Convenience Yield-Based Pricing of Commodity Futures

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1. The objectives and results

Background & Motivation

- Convenience yield (CY) is often used to describe the value to hold commodities as is explained in e.g., Geman(2005).

- CY is useful to represent the linkage between commodity spot and futures prices.

- A general concept of stochastic discount factor (SDF) is developed to determine asset prices including commodity futures prices written on the spot prices. The advantage will be free from market completeness.

- CY may play an alternative role of SDF to price commodity futures traded in incomplete markets.

Literature Survey

- Risk neutral valuation is applied to price commodity derivatives including commodity futures e.g., Schwartz and Smith(2000), Yan(2002), Casassus and Collin-Dufresne(2005), Korn(2005), Eydeland and Wolyniec (2003).

- The application does not necessarily seem satisfactory because commodity market is incomplete by the illiquidity while risk neutral valuation assumes complete markets.

- A familiar SDF selection is a utility-based approach. Davis(2001) and Cao and Wei(2000) use the method to price weather derivatives. But it depends on utility function and optimal consumption.

- Cochrane and Saarequejo(2000) introduced good-deal bounds (GDB) that can avoid the problem of utility based method and incorporate market incompleteness into commodity futures pricing.

- The maximum Sharpe ratio is a key to represent the degree of market incompleteness

- The GDB method is still dissatisfactory because the maximum Sharpe ratio binding SDF must be given exogenously.

The objectives

We try to find an incomplete market pricing formulae for commodity futures that can determine the maximum Sharpe ratio endogenously.

Using the pricing method, we conduct empirical analyses of crude oil, heating oil, and natural gas futures traded on the NYMEX in order to assess the incompleteness of energy futures markets.

We apply the market price of risk embedded in energy futures markets to the Asian call option pricing on crude oil futures.

The results

(1) We propose a convenience yield-based pricing for commodity futures that can endogenously determine the maximum Sharpe ratio. It can embed the incompleteness of commodity futures markets in convenience yield.

(2) Empirical analyses of crude oil, heating oil, and natural gas futures on the NYMEX demonstrate that the fluctuation from incompleteness partly comes from convenience yield.

(3) It is shown that the maximum Sharpe ratio is obtained from the NYMEX data.

(4) We numerically price the Asian call option using the maximum Sharpe ratio estimated from crude oil futures prices.

2. The convenience yield-based pricing of commodity futures

Model Setup

The relationship between spot and futures prices is expressed by two ways: CY and SDF.

SDF can incorporate incompleteness of the market into futures pricing, which will be applicable to CY.

Convenience Yield (CY)

We employ Gibson Schwartz (1990) two-factor model based on CY (δ) to represent the spot price (S).

$$\frac{dS_t}{S_t} = (\mu - \delta_t)dt + \sigma_1 dw_t, \quad (1)$$

$$d\delta_t = \kappa(\alpha - \delta_t)dt + \sigma_2 du_t, \quad (2)$$

where $E_t[dw_t du_t] = \rho dt$. Using Ito's lemma to equation (1), we obtain

$$S_T = S_t e^{(\mu-\alpha-\frac{1}{2}\sigma_1^2)(T-t)+\frac{1}{\kappa}(1-e^{-\kappa(T-t)})(\delta_t-\alpha)+\int_t^T(\sigma_1+\frac{\sigma_2\rho}{\kappa}(1-e^{-\kappa(T-s)}))dw_s+\int_t^T(\frac{\sigma_2\sqrt{1-\rho^2}}{\kappa}(1-e^{-\kappa(T-s)}))dz_s}. \quad (3)$$

We assume that the fluctuation due to CY is spanned by both of complete and incomplete parts:

$$du_t = \rho dw_t + \sqrt{1-\rho^2} dz_t. \quad (4)$$

Stochastic Discount Factor (SDF)

Commodity markets may demonstrate incompleteness because of the illiquidity.

Following Cochrane and Saarequejo(2000), we assume that SDF at time t (Λ_t) is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -rdt - \phi dw_t - \nu dz_t. \tag{5}$$

ϕ and ν are referred to as complete and incomplete market price of risks (CMPR and IMPR), respectively.

Using Ito's lemma to equation (5), we obtain

$$\frac{\Lambda_T}{\Lambda_t} = e^{-(r+\frac{1}{2}\phi^2+\frac{1}{2}\nu^2)(T-t)-\int_t^T \phi dw_s - \int_t^T \nu dz_s}. \tag{6}$$

Linkage between CY and SDF

Using SDF, the futures prices F_t^T are in general represented as follows:

$$F_t^T = E_t\left[\frac{\frac{\Lambda_T}{\Lambda_t} S_T}{E_t\left[\frac{\Lambda_T}{\Lambda_t}\right]}\right]. \tag{7}$$

The convenience yield-based pricing of commodity futures (CY-based model)

We have the futures price as follows:

$$F_t^T = S_t e^{\Upsilon(t,T)-\Omega(t,T)\delta_t}, \tag{8}$$

$$\begin{aligned}\Upsilon(t,T) = {} & (r-\alpha+\frac{\sigma_2^2}{2\kappa^2}-\frac{\sigma_1\sigma_2\rho}{\kappa}+\phi\frac{\sigma_2\rho}{\kappa}+\frac{\nu\sigma_2\sqrt{1-\rho^2}}{\kappa})(T-t)+\frac{\sigma_2^2}{4\kappa^3}(1-e^{-2\kappa(T-t)}) \\ & +(\alpha\kappa+\rho\sigma_1\sigma_2-\frac{\sigma_2^2}{\kappa}-\phi\sigma_2\rho-\nu\sigma_2\sqrt{1-\rho^2})\frac{1-e^{-\kappa(T-t)}}{\kappa^2},\end{aligned} \tag{9}$$

$$\Omega(t,T) = \frac{1-e^{-\kappa(T-t)}}{\kappa}. \tag{10}$$

Model Implication

The point of this model is the inclusion of complete and incompleteness parameter of CMPR (ϕ) and IMPR (ν) into spot-futures price relationship.

We obtained a futures pricing method using convenience yield-based incompleteness parameter ν, not relying on risk neutral measure.

3. Empirical studies for energy prices

Data

We use the daily closing prices of WTI crude oil (WTI), heating oil (HO), and natural gas (NG) futures traded on the NYMEX.

Each futures product includes six delivery months – from one month to six months.

The covered time period is from April 3, 2000 to March 31, 2008.

The data are obtained from Bloomberg.

Incompleteness check using Kalman Filter

By examining the relationship between CMPR ϕ and IMPR ν, we try to find the degree of incompleteness of energy markets.

The parameters of the CY-based pricing are estimated using the Kalman filter.

Both log transformed spot prices (x_t) and CYs (δ_t) are unobservable while log transformed futures prices (y_t) are observable.

Time and measurement update equations

Time update equations:

$$x_t = x_{t-1} - \Delta t \delta_t + (\mu - \frac{1}{2}\sigma_1^2)\Delta t + \sigma_1 \epsilon_t \equiv f_1(x_{t-1}, \delta_{t-1}, \epsilon_t). \quad (11)$$

$$\delta_t = (1 - \kappa \Delta t)\delta_{t-1} + \kappa \alpha \Delta t + \sigma_2 \eta_t \equiv f_2(x_{t-1}, \delta_{t-1}, \eta_t). \quad (12)$$

The measurement update equation is obtained from the futures-spot price relationship.

$$y_t = x_t - \Omega(t, T)\delta_t + \Upsilon(t, T) + \xi_t \equiv h_1(x_t, \delta_t, \xi_t). \quad (13)$$

Note that $V[\xi_t] = \text{diag}[m_1, m_2, m_3, m_4, m_5, m_6]$ (Diagonal matrix).

Maximum likelihood estimation (WTI)

The parameters of the CY-based pricing for crude oil are estimated by the maximum likelihood method:

	μ	σ_1	κ	α	σ_2	ρ	ν
Est	0.563	0.544	1.629	0.093	0.636	0.857	-1.404
(S.E.)	0.000	0.001	0.001	0.002	0.000	0.002	0.000
	m_1	m_2	m_3	m_4	m_5	m_6	
Est	2.197E-4	1.628E-5	1.000E-6	1.000E-6	1.000E-5	7.753E-6	
(S.E.)	1.854E-5	3.004E-6	1.893E-6	1.312E-6	2.747E-5	3.655E-6	
LL	5.461E+4						
AIC	-1.092E+5						
SIC	-1.092E+5						

All parameters except m_3, m_4, and m_5 for WTI are statistically significant.

Maximum likelihood estimation (WTI)

Since ρ is 0.857, the fluctuation from incompleteness is partly owed to CY.

The incompleteness of crude oil market (IMPR) is calculated as $|\nu| = 1.404$ using the market data.

Maximum likelihood estimation (HO)

	μ	σ_1	κ	α	σ_2	ρ	ν
Est	0.568	0.575	1.358	0.069	0.883	0.745	-1.041
(S.E.)	0.196	0.017	0.062	0.249	0.034	0.081	0.234
	m_1	m_2	m_3	m_4	m_5	m_6	
Est	3.619E-4	1.000E-5	2.936E-5	1.000E-5	1.181E-4	6.920E-4	
(S.E.)	6.725E-5	1.889E-5	1.810E-5	2.531E-5	3.485E-5	1.719E-4	
LL	4.325E+4						
AIC	-8.648E+4						
SIC	-8.651E+4						

The parameters except α, m_2, m_3, and m_4 are statistically significant.

Similar to crude oil, we also obtained the existence of incompleteness from CY and the heating oil IMPR using the market data.

Maximum likelihood estimation (NG)

	μ	σ_1	κ	α	σ_2	ρ	ν
Est	0.361	0.995	0.617	-0.416	2.061	0.829	-0.749
(S.E.)	0.299	0.022	0.089	0.660	0.081	0.012	0.252
	m_1	m_2	m_3	m_4	m_5	m_6	
Est	3.314E-3	5.368E-5	1.198E-3	1.033E-3	3.278E-5	2.884E-3	
(S.E.)	1.199E-4	2.124E-5	3.595E-5	3.133E-5	1.393E-5	9.812E-5	
LL	3.083E+4						
AIC	-6.163E+4						
SIC	-6.166E+4						

The parameters except μ and α are statistically significant.

We also obtained the existence of incompleteness from CY and the natural gas IMPR using the market data.

Incompleteness Assessment of Energy Futures Markets

	CMPR $\|\phi\|$	IMPR $\|\nu\|$	Max SR $A = \sqrt{\nu^2 + \phi^2}$
Crude oil	0.924	1.404	1.681
Heating oil	0.883	1.041	1.365
Natural gas	0.302	0.749	0.807

Note that ϕ is calculated as $\phi = \frac{\mu - r}{\sigma_1}$ assuming $r = 0.06$.

The IMPR for crude oil (1.404) is a little greater than the CMPR (0.924). It implies that the crude oil market should be spanned by both complete and incomplete markets.

The derivative pricing on crude oil prices requests the maximum Sharpe ratio of 1.681, which is about twice as large as the CMPR (0.924) based on the GDB.

For heating oil and natural gas, the Sharpe ratios take 1.365 and 0.807, respectively.

Empirical Study Implications

It was shown that the fluctuation from market incompleteness is partly owed to the fluctuation from convenience yield.

We could obtain the IMPR from the NYMEX market data.

4. Application of CY-based model to energy derivative pricing

Asian call option on energy futures prices

The illiquid futures products are considered as the same to the newly introduced derivative in the sense that futures products have no trading maturity period and new products have no trading volume.

IMPR ν obtained from futures market will be useful to price the newly introduced derivative products written on the same underlying asset

We price Asian call option on energy futures prices using the IMPR ν estimated from the NYMEX data.

Asian call option on energy futures prices

In general, GDB pricing is expressed by

$$\underline{C}_t = E_t \int_{s=t}^{T} \frac{\underline{\Lambda}_s}{\underline{\Lambda}_t} x_s ds + E_t\left(\frac{\underline{\Lambda}_T}{\underline{\Lambda}_t} x_T\right), \quad \frac{d\underline{\Lambda}_t}{\underline{\Lambda}_t} = -r dt - \phi dw_t \mp \nu dz_t, \tag{14}$$

where $\mp$ represents lower and upper price boundaries, respectively.

We assume the average i-month futures price from times 0 to $\bar{T}$ as

$$I = \frac{1}{\bar{T}} \int_0^{\bar{T}} F^i(S, \delta, t) dt. \tag{15}$$

We set $\frac{d\underline{C}}{\underline{C}} = \mu_{\underline{C}} dt + \sigma_{\underline{C}w} dw + \sigma_{\underline{C}z} dz$. GDB pricing is transformed into

$$\mu_{\underline{C}} - r + \sigma_{\underline{C}w} \mp \sigma_{\underline{C}z} \nu = 0, \tag{16}$$

where $\mp$ represents lower and upper price boundaries, respectively. Note that $x_s = 0$.

Asian call option on energy futures prices

Applying Ito's lemma to a price boundary $\underline{C}(S,\delta,I,t)$, we have

$$\mu_{\underline{C}} = \frac{1}{\underline{C}}\left\{\frac{\partial \underline{C}}{\partial t} + (\mu-\delta)S\frac{\partial \underline{C}}{\partial S} + \kappa(\alpha-\delta)\frac{\partial \underline{C}}{\partial \delta} + \frac{1}{2}\sigma_1^2 S^2\frac{\partial^2 \underline{C}}{\partial S^2}\right.$$
$$\left. + \frac{1}{2}\sigma_2^2\frac{\partial^2 \underline{C}}{\partial \delta^2} + \rho\sigma_1\sigma_2 S\frac{\partial^2 \underline{C}}{\partial \delta\partial S} + \frac{1}{\bar{T}}F\frac{\partial \underline{C}}{\partial I}\right\},$$
$$\sigma_{\underline{C}w} = \frac{1}{\underline{C}}\left\{\mu S\frac{\partial \underline{C}}{\partial S} + \rho\sigma_2\frac{\partial \underline{C}}{\partial \delta}\right\},$$
$$\sigma_{\underline{C}z} = \frac{1}{\underline{C}}\left\{\sigma_2\sqrt{1-\rho^2}\frac{\partial \underline{C}}{\partial \delta}\right\}.$$

Partial differential equation

$$-r\underline{C} + \frac{\partial \underline{C}}{\partial t} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 \underline{C}}{\partial S^2} + \frac{1}{2}\sigma_2^2 \frac{\partial^2 \underline{C}}{\partial \delta^2} + \rho\sigma_1\sigma_2 S \frac{\partial^2 \underline{C}}{\partial \delta \partial S} + \frac{dI}{dt}\frac{\partial \underline{C}}{\partial I}$$
$$= (\delta - r)S\frac{\partial \underline{C}}{\partial S} + \left(\phi\rho\sigma_2 - \kappa(\alpha - \delta) + k\nu\sigma_2\sqrt{1-\rho^2}\,\mathrm{sgn}\left(\frac{\partial \underline{C}}{\partial \delta}\right)\right)\frac{\partial \underline{C}}{\partial \delta},$$

with the terminal payoff: $\underline{C}(S, \delta, I, \bar{T}) = f(I_{\bar{T}})$, where $k = -1$ and $+1$ generate the upper and lower price boundaries, respectively.

To obtain the GDB prices of the Asian call option, we set the payoff at maturity to be $f(I_{\bar{T}}) = \max(I_{\bar{T}} - K, 0)$ and, following Ingersoll(1987), $\frac{dI}{dt}$ to be: $dI = \frac{1}{\bar{T}}F(S, \delta, t)dt$.

Asian call option price

We computed Asian call option prices written on 1-month crude oil futures prices assuming that the strike price is 70 USD, the delta is zero, and interest rate is set to 6 %.

Futures Prices	70	80	90	100	110	120	130
Upper Price	1.47	7.31	17.40	26.36	35.23	44.15	52.71
No Risk Prem.	1.45	7.25	17.34	26.30	35.15	44.07	52.62
Lower Price	1.43	7.20	17.27	26.24	35.08	44.00	52.54
Upper Premium	0.02	0.05	0.07	0.06	0.07	0.08	0.08
Lower Premium	0.02	0.05	0.07	0.06	0.08	0.08	0.08
UP/NRPP (%)	1.08	0.72	0.39	0.24	0.21	0.17	0.16
LP/NRPP (%)	1.07	0.72	0.39	0.23	0.21	0.17	0.16

Outcomes from Asian call option prices

Both upper and lower risk premiums are small enough comparing with the level of the option prices. It may be easy to use for practitioners in the sense that the option price is priced using small price range.

We were able to obtain the risk premium based on the incompleteness of energy futures market implied from the CY-Based pricing method we proposed.

4. Conclusions and directions for future research

Conclusions

(1) We have proposed a convenience yield-based pricing for commodity futures, which embeds the incompleteness of commodity futures markets in convenience yield.

(2) Empirical analyses of crude oil, heating oil, and natural gas futures on the NYMEX showed that the fluctuation from incompleteness is partly owed to convenience yield.

(3) It was shown that the additional Sharpe ratio, which represents the degree of market incompleteness and is also used for derivative pricing written on energy prices, is obtained from the NYMEX data.

(4) We numerically priced the Asian call option using market price of risk estimated from crude oil futures prices.

Directions for future research

This paper only dealt with energy futures due to the availability of data. The concept in this paper can be extended to other commodity futures like agricultural futures. These empirical studies may be the next direction for our future researches.

Thank you.

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