The Information Premium in Electricity Markets

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Table of contents

Motivation

What is the relationship between spot and forward?

Risk Premium

The risk premium is the difference between the forward price and the expected spot price:

$$R(t,T) = F(t,T) - \mathbb{E}[S(T)|\mathcal{F}_t]$$

where $\mathcal{F}$ is the historical filtration.

- *Risk premium* exists for all underlyings, in particular electricity
- Several ideas to explain its shape and size:
  - Changing to risk-neutral measure, assuming $F(t,T) = \mathbb{E}^{\mathbb{Q}}[S(T)|\mathcal{F}_t]$
  - Bessembinder and Lemmon: Equilibrium approach
  - Benth, Cartea, Kiesel: Influence of market power
- Here, we will introduce an information approach

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Motivation 1/4

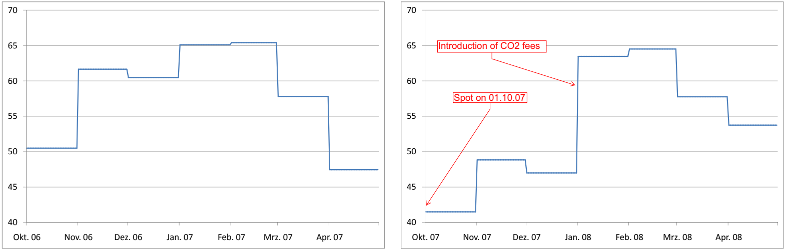

Figure: EEX Forward prices observed on 01/10/06 (left) and 01/10/07 (right)

- Typical winter and bank holidays behaviour in both graphs
- General upward shift in 2008

$\Rightarrow$ 2nd phase of CO_2 certificates

Motivation 2/4

- Future information is incorporated in the forward price
- ... but not necessarily in the spot price due to non-storability
- ... buy-and-hold strategy does not work

Motivation 3/4

- Assuming the *Efficient Markets Hypothesis* holds one has the well-known relation between spot and forward:

$$F(t,T) = \mathbb{E}^{\mathbb{Q}}[S_T | \mathcal{F}_t]$$

- Not sufficient: natural filtration $\mathcal{F}_t = \sigma(S_s, s \le t)$

- Idea: enlarge the filtration!

- ... by information about the spot at some future time T_Υ
- Info could be that spot will be in certain interval...
- ... or the value of a correlated process (temperature)

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Motivation 4/4

Filtrations

- $\mathcal{F}_t$ - the historical filtration
- $\mathcal{H}_t$ - complete information, i.e. $\mathcal{H}_t = \mathcal{F}_t \vee \sigma(S(T_\Upsilon))$
- $\mathcal{G}_t$ - the filtration of all information publicly available to the market

- Hence, we have the relation $\mathcal{F}_t \subseteq \mathcal{G}_t \subseteq \mathcal{H}_t$
- In the following we will consider the observed forward as

$$F(t,T) = \mathbb{E}^{\mathbb{Q}}[S(T)|\mathcal{G}_t]$$

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The information premium 1/2

- Quantify the influence of future information using:

Information Premium

The information premium is defined to be

$$I(t, T) = \mathbb{E}[S_T|\mathcal{G}_t] - \mathbb{E}[S_T|\mathcal{F}_t]$$

i.e. the difference between the prices of the forward under $\mathcal{G}$ and $\mathcal{F}$.

The information premium 2/2

Lemma

The information premium is orthogonal to the space $L^2(\mathcal{F}_t, \mathbb{P})$.

Proof:

$$\mathbb{E}[I_{\mathcal{G}}(t,T) \mid \mathcal{F}_t] = \mathbb{E}[\ \mathbb{E}[S(T) \mid \mathcal{G}_t] - \mathbb{E}[S(T) \mid \mathcal{F}_t]\ \mid \mathcal{F}_t] = 0$$

- Result valid for all measures equivalent to $\mathbb{P}$
- Usual method to attain the *Risk Premium* is a measure change
- This is not possible for the *Information Premium*

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The information premium with delivery period

Spot price model and forward price with delivery

Two-factor arithmetic Spot Price

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

$$X(T) = e^{-\alpha(T-t)}X(t) + \sigma \int_t^T e^{\alpha(T-s)}dW(s)$$

$$Y(T) = e^{-\beta(T-t)}Y(t) + \int_t^T e^{\beta(T-s)}dL(s)$$

where $\Lambda(t)$ is deterministic, $W(t)$ a BM, $L(t)$ a Lévy process.

- The forward price with delivery in $[T_1, T_2]$ is then given by

$$F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \mathbb{E}\left[\int_{T_1}^{T_2} S(u)du \,\middle|\, \mathcal{F}_t\right]$$

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Enlargement of filtrations

- We will demonstrate how to calculate the information premium in this model
- We will enlarge the historical filtration of Lévy process L_t...
- ... with future information about the value L_{T_Υ}
- *Grossissements de filtrations*:
  - Developed by French Mathematicians (Jeulin, Yor) in the 1980s
  - First theorem by Itō in 1976

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Itō's theorem for Lévy processes and additional incomplete information

Theorem

Let L_t be a Lévy process and $\mathcal{G}_t \subseteq \mathcal{H}_t = \mathcal{F}_t \vee \sigma(L_{T_\Upsilon})$. Then

1. *L is still a semimartingale with respect to $\mathcal{G}_t$*
2. *if $\mathbb{E}[|L_t|] < \infty$ then*

$$\xi(t) = L_t - \int_0^{t \wedge T_\Upsilon} \frac{\mathbb{E}[L_{T_\Upsilon} - L_s | \mathcal{G}_s]}{T_\Upsilon - s} \, ds$$

is a $\mathcal{G}_t$-martingale.

Future Lévy information 1/3

- The info premium is

$$I_{\mathcal{G}}(t, T_1, T_2; T_\Upsilon) = F_{\mathcal{G}}(t, T_1, T_2) - F_{\mathcal{F}}(t, T_1, T_2)$$

- Brownian motion terms as well as X_t and Y_t terms cancel (both filtrations coincide), thus

$$I_{\mathcal{G}}(t, T_1, T_2; T_\Upsilon) = \frac{1}{T_2 - T_1} \mathbb{E}\left[\int_{T_1}^{T_2} \int_t^u e^{-\beta(u-s)} dL(s) du | \mathcal{G}_t\right] - \frac{1}{T_2 - T_1} \hat{\beta}(t, T_1, T_2) \phi'(0)$$

- where $\hat{\beta}$ is some deterministic function (> 0) and ϕ is the log-moment-generating function of L_1

Future Lévy information 2/3

- We now apply Itō's theorem

(remember $\xi(t) = L(t) - \int_0^t \frac{\mathbb{E}[L(T_\Upsilon) - L(s)|\mathcal{G}_s]}{T_\Upsilon - s}\, ds$ is a $\mathcal{G}$-martingale)

$$
\begin{aligned}
\mathbb{E}&\left[\int_{T_1}^{T_2}\int_t^u e^{-\beta(u-s)} dL(s) du|\mathcal{G}_t\right] \\
&= \mathbb{E}\left[\int_{T_1}^{T_2}\int_t^u e^{-\beta(u-s)} \frac{\mathbb{E}[L_{T_\Upsilon} - L_s|\mathcal{G}_s]}{T_\Upsilon - s}\, ds\, du|\mathcal{G}_t\right] \\
&= \dots \\
&= \frac{\mathbb{E}[L_{T_\Upsilon} - L_t|\mathcal{G}_t]}{T_\Upsilon - t}\, \hat{\beta}(t, T_1, T_2)
\end{aligned}
$$

Future Lévy information 3/3

- Collecting terms yields

$$I_{\mathcal{G}}(t, T_1, T_2; T_\Upsilon) = \frac{1}{T_2 - T_1}\, \hat{\beta}(t, T_1, T_2) \left(\frac{\mathbb{E}[L_{T_\Upsilon} - L_t | \mathcal{G}_t]}{T_\Upsilon - t} - \phi'(0) \right)$$
$$= \frac{1}{T_2 - T_1} \frac{\hat{\beta}(t, T_1, T_2)}{T_\Upsilon - t} \left(\mathbb{E}[L_{T_\Upsilon} | \mathcal{G}_t] - \mathbb{E}[L_{T_\Upsilon} | \mathcal{F}_t] \right)$$

- Sign of the premium depends on $\mathbb{E}[L_{T_\Upsilon} | \mathcal{G}_t] - \mathbb{E}[L_{T_\Upsilon} | \mathcal{F}_t]$
- ... which matches the intuition:
  - i.e. CO_2 certificates: positive premium $\Rightarrow \mathbb{E}[L_{T_\Upsilon} | \mathcal{G}_t] > \mathbb{E}[L_{T_\Upsilon} | \mathcal{F}_t]$

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Summary so far:

- We have seen an idea of what is possible theoretically
- As a by-product we thought about how to take expectations for this spot model and with delivery periods
- Remember also the orthogonality result of the information premium $\mathbb{E}[I_{\mathcal{G}}(t,T) \mid \mathcal{F}_t] = 0$

Empirical Study - ”showing” the information premium exists

Showing the existence of the info premium 1/2

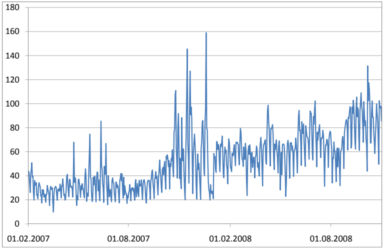
- Agenda:
  1. Calibrate the spot model to observed data (EEX)
  2. Calculate expectations under $\mathbb{P}$
  3. Conduct for each class of month-forwards a constant distance-minimising change of measure (ls-sense)
  4. Calculate expectation under $\mathbb{Q}$
  5. Assume observed forward price $\hat{F}(t, T_1, T_2)$ is $F^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$
  6. For the life-time of different forwards calculate

$$\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2) = \hat{F}(t, T_1, T_2) - F^{\mathbb{Q}}_{\mathcal{F}}(t, T_1, T_2)$$

(where the January 2008 forward will be our main example)

Spot Calibrating

- EEX spot from 01/02/2007 to 30/10/2008
- Includes CO_2-date 01/01/08 as midpoint



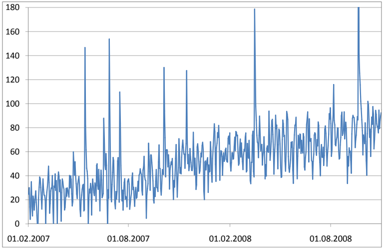


Figure: Spot and Simulation for data set

Expectations and change of measure

- Prices under $\mathbb{P}$ and $\mathbb{Q}$ and observed January 2008 forward

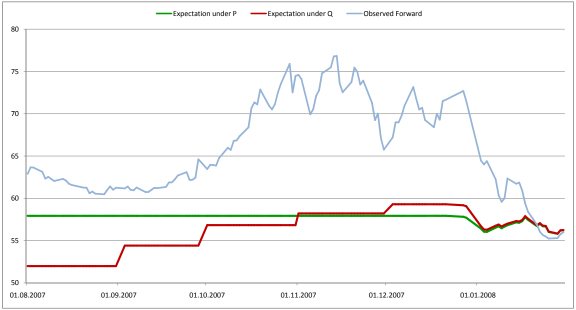


Figure: Observed, $\mathbb{E}^{\mathbb{P}}$ and $\mathbb{E}^{\mathbb{Q}}$ Prices

The information premium?

- The residual $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2) = \hat{F}(t, T_1, T_2) - F^{\mathbb{Q}}_{\mathcal{F}}(t, T_1, T_2)$
  - is positive approx. between 5 and 20 €
  - converges in the delivery period

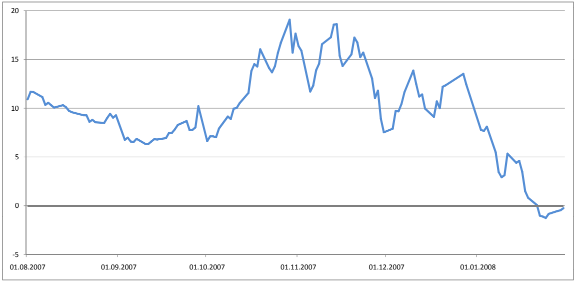


Figure: The residual $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$ for the January 2008 forward

Showing the existence of the info premium 2/2

- $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$ is our best guess for $I(t, T_1, T_2)$!
- We need to show that:
  1. $\hat{I}^{\mathbb{Q}}_{\mathcal{G}} \neq 0$
  2. $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$ is not $\mathcal{F}_t$-measurable, i.e. $\mathbb{E}[\hat{I}^{\mathbb{Q}}_{\mathcal{G}} | \mathcal{F}_t] = 0$
- We will consider Nov07, Jan08, Mar08 and Aug08 contracts (lifetime before, during and after 01/01/08)

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$\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$ non-zero?

- $\hat{I}^{\mathbb{Q}}_{\mathcal{G}} \neq 0$
  - All four series are clearly not white-noise
  - Ljung-Box test rejected at all levels

How do we show non-measurability?

- We want to show $\mathbb{E}[\hat{I}_{\mathcal{G}}^{\mathbb{Q}}|\mathcal{F}_t] = 0$
  - Consider Hilbert space $L^2(\mathcal{F}, \mathbb{Q})$
  - Try to express $\hat{I}_{\mathcal{G}}^{\mathbb{Q}}$ in terms of a countable basis of the spot...
  - ... by means of regression from S onto $\hat{I}_{\mathcal{G}}^{\mathbb{Q}}$
  - Non-measurability $\Rightarrow$ Bad regression results!
- For now, let $\mathcal{B} = \{x^i : i \in \mathcal{I}\}$ the polynomial basis
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Regression results

- Regression: $\triangle \hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2) = \sum_{i=1}^{N} c_i \triangle S^i_t + \epsilon(t)$

Regression results for $N = 10$

	Nov 07	Jan 08	Mar 08	Aug 08
R^2	0.14	0.07	0.03	0.07
$F - stat$	1.47	0.65	0.35	0.75

  - F-value for 95% is 1.88, thus we cannot reject $c_1 = \ldots = c_N = 0$
  - Increasing N does not alter the results
  - Contracts living on 01/01/08 show more extreme results!
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Discussion

- Size of $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$ for Jan08:
  - 2007: CO_2 price practically zero
  - 2008: around 22 €
  - assume $0.7 tCO_2/MWh$ efficiency rate
  - $\Rightarrow$ info premium should be around $0.7 \cdot 22$ € ≈ 15 €
  - $\Rightarrow$ which $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$ is!
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Conclusion and future research

- Conclusion:
  - Introduced the notion of the information premium to explain the spot-forward relationship
  - Briefly showed that theoretical results are possible
  - Provided algorithm to empirically check existence of info premium
  - ... which yields a time series that makes sense in shape and value
  - ... which explains what happened when CO_2 fees were introduced

References

- R. B-K, F. E. Benth, R. Kiesel *An empirical study of the information premium on electricity markets*, to appear

- F. E. Benth and Th. Meyer-Brandis *The information premium for non-storable commodities*, Journal of Energy Markets, 2009

Thank you for your attention...

Appendix I - Spot calibration

	Mean	Stnd. Dev.	Skewness	Kurtosis
Observed	52.18	23.94	0.69	0.28
Simulated	52.93	26.20	1.36	8.93

Table: First four moments of the original series and of the simulated paths

Parameter	α	σ	β	λ	p	q	η_1	η_2
Value	0.538	11.108	0.786	0.034	0.955	0.045	0.019	0.027

Table: Fitted parameter values for the data set 01/02/2007 until 30/10/2008

Appendix II - Change of measure

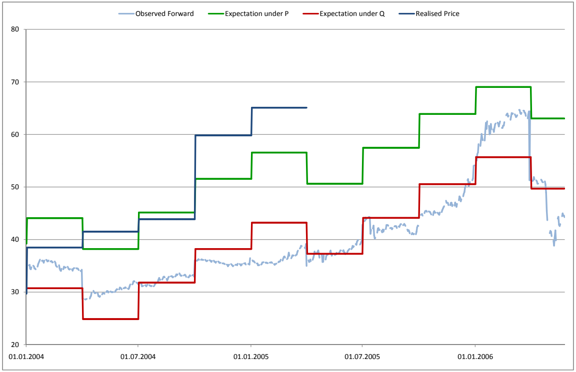

Figure: Observed, $\mathbb{E}^{\mathbb{P}}$ and $\mathbb{E}^{\mathbb{Q}}$ Six-month-forward Prices

Appendix III - Regression Results

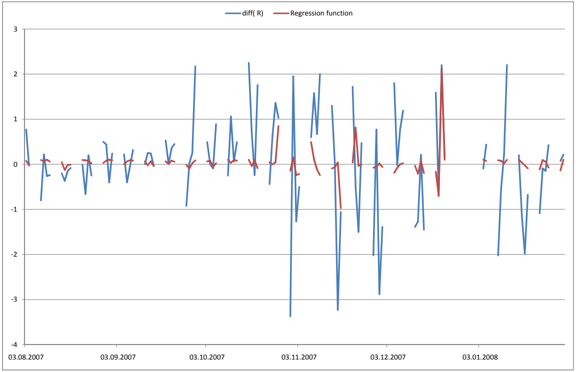

Figure: $\triangle \hat{I}_{\mathcal{G}}^{\mathbb{Q}}$ and regression function

Appendix IV - Forward price formula with delivery

- Risk-neutral valuation formula yields:

$$0 = e^{-rt}\mathbb{E}^{\mathbb{Q}}\left[\int_{T_1}^{T_2} e^{-r(u-t)}(S(u) - F(t, T_1, T_2))du|\mathcal{F}_t\right]$$

- If settlements only take place at the final date T_2 one gets

$$0 = e^{-rt}\mathbb{E}^{\mathbb{Q}}\left[\int_{T_1}^{T_2} (S(u) - F(t, T_1, T_2))du|\mathcal{F}_t\right]$$

- and finally for the futures price:

$$F(t, T_1, T_2) = \mathbb{E}^{\mathbb{Q}}\left[\int_{T_1}^{T_2} \frac{1}{T_2 - T_1} S(u)du|\mathcal{F}_t\right]$$

Appendix V - LSMC vs BKBK

Method	Classical LSMC	New Method
Time	fixed t	$t_k \in [t_0, T_n]$
Regressor	Simulated X_t	stationary $\triangle X_{t_k}$ $\forall k$
Regressand	Simulated $F(X_{t+1})$	stationary $\triangle F(t_k)$ $\forall k$
Goal	Value of cond. expectation	Quality of regression

Table: Comparison of methods