Gas storage valuation using a multi-factor price process
What drives the value of gas storage?

- Maximum volume
- Max. injection/withdrawal rates
- Operational limitations

- Forward curve
- Price volatility
- Cost injection/withdrawal
- Interest rate

- Physical aspect
- Economic aspect
## Gas storage valuation: market perspective

<table>
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<tr>
<th>Strategy</th>
<th>What do we trade?</th>
<th>Multi-factor price process</th>
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<tr>
<td>Rolling intrinsic</td>
<td>Complete forward curve</td>
<td>Easy to implement</td>
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<tr>
<td>Spot-based</td>
<td>(In principle) only spot action</td>
<td>Possible! Current talk</td>
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Boogert & De Jong - Gas Storage Valuation Using a Monte Carlo Method (Journal of Derivatives, Spring 2008)

Adjust Least-Squares Monte Carlo to gas storage valuation
- Use 1-factor price process for simulation
- Include only spot prices into regression

Key results:
- Values converge well for few paths
- Results stable when using different basis functions (powers and b-splines)
Multi-factor price process

Multi-factor needed to capture realistic price behavior
Multi-factor relevant for risk management
Different specifications available

Our price process contains three factors:
1. Spot factor (as 1-factor Schwartz model used in starting point)
2. Long-term factor
3. Winter-summer factor

Note: this model has not been published in the literature so far
Gas storage pricing problem

In each time step:
find optimal decision dependent on price and volume.

Possible decisions:
• Inject
• Do nothing
• Withdraw

American option: compare continuation value and exercise value
Gas storage: compare different continuation values
Least-Squares Monte Carlo – one factor storage

Determine relevant state variables
Generate large number of scenarios
Work backwards, derive exercise boundaries by estimating 
  continuation values as a function of current prices and other 
  state variables

\[ E[Y \mid X] = c_1 f_1(X) + c_2 f_2(X) + \ldots \]

\[ \text{for basis functions } f \text{ by least-squares regression} \]
Multi factor optimization

Original Longstaff-Schwartz article already considered derivatives based on multi factor price process

With two exceptions [KYOS (2006), Li (2007)], this appears not to have been picked up in gas market

LS example: value cancelable index amortizing swaps
• Swap term structure driven by two independent processes: X Y
• Use 9 basis functions
  • Constant
  • First three powers of value underlying non-cancelable swap
  • $X, X^2, Y, Y^2, XY$
Multi-factor optimization for gas storage

State variables

- Three-factor price process delivers three factors (spot X, long-term Y, winter-summer Z)
- Use price state variables X Y Z in regression
- Do not use volume state variable in regression

Basis functions

- Use more spot than long-term & winter-summer in basis
- Use limited number of basis functions
- Alternative: use greedy approach: take as many basis functions as possible while keeping basis regular
Questions

1. Impact of using 1 factor instead of 3 factor price process?
2. Impact of using 1 factor instead of 3 factors in optimization?
3. Impact of basis functions (different families?) and different number of simulations?
Consider simple storage: inject 1, withdraw 1, max volume 100 units

Consider three situations:
- Use 1 factor price process and 1 factor optimization
- Use 3 factor price process and 1 factor optimization
- Use 3 factor price process and 3 factor optimization

Consider 60 different seeds and five different basis functions
- Powers
- Chebyshev of the first kind A
- Laguerre
- Legendre
- Hermite A
Spot approach: impact multi-factor and hedge

10000 simulations, power basis function, 1 seed
Impact of basis function + convergence

Chebyshev and Hermite polynomials under perform
Legendre, Laguerre and power perform similar
Impact of changing market parameters

- ST vol
- LT vol
- WS vol
- Mean-reversion
Impact of operational parameters

Fast storage: inj = 2 / wd = 5 / vol = 100 (earlier inj = wd = 1 / vol = 100)
Conclusion

- When using multi factor price process:
  - Not sufficient to use one factor within optimization
  - Simple power basis performs well (Hermite/Chebyshev not)

- Practical results:
  - Adding factors increases expected standard deviation, but does not really change expected value
  - Static hedge improves no-hedge approach
  - Not all volatility is good for storage operator