A comparison of extended electricity price models considering the impact of wind energy feed-in

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1. Background

2. Impact of wind power feed-in on electricity prices

3. Modelling electricity prices via stochastic processes

4. Simulation of wind-power feed-in

5. Extended electricity price modeling considering the impact of wind power feed-in

6. Conclusions and Outlook
Background

- High uncertainties in energy markets due to liberalization and structural changes
- Electricity prices have become very volatile

\[ \text{Stochastic simulation to capture these volatilities} \]

- Volatile feed-in of large amount wind power into grid
- Wind power feed-in (WPF) has a significant impact on the electricity price
- Long-term effects on power plant mix

\[ \text{Evaluation of new investments need to consider uncertain prices and WPF} \]

\[ \text{Therefore a basic knowledge of the interrelations is necessary} \]
Impact of wind power feed-in (WPF) on electricity prices

- historical couples of electricity prices and WPF (2006-2009)

→ the electricity price declines by 1,47 €/MWh on an average per Gigawatt wind power feed-in
→ reason: merit order effect of WPF
→ the average price reduction of 1,47 €/MWh per GW does not explain extreme price decreases of up to -500€/MWh
Impact of wind power feed-in (WPF) on electricity prices

- change of electricity prices depending on the current load

-governmental intervention in the electricity market

- the average price change caused by WPF varies according to the demand level
- possible cause: irregular structure of the merit order curve

\[
\text{average change in electricity price per GW of wind energy feed-in [€/MWh]}
\]

\[
\beta \alpha + \beta = \text{load [GW]}
\]

\[
\text{price change } \alpha_L \text{ dependent on load}
\]

\[
\text{price change without consideration of load}
\]
Modeling electricity prices via stochastic prices

- basic model for power price simulation:

  - Division of the power price into a deterministic and a stochastic components
  - Simulation of the stochastic portion via models of financial mathematics

  ![Diagram](https://via.placeholder.com/150)

- historical power price time series (logarithmized)
- Elimination of the deterministic components
  - Trend
  - annual, weekly, daily cycle
- Stochastic component of the power price
- Simulation of the stochastic component
- simulated power price time series (logarithmized)
- Addition of the deterministic components
  - Trend
  - annual, weekly, daily cycle
- simulated stochastic part of electricity prices
Modelling electricity prices via stochastic prices

Modelling of the stochastic component:

Base regime:

- **Autoregressive mean-average (ARMA(p,q))- process**
  - Assumption: Price $X_{t}^{s}$ depend on the last $p$ prices and $X_{t-p}$ and $q$ innovations $\varepsilon_{t-p}$
  $X_{t}^{BASE,s} = \sum_{i=1}^{p} \alpha_{i}X_{t-i}^{R} + \sum_{j=1}^{q} \beta_{j}\varepsilon_{t-j} + \varepsilon_{t}$
  - Parameters are estimated via MLE (Garch-Toolbox in MATLAB)
  - Innovations: $\varepsilon_{t} \sim$ Laplace($\mu_{\varepsilon},b_{\varepsilon}$)

- **Integrated ARMA (ARIMA)-process**
- **GARCH-Process**
- **Mean-Reversion process**
Jump regime: extension of the base processes

\[ X_{t}^{s,\text{JUMP}} = X_{t}^{s,\text{BASE}} + \ln J_{t} \]

\[ \ln J_{t} \sim N(\mu_{\ln J}, \sigma_{\ln J}^2) \]

- Prices in \( X_{t}^{s} \) beyond a confidence interval \([\mu - 3 \cdot \sigma, \mu + 3 \cdot \sigma]\) are declared as jumps.
- Calculation of switching probabilities as the relative frequency of price switches between the one in and out the confidence interval:

\[
P_{12} = \frac{\text{card}\{l \mid X_{l,t}^{SR} \in [\mu - 3\sigma, \mu + 3\sigma] \land X_{l,t+1}^{SR} \in (\mu + 3\sigma, \ln 3000]\}}{\text{card}\{l \mid X_{l,t}^{SR} \in [\mu - 3\sigma, \mu + 3\sigma]\}}
\]

\[
T = \begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{bmatrix}
\]

- Differentiation of switching probabilities for summer weekdays, winter weekdays and weekends.

\[ \rightarrow \text{Simulation of the stochastic component } X_{t}^{s} \text{ with the regime-switching model and addition of the deterministic ones} \]

\[ \rightarrow \text{Simulated electricity price paths} \]
Modeling electricity prices via stochastic prices – simulation results

*Simulated prices* vs. *Historical spot prices*:
- **Weekly cycle**
- **Long-term average**
- **Jump/spike**

*Simulated PDC* vs. *Historical PDC*:
- **Daily cycle**
Modeling electricity prices via stochastic prices – simulation results

<table>
<thead>
<tr>
<th>Stochastic Model</th>
<th>MRSE [€/MWh]</th>
<th>R2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean-Reversion (MR)</td>
<td>8.51</td>
<td>6.11</td>
</tr>
<tr>
<td>ARMA(1,1), *(5,5)</td>
<td>8.63,</td>
<td>6.86,</td>
</tr>
<tr>
<td></td>
<td>8.62</td>
<td>7.12</td>
</tr>
<tr>
<td>ARIMA(1,1,1), *(5,1,5)</td>
<td>8.15,</td>
<td>5.91,</td>
</tr>
<tr>
<td></td>
<td>8.19</td>
<td>5.55</td>
</tr>
<tr>
<td>GARCH(1,1),*(5,5)</td>
<td>15.10,</td>
<td>9.03,</td>
</tr>
<tr>
<td></td>
<td>17.85</td>
<td>11.07</td>
</tr>
<tr>
<td>MR without RS</td>
<td>19.92</td>
<td>17.52</td>
</tr>
<tr>
<td>GARCH without RS</td>
<td>109.18,</td>
<td>102.01,</td>
</tr>
<tr>
<td></td>
<td>111.84</td>
<td>102.97</td>
</tr>
<tr>
<td>ARIMA without deseasonalizing</td>
<td>9.94,</td>
<td>14.92,</td>
</tr>
<tr>
<td></td>
<td>21.96</td>
<td>13.00</td>
</tr>
</tbody>
</table>
Simulation of wind power feed-in

- Overview of the simulation model

- Estimation of Laplace-distribution parameters depending on the utilization level

- Deseasonalisation

- Analysis of change rates:
  - Laplace / exp. Distributed
  - Direction depends on own history
  - Amount depends on utilization level

- Probabilities for the direction of changes

- Recursive simulation

- Current level

- Distribution parameter

- Height of change

- Direction of change

- Current capacity utilization

- Current change rate

- Simulated capacity utilization without season

- Reseasonalisation

- Simulated capacity utilization with season
Simulation of wind energy feed-in (WEF)

- utilization levels are classified by height
- the distribution of change rates is analysed separately for each class and parameters are determined
- with the help of these distribution parameters the height of the change is determined as a random number dependent on the recent utilization level
• parameters $\mu^+$ und $\mu^-$

$\mu_{t+1}^+ = -5,2 \cdot 10^{-4} \cdot Niv(X_t)^2 + 0,055 \cdot Niv(X_t) + 0,83$

$\mu_{t+1}^- = 0,042 \cdot Niv(X_t) + 0,38$

$\rightarrow \mu^+$ and $\mu^-$ correspond to the mean of the positive and negative changes, that were moved by the modal value

$\rightarrow$ the height of negative change rates grows with increasing utilization level

$\rightarrow$ the height of positive change rates reaches its maximum at medium utilization rates
Simulation approach

→ Recursive simulation of the hourly capacity utilization $X_t^{\text{sim}}$ based on the model of the change rates $\Delta X_t^{\text{sim}}$

$$X_t^{\text{sim}} = X_{t-1}^{\text{sim}} + \Delta X_{t-1}^{\text{sim}}$$
Simulation of wind power feed-in (WPF)

- Simulation results for an actual installed capacity of 26 GW wind power
Extended modeling of power prices including the impact of wind power feed-in

- extension of the base model

**Elimination of the deterministic components**
- Trend
- annual, weekly, daily cycle

**Addition of the deterministic components**
- Trend
- annual, weekly, daily cycle

historical power price time series

stochastic component

wind power feed-in

remaining stochastic price component

Simulation of the remaining stochastic price component via financial models

simulated stochastic power price path

Addition of the impact of wind power feed-in

simulated time series of wind energy feed-in

Simulation of wind power feed-in levels

\[ SP_{hist, stoch} = SP_{hist} - SP_{hist, det} \]

\[ -\Delta SP(WEF_{hist}) \]

impact of the WPF is included to the stochastic, but explainable price component and thus it is treated like a deterministic price component.
Modeling electricity prices under volatile WPF – simulation results

Comparison of the electricity price simulation results **without** and **with** consideration of wind power simulation:

<table>
<thead>
<tr>
<th></th>
<th>2009</th>
<th></th>
<th>2008</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W/O Wind</td>
<td>with Wind</td>
<td>HISTORICAL</td>
<td>W/O Wind</td>
</tr>
<tr>
<td>MRSE [€/MWh]</td>
<td>8.51</td>
<td>6.67</td>
<td>-</td>
<td>6.11</td>
</tr>
<tr>
<td>MAPE</td>
<td>13.93%</td>
<td>10.64%</td>
<td>-</td>
<td>6.40%</td>
</tr>
<tr>
<td>R²</td>
<td>36.60%</td>
<td>38.38%</td>
<td>-</td>
<td>52.53%</td>
</tr>
<tr>
<td>Mean [€/MWh]</td>
<td>41.25</td>
<td>38.86</td>
<td>38.85</td>
<td>67.65</td>
</tr>
<tr>
<td>σ [€/MWh]</td>
<td>23.94</td>
<td>20.16</td>
<td>19.41</td>
<td>33.87</td>
</tr>
<tr>
<td>skewness</td>
<td>1.49</td>
<td>-1.16</td>
<td>-3.23</td>
<td>2.01</td>
</tr>
<tr>
<td>kurtosis</td>
<td>20.08</td>
<td>8.66</td>
<td>83.90</td>
<td>37.85</td>
</tr>
</tbody>
</table>

→ the consideration of the impact of wind power leads to significant improvement of the price simulation
Summary

- The simulation results of the mean-reversion and the ARMA-models do not differ significantly, both models are suitable for electricity price simulation.

- However, the GARCH approach is less suitable, the MRSE is very high in this case.

- Regime switching approach improves the simulation immensely, the error is reduced by more than half.

- Consideration of the seasonal components leads also to a significant improvement of the electricity price simulation.

- Impact of wind power:
  - Depends strongly on the actual load level.
  - Wind power feed-in can be simulated via Laplace-distributed change rates.
  - The separation of the stochastic component into a “wind power driven” one and remaining stochastic one improves also the electricity price simulation.
Thank you!

Questions?
Impact of wind energy feed-in (WEF) on electricity prices

- structure of the merit order

- exponential structure, jumps and constant levels
- depending on merit order characteristics equal WEF can cause different price reductions
Impact of wind energy feed-in (WEF) on electricity prices

- Comparison of the merit order structure and price reductions

→ merit order structure and extreme market situations explain fluctuations within price reductions by WEF
Modeling electricity prices via stochastic prices

Modeling of the deterministic parts:

Logarithmised prices  
(2002 - 2009)

Trend of price logs

Annual cycle

Weekly cycle

Daily cycle

Stochastic residues

$X_t = \ln p_t$

$X_t^{trend} = X_0 + \lambda \cdot t$

$X_{annual\ cycle}^d(t) = \alpha_{dh} + \beta_{dh} \cos \left( 2\pi \frac{t - \tau}{8760} \right) + \gamma_{dh} \sin \left( 2\pi \frac{t - \tau}{8760} \right)$

$X_{weekly\ cycle}^t = \alpha + \beta \sin \left( \frac{\pi \cdot t}{168} - \varphi \right)$

$X_{daily\ cycle}^{i,season} = \frac{24}{T} \sum_{t=0}^{(T/24)-1} X_{i+24t,season}$

$\forall \ i \in \{1,2,\ldots,24\} \land \forall season \in \{\text{winter, spring, summer, autumn}\}$

$X_t^s = X_t - X_t^{trend} - X_t^{annual\ cycle} - X_t^{weekly\ cycle} - X_t^{daily\ cycle}$

- Negative prices are set to 0.01€/MWh
- Log. leads to variance stabilization
Modelling electricity prices

Modelling of the stochastic component:

Base regime:

Mean-reversion process

- Assumption: prices return to the long-term mean $\mu$ with the speed $\kappa$

$$dX_t = \kappa(\mu - X_t) \cdot dt + \sigma \cdot dW_t$$

- Wiener Process $dW_t = \varepsilon_t \, dt^{1/2}$, whereas $\varepsilon_t$ is a standard normally distributed error term

- Exact solution:

$$E(X_t) = ae^{-\theta t} + \mu(1 - e^{-\theta t})$$

$$\text{Cov}(X_s, X_t) = \frac{\sigma^2}{2\theta} (e^{-\theta|s-t|} - e^{-\theta(s+t)})$$

$$X_t \sim \mathcal{N}(ae^{-\theta t} + \mu(1 - e^{-\theta t}), \frac{\sigma^2}{2\theta}(1 - e^{-2\theta t}))$$

- Parameter estimation via MLE
• Analysis of change rates: dependencies

→ Dependency of change rates on historical values (Autocorrelation):
  - Direction of a change depends on previous change
  - Probability, that change will be positive or negative, results from how many directly preceding changes were positive or negative
  - With historical date probabilities can be determined and thus for each hour in the simulation time frame the direction can be provided

→ Dependency of change rates on historical values of capacity utilization:
  - historical capacity utilization determine the amount of the following change
  - this history is described by the level of utilization, defined as the moving average of the past 11 hours
Simulation of wind energy feed-in (WEF)

• parameter m

\[ m = -0.024 \cdot Niv(X) + 0.10 \]

→ m corresponds to the mode of change rates
→ the higher the utilization level, the smaller (or more negative) the average change amount
Simulation of wind energy feed-in (WEF)

- Modelling the change rates $\Delta x_t^{\text{sim}}$

$$
\Delta X_t^{\text{sim}} = \begin{cases} 
  e_t + m_t & , l_t = 1 \\
  -e_t + m_t & , l_t = -1 
\end{cases}
$$

$$
e_t \sim \begin{cases} 
  \text{Exp}(\mu_t^+) & , l_t = 1 \\
  \text{Exp}(\mu_t^-) & , l_t = -1 
\end{cases}
$$

$$
m_t, \mu_t^+, \mu_t^- = f[Niv_t(X_t)]
$$

$t = 1, \ldots, N^{\text{sim}}$

- Amount of the change rate is generated with a exponentially distributed random number, that is moved by the modal value of the original Laplace distribution
- The direction of the change is determined by the series I of algebraic signs, that provides the direction of the change in each hour $t$