{a_1,a_2,a_3}{\arg\min} \sum_{m=1}^{12} \sum_{c=1}^{6} \left| \lambda(t_m) - \overline{\log F_c(t_m)} \right|$$

where t_m denotes the middle of the month number m, $\overline{\log F_c(t_m)}$ is the average log price taken over the sample period of delivery in month number m for the monthly contract with Pos c. The parameter estimates are $\hat{a}_1 = 3.659$, $\hat{a}_2 = 0.185$ and $\hat{a}_3 = 5.745$.

Model Specification

We restate our model here for convenience

$$
\begin{aligned}
s(t) =& \lambda(t) + q(t) + \eta(t), \\
d\eta(t) =& \mu dt + \sigma_\eta dW_\eta, \\
dq(t) =& [-\gamma q(t) + \mathbf{\Gamma X}(t)]dt + \sigma_q dW_q, \\
d\mathbf{X}(t) =& -\mathbf{\Delta X}(t)dt + \mathbf{B_X} d\mathbf{W_X}.
\end{aligned}
$$

For the ICE data we assume that $\lambda(t) \equiv 0$.

Model Specification

We fit 4 models to the ICE and Nord Pool data. More specifically, the models are obtained by choosing the state variables and parameters in the following way:

$$\mathbf{X}(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}',$$

$$\mathbf{\Delta} = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix},$$

$$\mathbf{B_X} = \begin{bmatrix} \sigma_{x_1} & 0 \\ 0 & \sigma_{x_2} \end{bmatrix},$$

$$d\mathbf{W}_X = \begin{bmatrix} dW_{x_1} & dW_{x_2} \end{bmatrix}',$$

Model Specification

The four models are then defined by setting

$$
\begin{aligned}
&\text{Model 1:} \quad \boldsymbol{\Gamma} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad \mathrm{E}[dW_q dW_\eta] = 0dt, \quad \mathrm{E}[dW_q d\mathbf{W_X}] = \mathbf{K}dt, \\
&\text{Model 2:} \quad \boldsymbol{\Gamma} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \mathrm{E}[dW_q dW_\eta] = 0dt, \quad \mathrm{E}[dW_q d\mathbf{W_X}] = \mathbf{K}dt, \\
&\text{Model 3:} \quad \boldsymbol{\Gamma} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad \mathrm{E}[dW_q dW_\eta] = 0dt, \quad \mathrm{E}[dW_q d\mathbf{W_X}] = \mathbf{0}dt, \\
&\text{Model 4:} \quad \boldsymbol{\Gamma} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad \mathrm{E}[dW_q dW_\eta] = \rho dt, \quad \mathrm{E}[dW_q d\mathbf{W_X}] = \mathbf{0}dt.
\end{aligned}
$$

Model Specification

For computational simplicity we assume that the covariance matrix H_t is diagonal. For the Nord Pool data the measurement error covariance matrix is assumed to be on the form

$$\mathbf{H} = \begin{bmatrix} \Sigma_W & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Sigma_M & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Sigma_Q & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \Sigma_Y \end{bmatrix},$$

where $\Sigma_W = \sigma_W^2 \mathbf{I}_{3\times3}$, $\Sigma_M = \sigma_M^2 \mathbf{I}_{6\times6}$, $\Sigma_Q = \sigma_Q^2 \mathbf{I}_{2\times2}$, and $\Sigma_Y = \sigma_Y^2 \mathbf{I}_{8\times8}$. For the ICE data we assume that one variance applies to all measurement errors for the log of futures prices

$$\mathbf{H} = \mathbf{I}_{30\times30}\sigma_{ICE}^2.$$

Results

ICE Brent Oil Futures

ICE Brent Oil Futures Parameter Estimates

Estimated model parameters ICE data

	Model 1		Model 2		Model 3		Model 4	
μ	-0.030	(0.001)	-0.037	(0.001)	-0.041	(0.001)	-0.051	(0.001)
γ	0.981	(0.024)	1.433	(0.027)	1.026	(0.005)	1.027	(0.005)
δ_1	6.421	(0.066)	1.434	(0.028)	-	(-)	-	(-)
δ_2	0.891	(0.024)	-	(-)	-	(-)	-	(-)
σ_η	0.191	(0.005)	0.192	(0.005)	0.192	(0.005)	0.238	(0.015)
σ_q	0.181	(0.005)	0.159	(0.005)	0.578	(0.005)	0.578	(0.005)
σ_{x_1}	0.534	(0.017)	0.165	(0.005)	-	(-)	-	(-)
σ_{x_2}	0.134	(0.004)	-	(-)	-	(-)	-	(-)
$\rho_{\eta,q}$	-	(-)	-	(-)	-	(-)	0.644	(0.061)
$\rho_{x_1,q}$	-0.383	(0.037)	0.263	(0.052)	-	(-)	-	(-)
$\rho_{x_2,q}$	0.347	(0.046)	-	(-)	-	(-)	-	(-)
logl.	136367.7		125165.6		108074.8		108095.9	

Table: Estimated model parameters of Model 1 – 4. Standard deviations given in parenthesis.

In Sample Fit

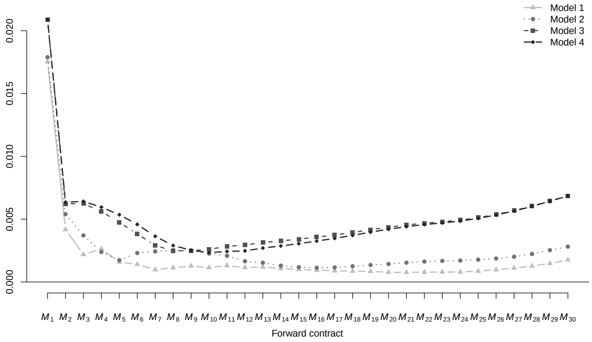

Figure: Root mean squared in-sample pricing errors using the ICE oil data spanning 07.02.2005 – 01.01.2008.

Out of Sample Fit

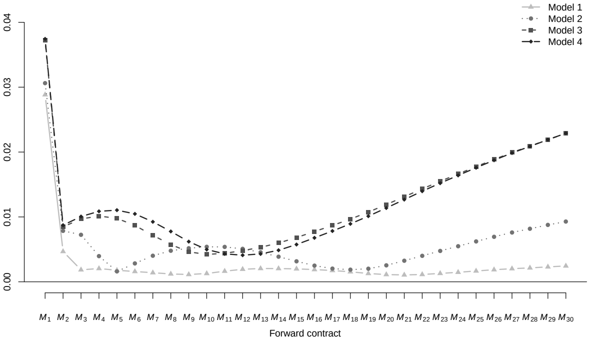

Figure: Root mean squared out-of-sample ICE pricing errors. The models are calibrated using data spanning 07.02.2005 to 01.01.2008, and out-of-sample pricing errors calculated for the period 02.01.2009 – 06.09.2010.

Volatility Term Structure

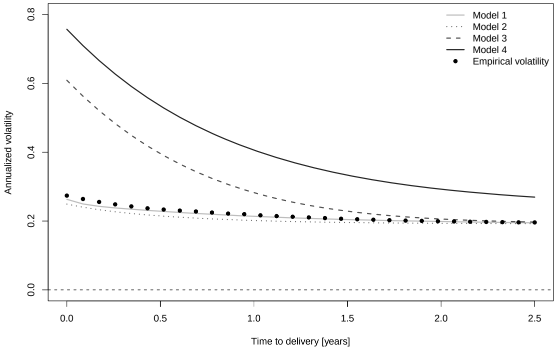

Figure: Term structure of volatility from the model for the ICE Brent data and observed instantaneous forward price volatility.

Results

Nord Pool Electricity Swaps

Nord Pool Electricity Swaps Parameter Estimates

Estimated model parameters Nord Pool data

	Model 1		Model 2		Model 3		Model 4	
μ	-0.006	(0.001)	-0.008	(0.001)	-0.009	(0.001)	-0.010	(0.001)
γ	3.286	(0.114)	3.204	(0.084)	2.094	(0.015)	2.095	(0.015)
δ_1	3.291	(0.110)	3.204	(0.084)	-	(-)	-	(-)
δ_2	0.373	(0.018)	-	(-)	-	(-)	-	(-)
σ_η	0.098	(0.004)	0.124	(0.004)	0.120	(0.004)	0.129	(0.006)
σ_q	0.448	(0.013)	0.416	(0.013)	1.448	(0.018)	1.448	(0.018)
σ_{x_1}	1.677	(0.080)	1.744	(0.074)	-	(-)	-	(-)
σ_{x_2}	0.704	(0.039)	-	(-)	-	(-)	-	(-)
$\rho_{\eta,q}$	-	(-)	-	(-)	-	(-)	0.423	(0.087)
$\rho_{x_1,q}$	-0.077	(0.054)	0.005	(0.058)	-	(-)	-	(-)
$\rho_{x_2,q}$	0.433	(0.038)	-	(-)	-	(-)	-	(-)
logl.	50778.29		48472.85		43997.92		44006.07	

Table: Estimated model parameters of Model 1 – 4. Standard deviations given in parenthesis.

In Sample Fit

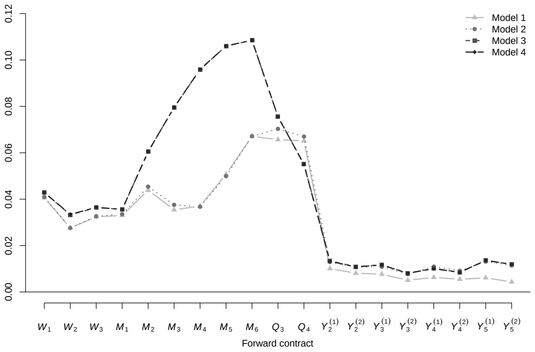

Figure: Root mean squared in-sample pricing errors using Nord Pool data spanning 01.01.2004 – 31.12.2008.

Out of Sample Fit

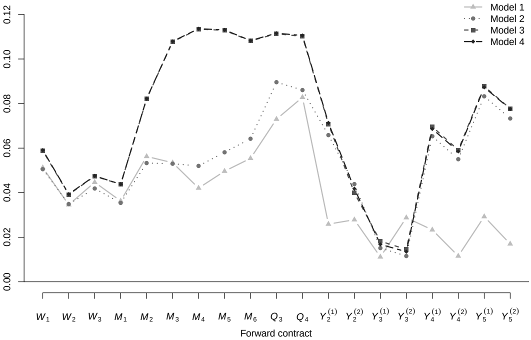

Figure: Root mean squared out-of-sample Nord Pool pricing errors. The models are calibrated using data spanning 01.01.2004 to 01.01.2008, and out-of-sample pricing errors calculated for the period 02.01.2009 – 30.04.2010.

Volatility Term Structure

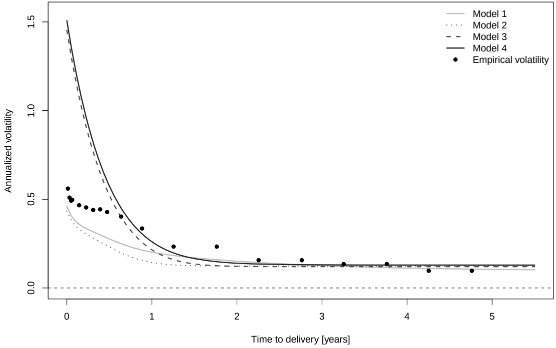

Figure: Term structure of volatility from the model for the Nord Pool data and the observed instantaneous swap price volatility.

Volatility Term Structure

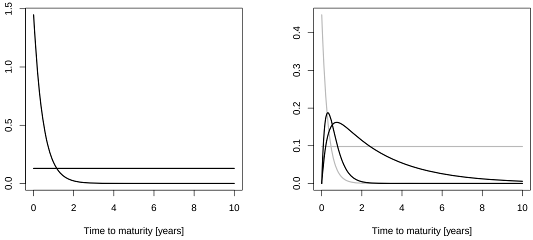

Figure: Left: scaled effects of the short- and long-term factors of Model 4. Right: scaled effects of the factors of Model 1. The plot is drawn for the Nord Pool data, and similar results are found for the ICE data.

Residual Variances

Residual variances

	Model 1	Model 2	Model 3	Model 4
		Nord Pool		
σ_W^2	0.0007	0.0007	0.0009	0.0009
σ_M^2	0.0019	0.0019	0.0069	0.0069
σ_Q^2	0.0056	0.0061	0.0059	0.0059
σ_Y^2	0.0003	0.0008	0.0009	0.0009
		ICE Brent		
σ_{ICE}^2	1.0369e-06	3.3549e-06	1.7219e-05	1.7216e-05

Table: Residual variances from the calibration of the models to the price of traded forwards at ICE and traded swaps at Nord Pool.

Conclusion

- Empirical results indicate that the classical two-factor model by Schwartz and Smith (2000) fail to accurately fit the observed forward prices and volatility structure in our data sets.
- Our model provides better in- and out of sample fit.
- The three and four factor models offer superior fit to observed volatility term structure.
- Model is easily estimated using the Kalman filter for the ICE Brent data.
- A simple approximation for the price of swap contracts can be used to calibrate the model in markets such as Nord Pool.