Numerical Methods for Pricing Energy Derivatives, including Swing Options, in the Presence of Jumps

Stelios Kourouvakalis, Senior Quantitative Analyst

RWE
The energy to lead

Motivation: Swing Options

- An electricity or gas **SUPPLIER** needs to be capable, at any point in time, to deliver the electricity or gas demanded by its customers.

Motivation: Swing Options

- An electricity or gas **SUPPLIER** needs to be capable, at any point in time, to deliver the electricity or gas demanded by its customers.
- Demand for electricity or gas by households usually depends on temperature

Motivation: Swing Options

- An electricity or gas **SUPPLIER** needs to be capable, at any point in time, to deliver the electricity or gas demanded by its customers.
- Demand for electricity or gas by households usually depends on temperature
  - How cold it is in winter so that radiators (electric or gas-fired) are turned on.

Motivation: Swing Options

- An electricity or gas **SUPPLIER** needs to be capable, at any point in time, to deliver the electricity or gas demanded by its customers.
- Demand for electricity or gas by households usually depends on temperature
  - How cold it is in winter so that radiators (electric or gas-fired) are turned on.
  - How warm it is in the summer so that air-conditioning turns on.

Motivation: Swing Options

- An electricity or gas **SUPPLIER** needs to be capable, at any point in time, to deliver the electricity or gas demanded by its customers.
- Demand for electricity or gas by households usually depends on temperature
  - How cold it is in winter so that radiators (electric or gas-fired) are turned on.
  - How warm it is in the summer so that air-conditioning turns on.
- Temperature cannot be predicted long in advance. Suppliers may have to deliver more or less **VOLUME** of electricity or gas, than what they have accounted for.

Motivation: Swing Options

- An electricity or gas **SUPPLIER** needs to be capable, at any point in time, to deliver the electricity or gas demanded by its customers.
- Demand for electricity or gas by households usually depends on temperature
  - How cold it is in winter so that radiators (electric or gas-fired) are turned on.
  - How warm it is in the summer so that air-conditioning turns on.
- Temperature cannot be predicted long in advance. Suppliers may have to deliver more or less **VOLUME** of electricity or gas, than what they have accounted for.
- The, unaccounted for, electricity or gas, has to be produced or purchased from the market and there is always a **PRICE** associated.

Motivation: Swing Options

- An electricity or gas **SUPPLIER** needs to be capable, at any point in time, to deliver the electricity or gas demanded by its customers.
- Demand for electricity or gas by households usually depends on temperature
  - How cold it is in winter so that radiators (electric or gas-fired) are turned on.
  - How warm it is in the summer so that air-conditioning turns on.
- Temperature cannot be predicted long in advance. Suppliers may have to deliver more or less **VOLUME** of electricity or gas, than what they have accounted for.
- The, unaccounted for, electricity or gas, has to be produced or purchased from the market and there is always a **PRICE** associated.
- This dependence on both **PRICE** and **VOLUME** is what lies at the heart of a Swing Option.

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
- The **cumulative volume**, $V(i)$, purchased up to time t_{i-1}, $i = 1, \ldots, n$, is the key variable.

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
- The **cumulative volume**, $V(i)$, purchased up to time t_{i-1}, $i = 1, \ldots, n$, is the key variable.
- One tree is used for **EVERY** possible value of the cumulative volume, $V(i)$.

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
- The **cumulative volume**, $V(i)$, purchased up to time t_{i-1}, $i = 1, \ldots, n$, is the key variable.
- One tree is used for **EVERY** possible value of the cumulative volume, $V(i)$.
- Pricing is done using the backward induction method.

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
- The **cumulative volume**, $V(i)$, purchased up to time t_{i-1}, $i = 1, \ldots, n$, is the key variable.
- One tree is used for **EVERY** possible value of the cumulative volume, $V(i)$.
- Pricing is done using the backward induction method.
- A grid that incorporates both jumps and mean-reversion is needed.

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
- The **cumulative volume**, $V(i)$, purchased up to time t_{i-1}, $i = 1, \ldots, n$, is the key variable.
- One tree is used for **EVERY** possible value of the cumulative volume, $V(i)$.
- Pricing is done using the backward induction method.
- A grid that incorporates both jumps and mean-reversion is needed.
  - We will use the model by Geman and Roncoroni, *Journal of Business* (2006)

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
- The **cumulative volume**, $V(i)$, purchased up to time t_{i-1}, $i = 1, \ldots, n$, is the key variable.
- One tree is used for **EVERY** possible value of the cumulative volume, $V(i)$.
- Pricing is done using the backward induction method.
- A grid that incorporates both jumps and mean-reversion is needed.
  - We will use the model by Geman and Roncoroni, *Journal of Business* (2006)
  - At each time-step, t_i, the density of the price of the underlying needs to be efficiently discretized: Bally, V., Pagès, G. & Printems, J., *Mathematical Finance* (2005)

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
- The **cumulative volume**, $V(i)$, purchased up to time t_{i-1}, $i = 1, \ldots, n$, is the key variable.
- One tree is used for **EVERY** possible value of the cumulative volume, $V(i)$.
- Pricing is done using the backward induction method.
- A grid that incorporates both jumps and mean-reversion is needed.
  - We will use the model by Geman and Roncoroni, *Journal of Business* (2006)
  - At each time-step, t_i, the density of the price of the underlying needs to be efficiently discretized: Bally, V., Pagès, G. & Printems, J., *Mathematical Finance* (2005)
  - Compare results obtained from the grid with Monte Carlo simulations.

Swing Option Pricing by "Forest of trees"

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
- The **cumulative volume**, $V(i)$, purchased up to time t_{i-1}, $i = 1, \ldots, n$, is the key variable.
- One tree is used for **EVERY** possible value of the cumulative volume, $V(i)$.
- Pricing is done using the backward induction method.
- A grid that incorporates both jumps and mean-reversion is needed.
  - We will use the model by Geman and Roncoroni, *Journal of Business* (2006)
  - At each time-step, t_i, the density of the price of the underlying needs to be efficiently discretized: Bally, V., Pagès, G. & Printems, J., *Mathematical Finance* (2005)
  - Compare results obtained from the grid with Monte Carlo simulations.
  - Swing Options pricing by Monte Carlo simulations: Barrera-Esteve, C., Bergeret, F., Dossal, C., Gobet, E., Meziou, A., Munos, R. & Reboul- Salze, D: *Methodology and Computing in Applied Probability* (2006).

Mathematical model for the spot electricity price under an equivalent martingale measure Q:

$$dE(t) = \theta_1[\widetilde{m}(t) - E(t^-)]\ dt\ + \sigma(t)dW(t) + h(t^-)\ln(J)\ dq(t) \tag{1}$$

where

$$\widetilde{m}(t) = \frac{1}{\theta_1}D\mu(t) + \mu(t) \tag{2}$$

- D denotes the derivative with respect to time
- $\mu(t)$ is a deterministic function and drives the seasonal part of the process
- θ_1 is the speed of mean reversion of the diffusion part
- $\sigma(t)$ is the volatility of the diffusion part
- $\ln(J)$ defines the size of the jump
- $W(t)$ is a $\boldsymbol{Q}$-Brownian motion
- $q(t)$ is a Poisson counter under $\boldsymbol{Q}$, with intensity $\lambda_J(t) = \theta_2 s(t)$

A closer look at the jump part of the process

- The function $h(t)$ is defined as

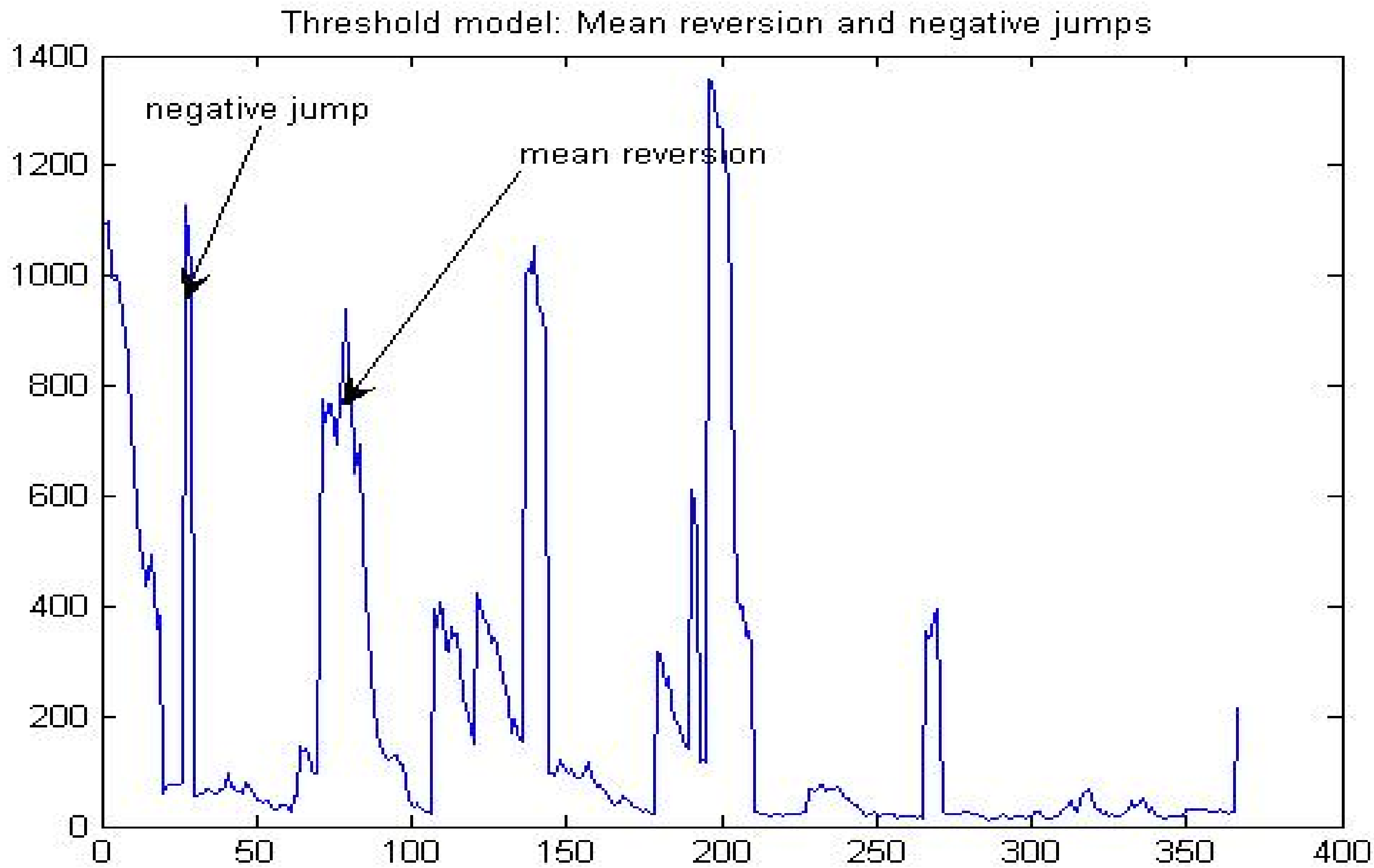
$$h(t) = \mathbf{1}_{\{E(t)<\mathcal{T}(t)\}} - \mathbf{1}_{\{E(t)\geq\mathcal{T}(t)\}}$$

  - If at the time of a jump τ, $E(\tau^-)$ is below the threshold $\mathcal{T}(\tau^-)$, then h will be equal to 1, producing a jump in the upwards direction
  - If $E(\tau^-)$ is above the threshold, then h will be equal to -1, producing a downward directed jump
  - $\mathcal{T}(t) = \mu(t) + \Delta$
- The function $\ln(J)$ defines the size of the jump and has density:

$$p\big(x, \theta_3, \psi\big) = \frac{\theta_3 e^{-\theta_3 x}}{1 - e^{-\theta_3 \psi}} \quad , 0 \leq x \leq \psi. \tag{3}$$

  - θ_3 is a parameter ensuring that p is a probability density function
  - ψ is the maximum jump size

Mean reversion and spikes in the Threshold Model



The solution of the model under Q

$$E(T) = D(t,T) \; + \; J(t,T) \tag{4}$$

where

$$D(t,T) = \mu(T) \; + \Big(E(t) - \mu(t) \Big) e^{-\theta_1 (T-t)} \; + \int_t^T \sigma(y) e^{-\theta_1 (T-y)} \, dW(y) \tag{5}$$

and

$$J(t,T) = e^{-\theta_1 T} \sum_{i=1}^{N(T-t)} e^{\theta_1 \tau_i} \, h(\tau_i^-) \; [\ln J]_i \tag{6}$$

- Choose a particular measure derived from the market prices of futures contracts.

Approximation of the continuous-time process

- The time interval $[t, T]$ is partitioned into n distinct subintervals using $n+1$ knots t_i
- $t =: t_0 < t_1 < \cdots < t_{n-1} < t_n := T$
- $t_{i+1} - t_i = \delta t$, for all i
- Start by $\widetilde{E}(t_0) := E(t_0)$
- Construct an approximating process that tracks the original process in each sub-interval

The approximating jump process: properties

- At most one jump allowed in each time interval

The approximating jump process: properties

- At most one jump allowed in each time interval
- Size of the jump: the same as the size of the first jump of the continuous-time process

The approximating jump process: properties

- At most one jump allowed in each time interval
- Size of the jump: the same as the size of the first jump of the continuous-time process
- Direction of the jump: Depends on the value of the underlying at the CENTER of the interval, if it moves SOLELY by mean-reversion from the beginning of the interval.

The approximating jump process: properties

- At most one jump allowed in each time interval
- Size of the jump: the same as the size of the first jump of the continuous-time process
- Direction of the jump: Depends on the value of the underlying at the CENTER of the interval, if it moves SOLELY by mean-reversion from the beginning of the interval.
- Direction of jump in the original process: Depends on the value of the underlying at a RANDOM time within the interval, if it moves SOLELY by mean-reversion + noise from the beginning of the interval.

The jump part of the approximating process

- Jump part of the original process

$$J(t_{u-\kappa},\ t_{u-\kappa+1}) = e^{-\theta_1 t_{u-\kappa+1}} \sum_{i=1}^{N[\Delta t(u-\kappa)]} e^{\theta_1 \tau_i}\ h(\tau_i^-)\ [\ln J]_i \tag{7}$$

- Jump part of the approximating process

$$\begin{aligned} \widetilde{J}(t_{m-\kappa},\ t_{m-\kappa+1}) := e^{-\theta_1 t_{m-\kappa+1}}\ e^{\theta_1 (t_{m-\kappa}+(\delta t/2))}\ h'(t_{m-\kappa} + \tfrac{\delta t}{2}) \\ \times\ [\ln J]_1\ \mathbf{1}_{\{N[\Delta t(m-\kappa)]\geq 1\}} \end{aligned} \tag{8}$$

- The function $h'(\alpha)$, for any $\alpha \in (t_{m-\kappa}, t_{m-\kappa+1}]$, is defined as:

$$h'(\alpha) := \mathbf{1}_{\{D_c(t_{m-\kappa},\alpha)<\mathcal{T}(\alpha)\}} -\ \mathbf{1}_{\{D_c(t_{m-\kappa},\alpha)\geq\mathcal{T}(\alpha)\}} \tag{9}$$

where $D_c(t_{m-\kappa}, \alpha)$ is defined as:

$$D_c(t_{m-\kappa}, \alpha) = \mu(\alpha)\ + \Big(\widetilde{E}(t_{m-\kappa}) - \mu(t_{m-\kappa})\Big)\ e^{-\theta_1(\alpha - t_{m-\kappa})} \tag{10}$$

The approximating process under Q

$$\widetilde{E}\Big[(t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big] = \widetilde{D}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big] + \widetilde{J}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big] \quad (11)$$

where

$$\widetilde{D}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big] = \mu(t_i+\delta t) + \Big(\widetilde{E}(t_i) - \mu(t_i)\Big) e^{-\theta_1 \delta t}$$

$$+ \sigma(t_i+\delta t)\ e^{-\theta_1 (t_i+\delta t)} \int_{t_i}^{t_i+\delta t} e^{\theta_1 y}\ dW(y) \quad (12)$$

and

$$\widetilde{J}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big] = e^{-\theta_1 \frac{\delta t}{2}}\ h'(t_i + \tfrac{\delta t}{2})\ [\ln J]_1\ \mathbf{1}_{\{N[\Delta t(i)] \geq 1\}} \quad (13)$$

Density of the components of the approximating process

- normal distribution with calculable mean and variance for the process

$$\widetilde{D}\Big[(t_i,\ t_i+\delta t) \,\Big|\, \widetilde{E}(t_i)\Big] = \mu(t_i+\delta t) + \Big(\widetilde{E}(t_i) - \mu(t_i)\Big) e^{-\theta_1 \delta t}$$

$$+ \sigma(t_i+\delta t)\ e^{-\theta_1 (t_i+\delta t)} \int_{t_i}^{t_i+\delta t} e^{\theta_1 y}\ dW(y)$$

- Conditional on the occurrence of at least one jump, the approximating jump process

$$\widetilde{J}\Big[(t_i,\ t_i+\delta t) \,\Big|\, \widetilde{E}(t_i)\Big] = e^{-\theta_1 \frac{\delta t}{2}}\ h'(t_i+\tfrac{\delta t}{2})\ [\ln J]_1\ \mathbf{1}_{\{N[\Delta t(i)]\geq 1\}} \qquad (14)$$

has a density given by

$$f_Y(y) = f_X\big(g^{-1}(y)\big)\ \left|\frac{d}{dy} g^{-1}(y)\right|$$

where $g(x) = h'(t_i+\frac{\delta t}{2}) e^{-\theta_1 \frac{\delta t}{2}}\ x$, and f_X is the density of the jump size.

Density of the approximating process

- Conditioning on an initial value $\widetilde{E}(t_i)$:

$$\widetilde{E}\Big[(t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big] = \widetilde{D}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big] + \widetilde{J}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big]$$

  – If no jump occurs then its density is defined from the density of

$$\widetilde{D}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big]$$

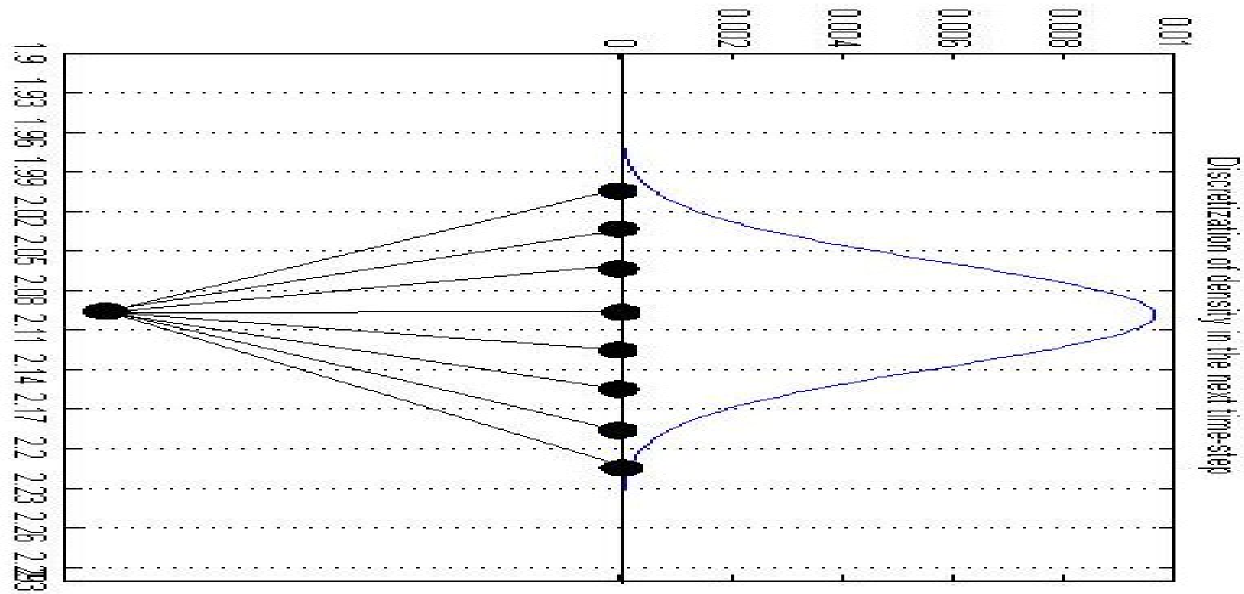
  – If at least one jump occurs its density is defined by the ***convolution*** of the densities of

$$\widetilde{D}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big]$$

  and

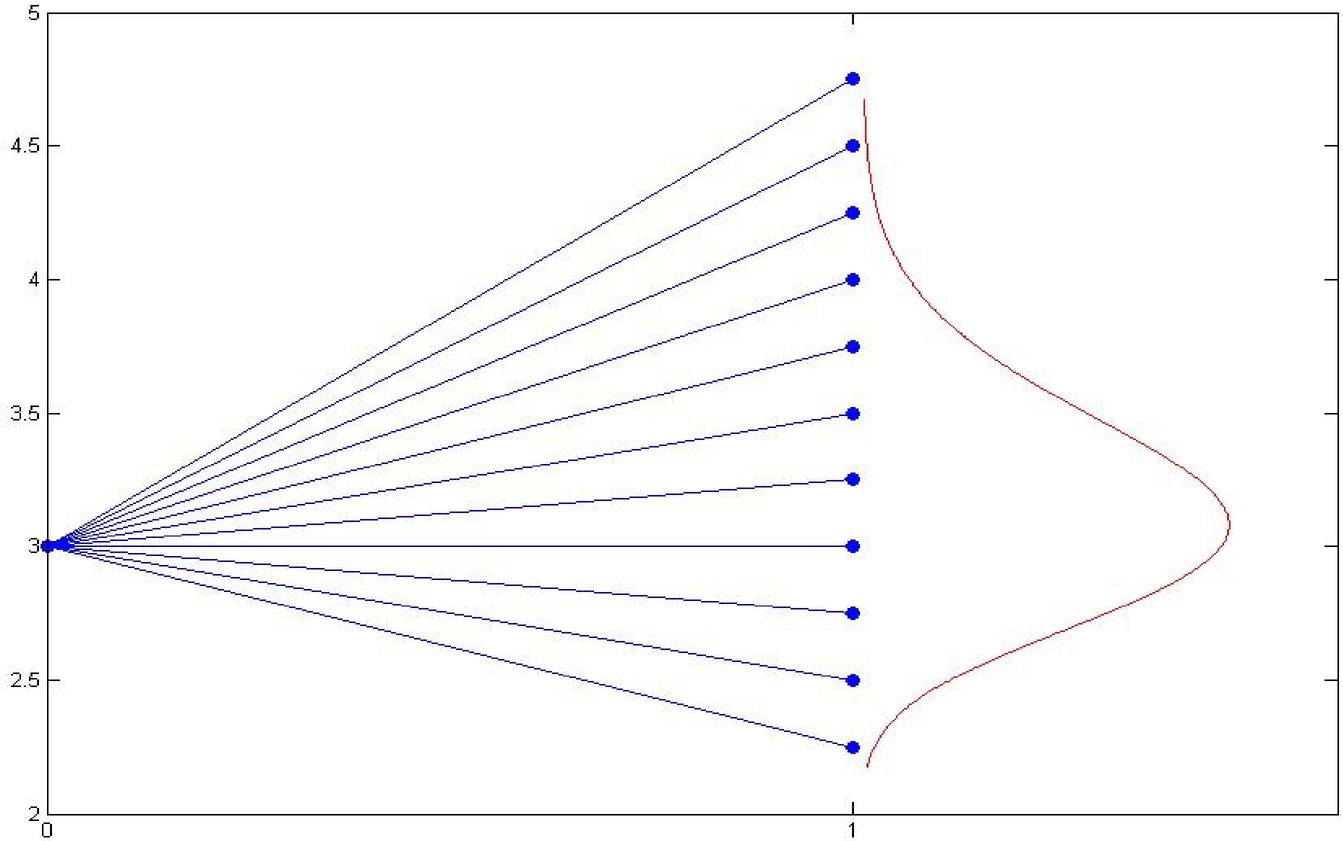
$$\widetilde{J}\Big[(t_i,\ t_i + \delta t) \,\big|\, \widetilde{E}(t_i)\Big]$$

Discretization of the density of a stochastic process one time-step ahead

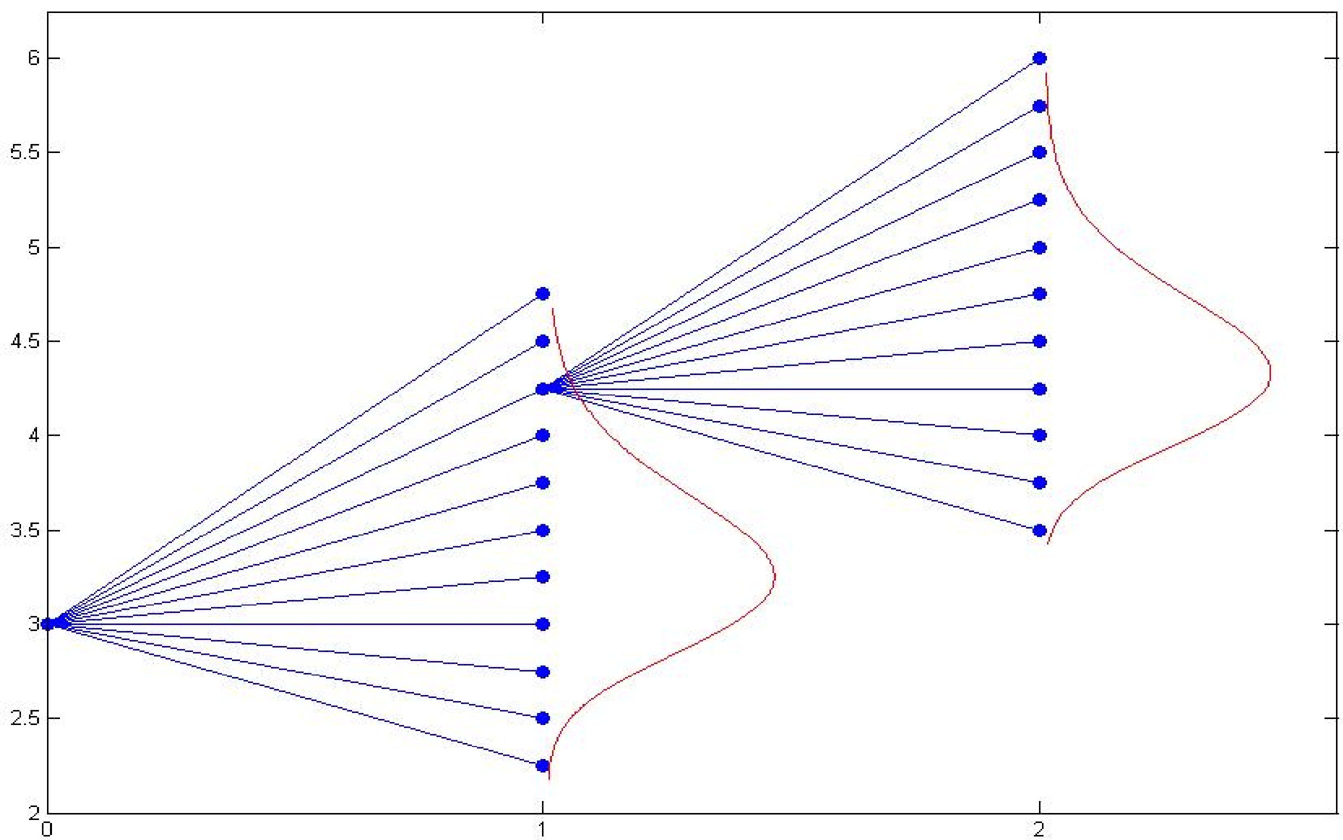


- The density is divided into sections
- The probability mass within a section is assigned to the transition probability from the starting node to the node in the middle of the section.
- A probability threshold Π prevents movements to sections with very low probability mass.

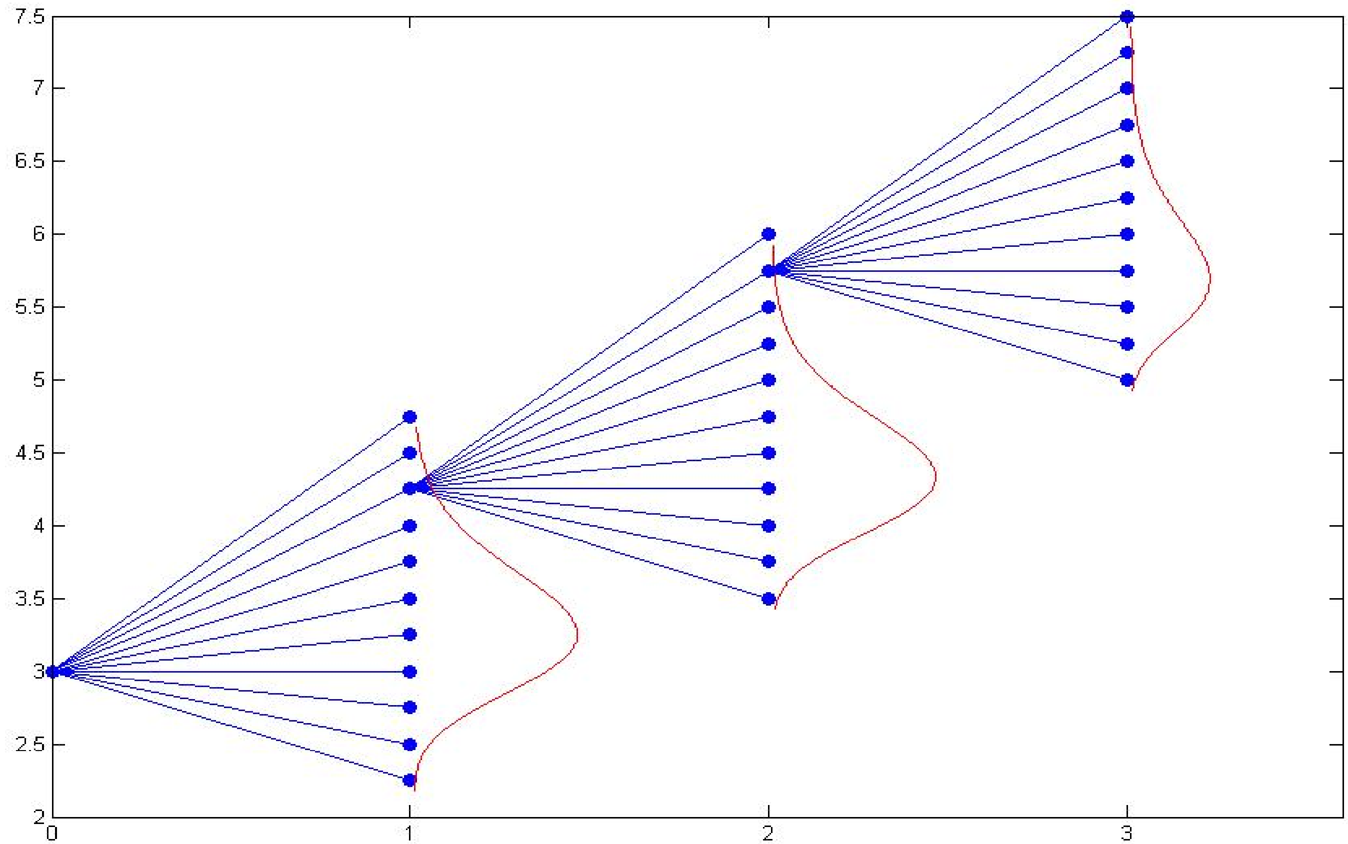
First step on the tree



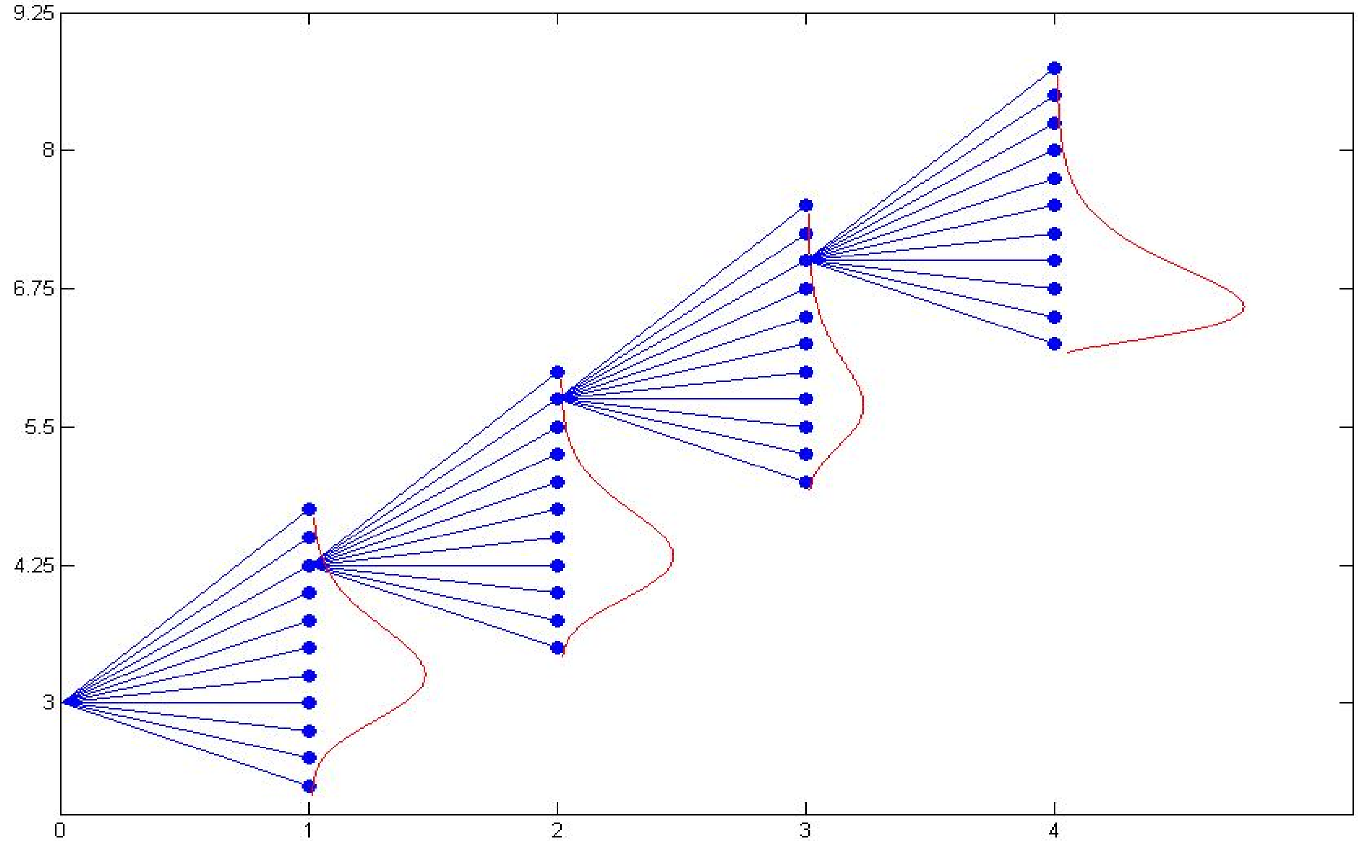
Second step: A different conditional probability distribution



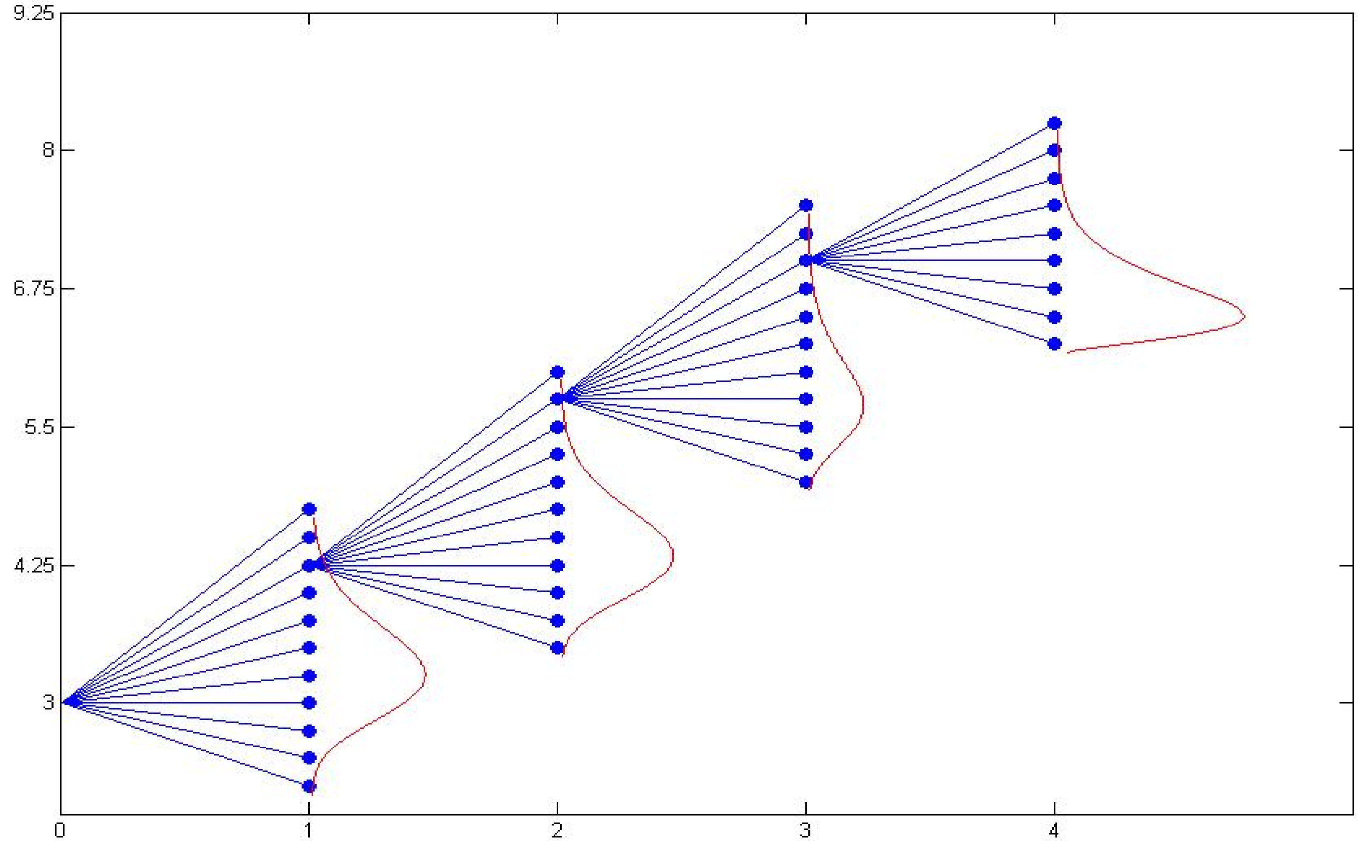
Third step: Mean reversion starts influencing the conditional distribution



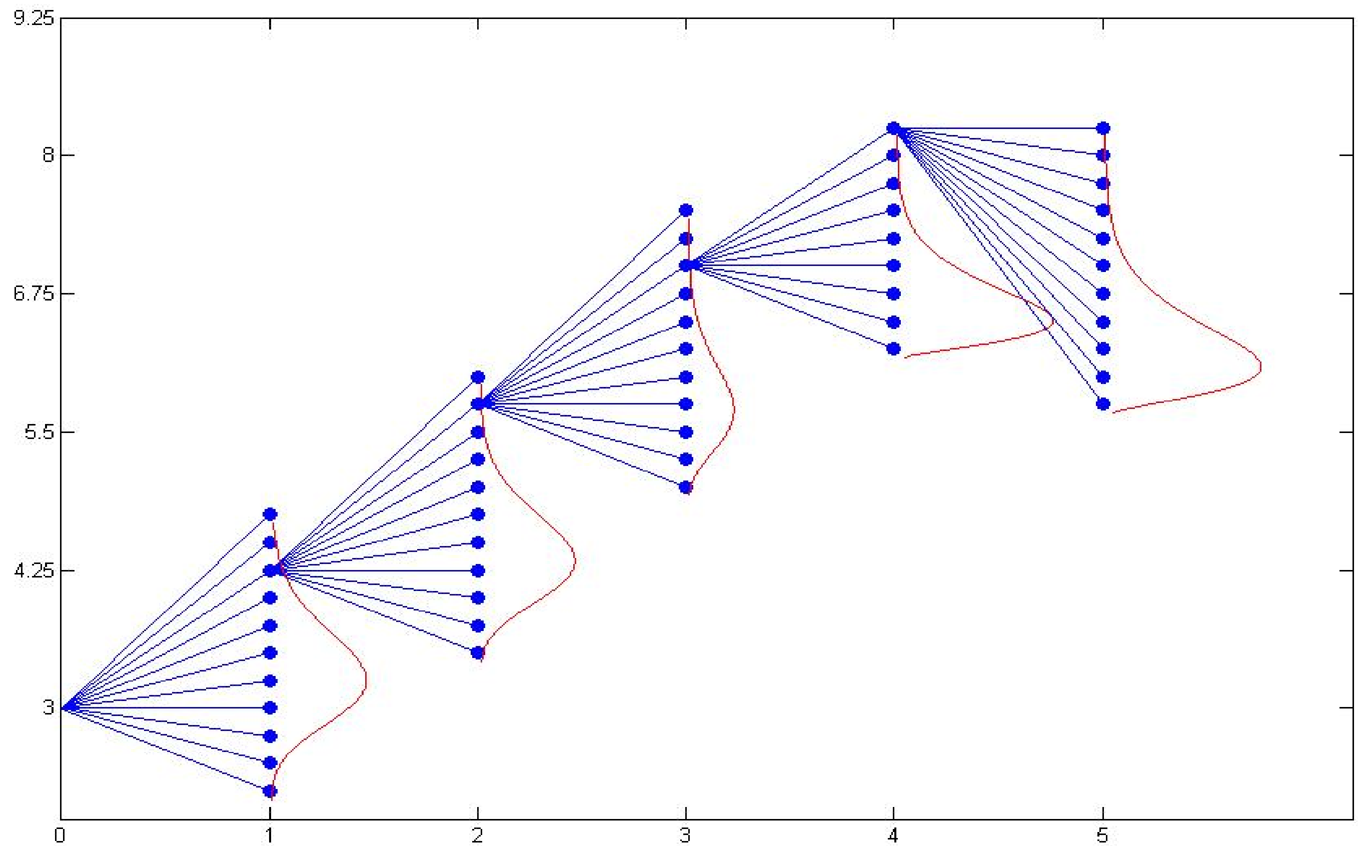
Fourth step: Strong mean reversion pull



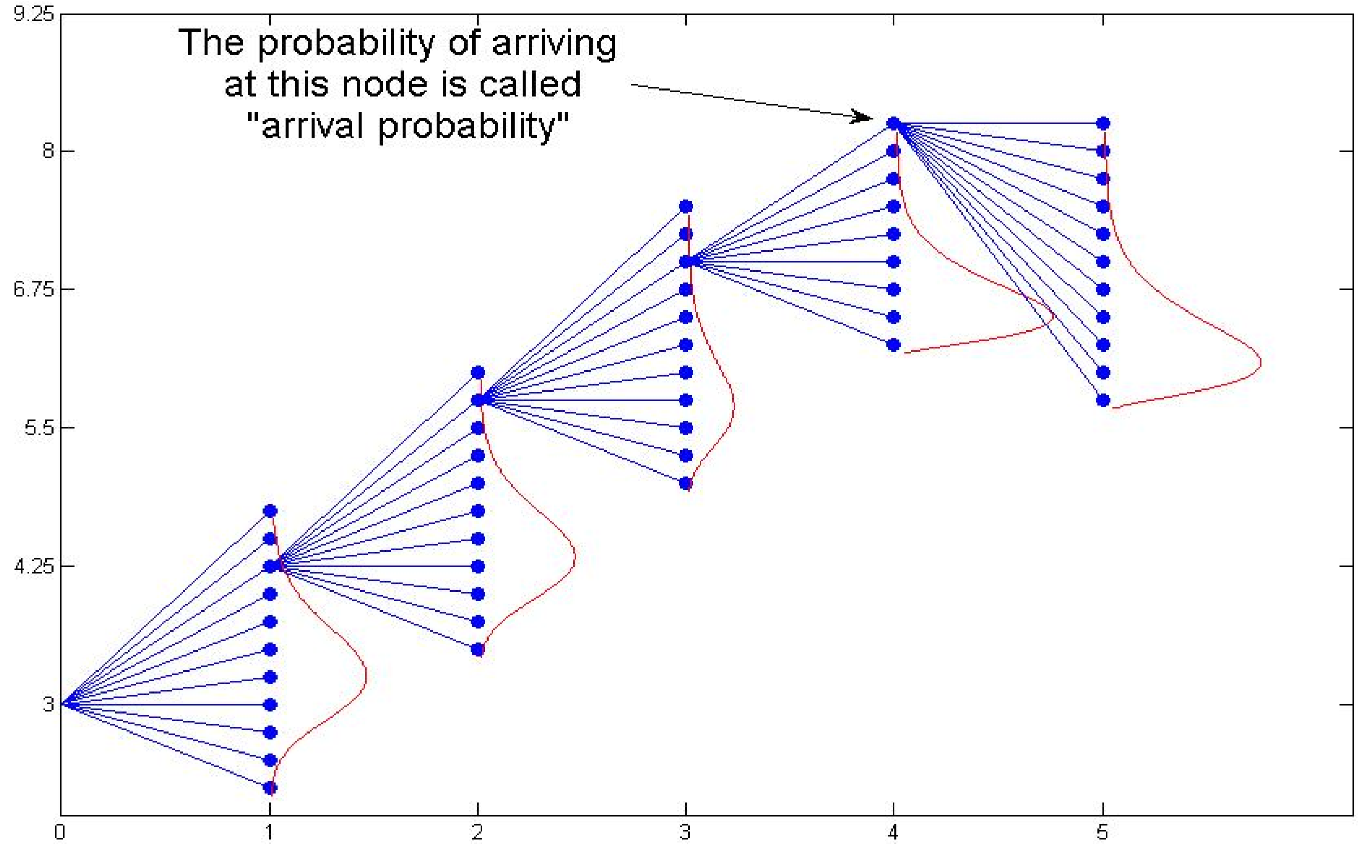
Some of the up movements have very low probability



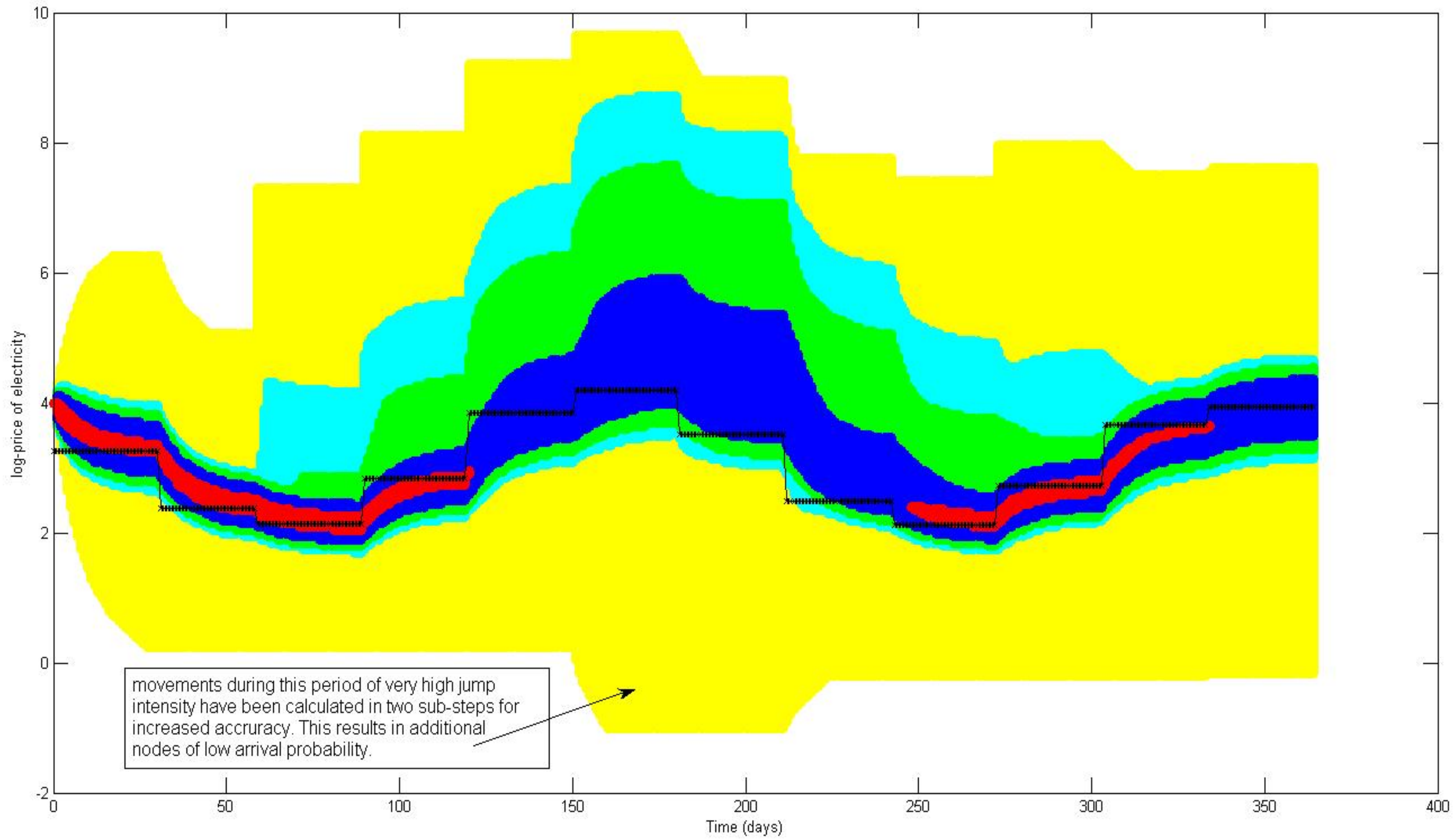
Mean reversion: Only downward movements



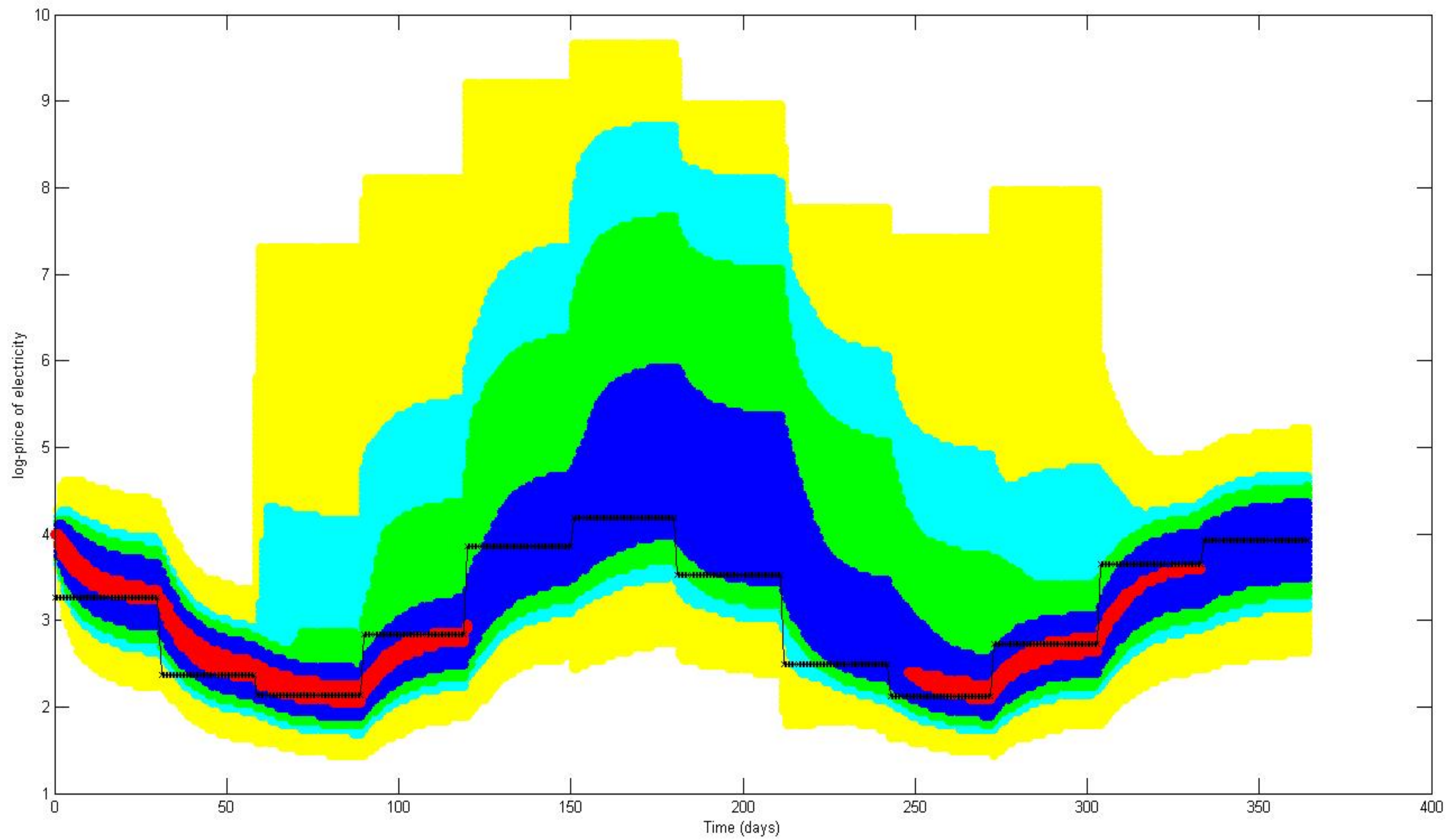
Arrival probability



A full one-year grid, time changing parameters



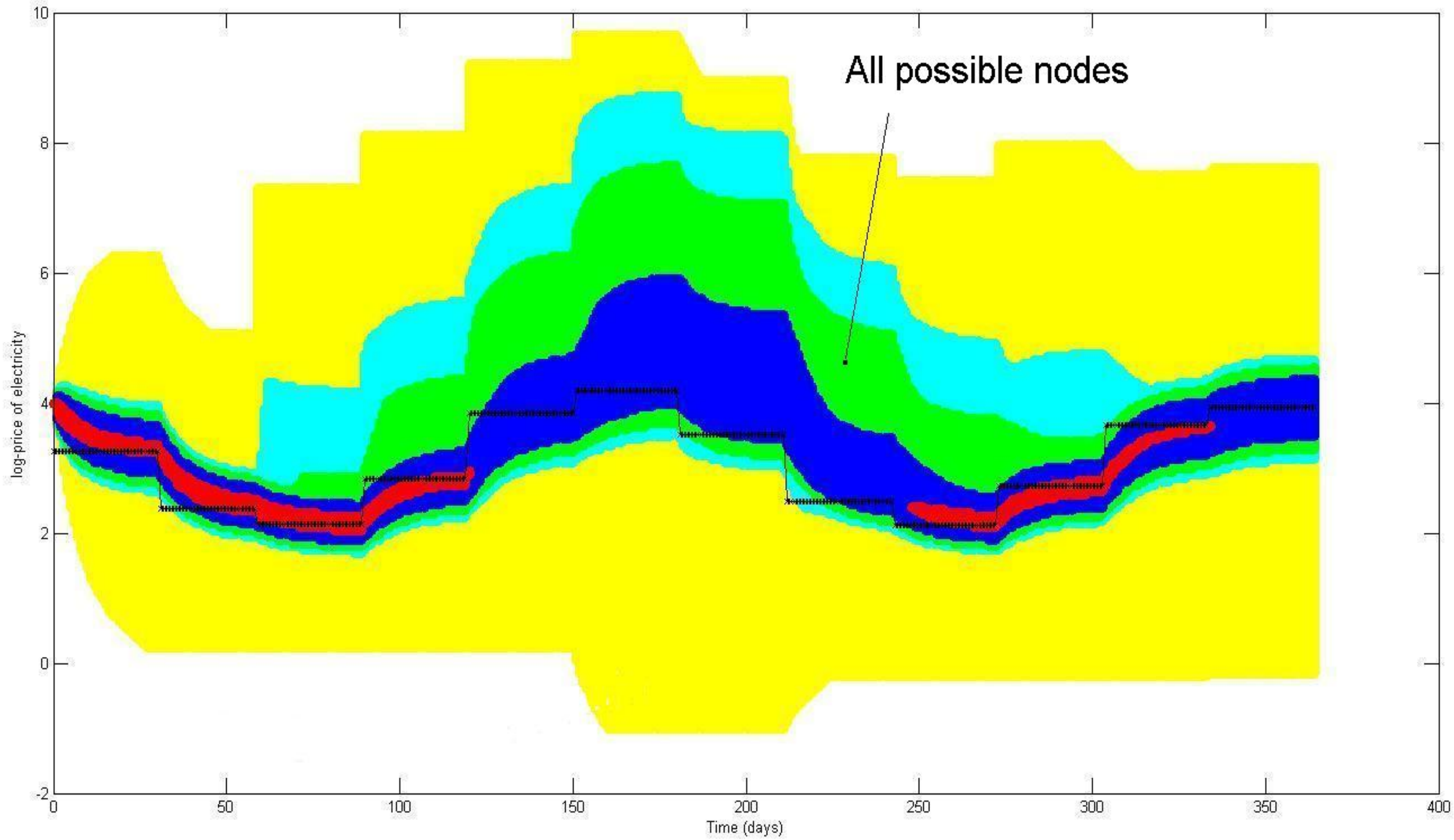
A full one-year grid, time changing parameters, "filtering" on



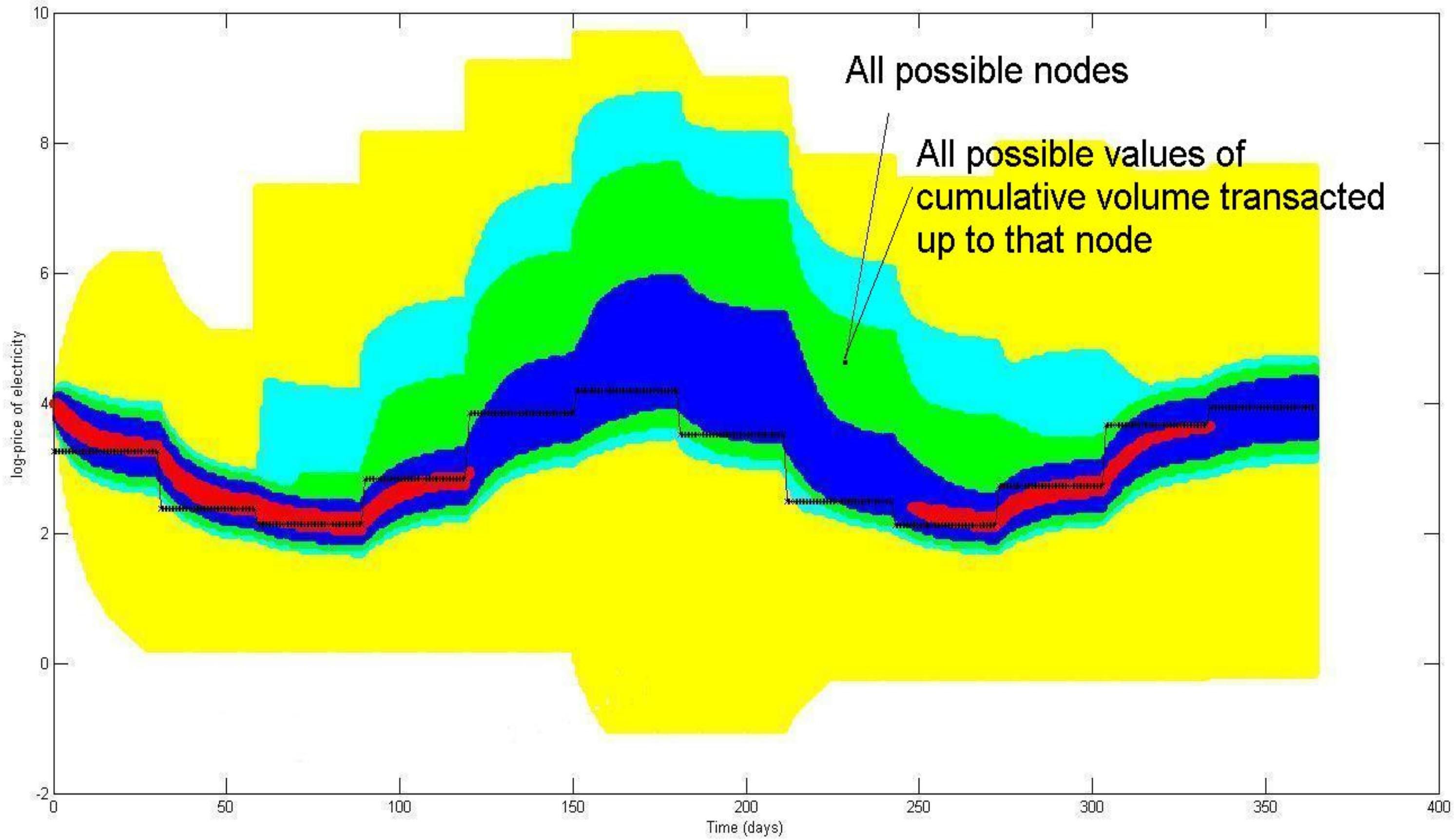
Grid applications: European style options, time changing parameters

Option matures on	Parameter values at maturity	method	running time (sec)	Strike = e^2 option price	Strike = e^3 option price	Strike = e^4 option price
31 Jan 2009	$\mu = 2.99$	Monte Carlo	40	13.77	1.93	0
	$\lambda_J = 0.0042$	Grid, all nodes included	0.8	13.76	1.95	0
	$\sigma = 1.3821$	Grid, filtering on	0.5	13.76	1.95	0
30 Apr 2009	$\mu = 3.65$	Monte Carlo	180	20.73	8.53	2.01
	$\lambda_J = 3.58$	Grid, all nodes included	10	20.75	8.51	2.06
	$\sigma = 1.4559$	Grid, filtering on	5.5	20.71	8.47	2.04
30 Jun 2009	$\mu = 3.25$	Monte Carlo	250	94.18	82.10	57.39
	$\lambda_J = 35.76$	Grid, all nodes included	24	93.86	81.49	57.23
	$\sigma = 1.5$	Grid, filtering on	13	93.80	81.43	57.19
31 Aug 2009	$\mu = 3.13$	Monte Carlo	325	35.75	23.36	11.01
	$\lambda_J = 12.52$	Grid, all nodes included	30	35.19	23.02	10.64
	$\sigma = 1.4410$	Grid, filtering on	17	35.16	22.99	10.63
31 Dec 2009	$\mu = 2.99$	Monte Carlo	430	12.30	1.36	0
	$\lambda_J = 0.0035$	Grid, all nodes included	37	12.31	1.35	0
	$\sigma = 1.3827$	Grid, filtering on	23	12.29	1.35	0

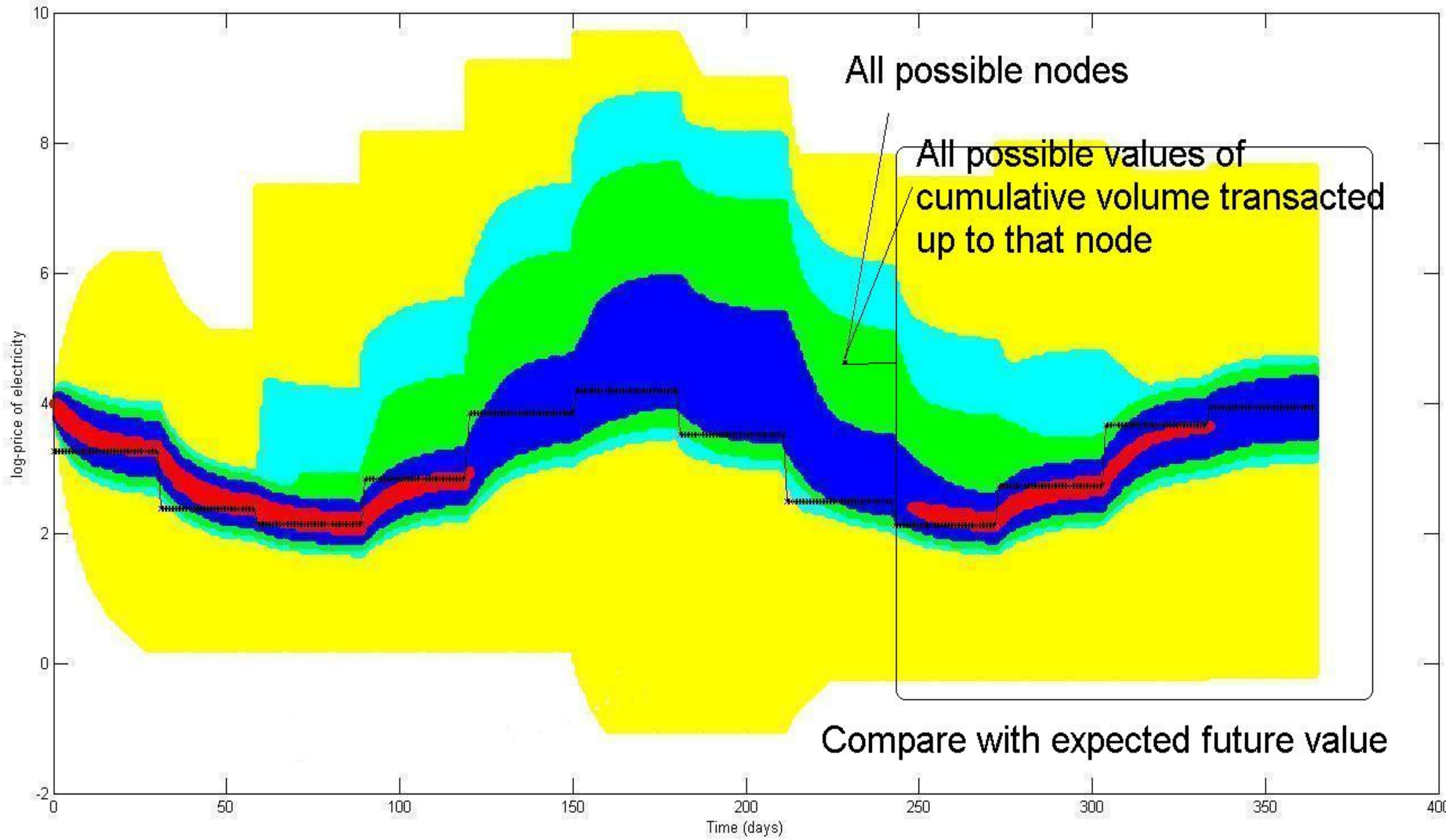
Swing option pricing on the tree



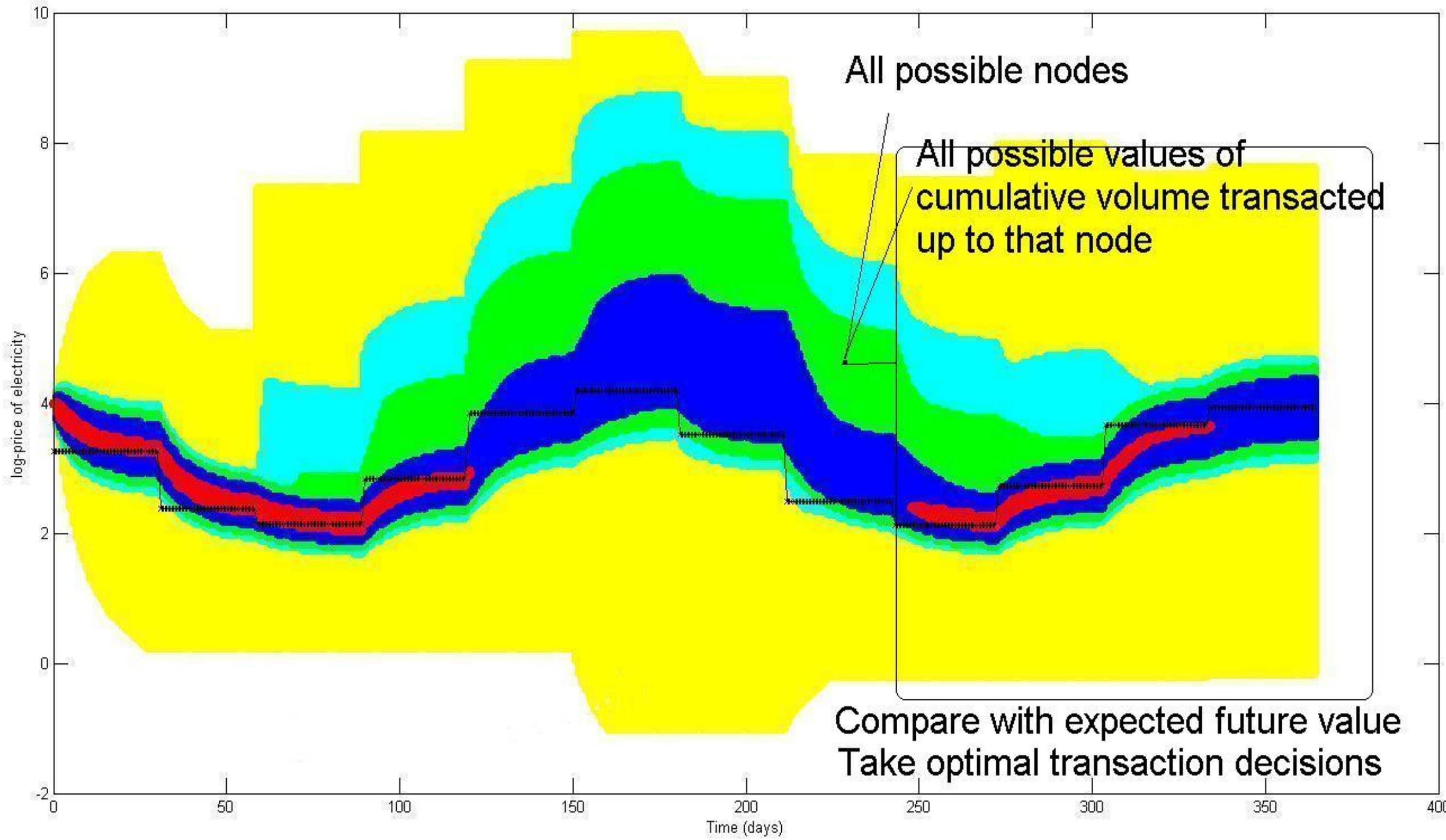
Swing option pricing on the tree



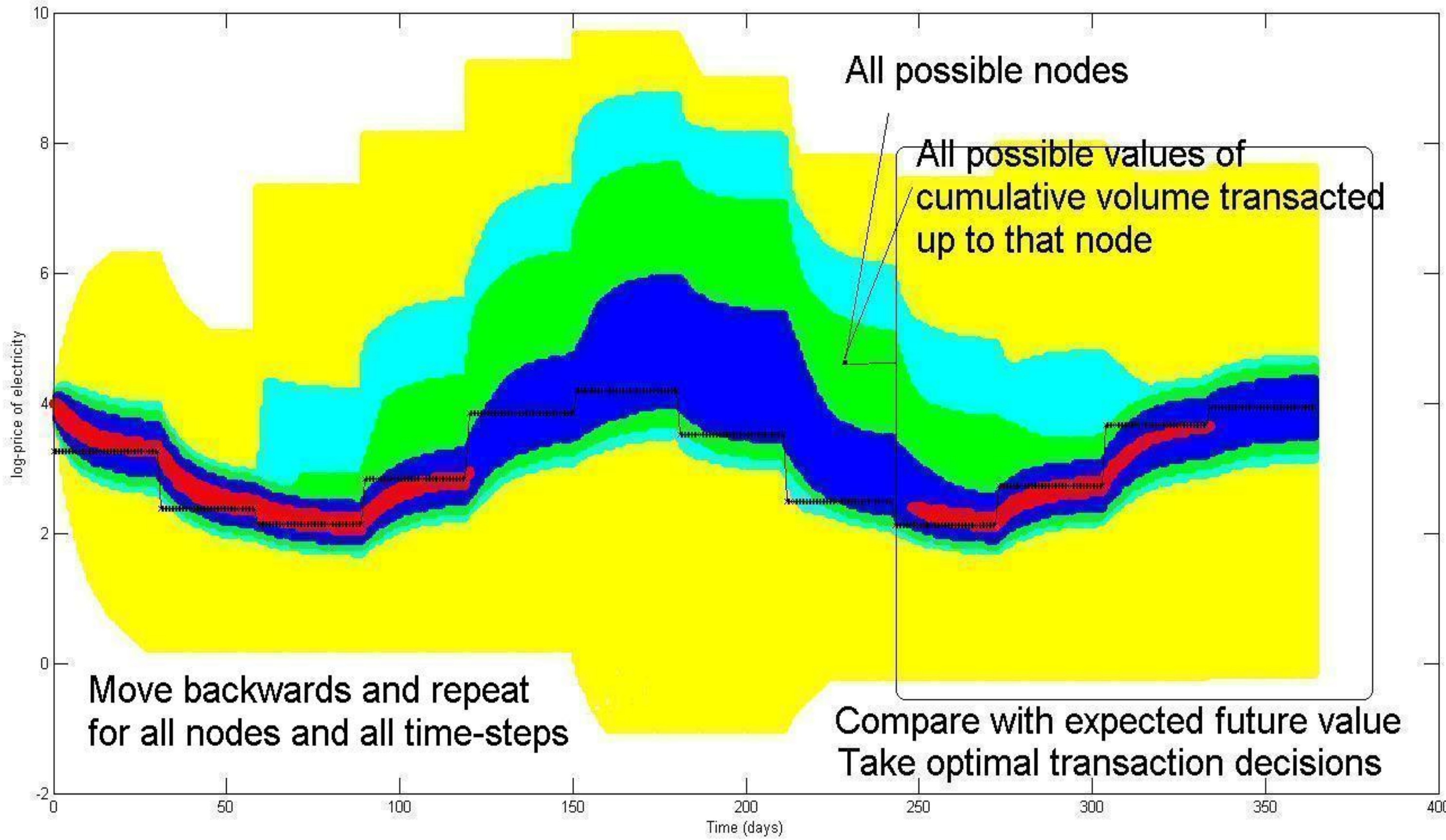
Swing option pricing on the tree



Swing option pricing on the tree



Swing option pricing on the tree



Grid and Monte Carlo methods for pricing swing options

Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:

Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
  - admissible cumulative volume,

Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
  - admissible cumulative volume,
  - value of the underlying,

Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
  - admissible cumulative volume,
  - value of the underlying,
  - time-step

Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
  - admissible cumulative volume,
  - value of the underlying,
  - time-step
- Monte Carlo method (Longstaff - Schwartz)

Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
  - admissible cumulative volume,
  - value of the underlying,
  - time-step

- Monte Carlo method (Longstaff - Schwartz)
  - possible values of the underlying are generated from $1,000$ paths

Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
  - admissible cumulative volume,
  - value of the underlying,
  - time-step
- Monte Carlo method (Longstaff - Schwartz)
  - possible values of the underlying are generated from $1,000$ paths
- Grid method

Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
  - admissible cumulative volume,
  - value of the underlying,
  - time-step
- Monte Carlo method (Longstaff - Schwartz)
  - possible values of the underlying are generated from $1,000$ paths
- Grid method
  - Possible values of the underlying are represented by the nodes of the grid at each time-step (about 200 nodes)

Swing option prices: Time varying parameters

Storage contract			parameters		Price	
Valuation date	Start date	End date	min	max	Grid	Monte Carlo
01-Jan-09	01-Jan-09	31-Mar-09	$\mu = 2.99$	$\mu = 3.11$	392.3	$[375,\ 410]$
			$\sigma = 1.38$	$\sigma = 1.43$		
			$\lambda_J = 10^{-4}$	$\lambda_J = 1.60$		
01-May-09	01-Jun-09	31-Jul-09	$\mu = 3.18$	$\mu = 3.25$	7214	$[6972,\ 7736]$
			$\sigma = 1.46$	$\sigma = 1.5$		
			$\lambda_J = 6.70$	$\lambda_J = 56$		

Swing option prices: Time varying parameters

Storage contract			parameters		Price	
Valuation date	Start date	End date	min	max	Grid	Monte Carlo
01-Jan-09	01-Jan-09	31-Mar-09	$\mu = 2.99$	$\mu = 3.11$	392.3	$[375,\ 410]$
			$\sigma = 1.38$	$\sigma = 1.43$		
			$\lambda_J = 10^{-4}$	$\lambda_J = 1.60$		
01-May-09	01-Jun-09	31-Jul-09	$\mu = 3.18$	$\mu = 3.25$	7214	$[6972,\ 7736]$
			$\sigma = 1.46$	$\sigma = 1.5$		
			$\lambda_J = 6.70$	$\lambda_J = 56$		

- A lot more paths are needed for the Monte Carlo method to produce smaller confidence intervals

Swing option prices: Time varying parameters

Storage contract			parameters		Price	
Valuation date	Start date	End date	min	max	Grid	Monte Carlo
01-Jan-09	01-Jan-09	31-Mar-09	$\mu = 2.99$	$\mu = 3.11$	392.3	$[375,\ 410]$
			$\sigma = 1.38$	$\sigma = 1.43$		
			$\lambda_J = 10^{-4}$	$\lambda_J = 1.60$		
01-May-09	01-Jun-09	31-Jul-09	$\mu = 3.18$	$\mu = 3.25$	7214	$[6972,\ 7736]$
			$\sigma = 1.46$	$\sigma = 1.5$		
			$\lambda_J = 6.70$	$\lambda_J = 56$		

- A lot more paths are needed for the Monte Carlo method to produce smaller confidence intervals
- For European options, 50,000 paths were needed in order to achieve narrow confidence intervals.

Swing option prices: Time varying parameters

Storage contract			parameters		Price	
Valuation date	Start date	End date	min	max	Grid	Monte Carlo
01-Jan-09	01-Jan-09	31-Mar-09	$\mu = 2.99$	$\mu = 3.11$	392.3	$[375,\ 410]$
			$\sigma = 1.38$	$\sigma = 1.43$		
			$\lambda_J = 10^{-4}$	$\lambda_J = 1.60$		
01-May-09	01-Jun-09	31-Jul-09	$\mu = 3.18$	$\mu = 3.25$	7214	$[6972,\ 7736]$
			$\sigma = 1.46$	$\sigma = 1.5$		
			$\lambda_J = 6.70$	$\lambda_J = 56$		

- A lot more paths are needed for the Monte Carlo method to produce smaller confidence intervals
- For European options, 50,000 paths were needed in order to achieve narrow confidence intervals.
- For European options the grid method worked very well with only 200 nodes, without filtering.

Swing option prices: Time varying parameters

Storage contract			parameters		Price	
Valuation date	Start date	End date	min	max	Grid	Monte Carlo
01-Jan-09	01-Jan-09	31-Mar-09	$\mu = 2.99$	$\mu = 3.11$	392.3	$[375,\ 410]$
			$\sigma = 1.38$	$\sigma = 1.43$		
			$\lambda_J = 10^{-4}$	$\lambda_J = 1.60$		
01-May-09	01-Jun-09	31-Jul-09	$\mu = 3.18$	$\mu = 3.25$	7214	$[6972,\ 7736]$
			$\sigma = 1.46$	$\sigma = 1.5$		
			$\lambda_J = 6.70$	$\lambda_J = 56$		

- A lot more paths are needed for the Monte Carlo method to produce smaller confidence intervals
- For European options, 50,000 paths were needed in order to achieve narrow confidence intervals.
- For European options the grid method worked very well with only 200 nodes, without filtering.
- The grid presents a very promising approach, achieving a good balance between accuracy and calculation time.

Thank you for your attention.