Numerical Methods for Pricing Energy Derivatives, including Swing Options, in the Presence of Jumps

Stelios Kourouvakalis, Senior Quantitative Analyst

RWE
The energy to lead
Motivation: Swing Options

- An electricity or gas **SUPPLIER** needs to be capable, at any point in time, to deliver the electricity or gas demanded by its customers.
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- The, unaccounted for, electricity or gas, has to be produced or purchased from the market and there is always a PRICE associated.
- This dependence on both PRICE and VOLUME is what lies at the heart of a Swing Option.
Swing Option Pricing by “Forest of trees”

- A tree (or grid) is constructed that discretizes the price movements of the underlying (gas or electricity), at each time-step throughout the duration of the contract.
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- The **cumulative volume**, $V(i)$, purchased up to time $t_{i-1}$, $i = 1, \ldots, n$, is the key variable.
- One tree is used for **EVERY** possible value of the cumulative volume, $V(i)$. 
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Mathematical model for the spot electricity price under an equivalent martingale measure $Q$:

$$
\begin{align*}
\frac{dE(t)}{dt} &= \theta_1[\tilde{m}(t) - E(t^-)] dt + \sigma(t)dW(t) + h(t^-) \ln(J) dq(t) \\
\end{align*}
$$

(1)

where

$$
\tilde{m}(t) = \frac{1}{\theta_1} D\mu(t) + \mu(t)
$$

(2)

- $D$ denotes the derivative with respect to time
- $\mu(t)$ is a deterministic function and drives the seasonal part of the process
- $\theta_1$ is the speed of mean reversion of the diffusion part
- $\sigma(t)$ is the volatility of the diffusion part
- $\ln(J)$ defines the size of the jump
- $W(t)$ is a $Q$-Brownian motion
- $q(t)$ is a Poisson counter under $Q$, with intensity $\lambda_J(t) = \theta_2 s(t)$
A closer look at the jump part of the process

- The function $h(t)$ is defined as

$$h(t) = 1\{E(t)<T(t)\} - 1\{E(t)\geq T(t)\}$$

- If at the time of a jump $\tau$, $E(\tau^{-})$ is below the threshold $T(\tau^{-})$, then $h$ will be equal to 1, producing a jump in the upwards direction
- If $E(\tau^{-})$ is above the threshold, then $h$ will be equal to -1, producing a downward directed jump
- $T(t) = \mu(t) + \Delta$

- The function $\ln(J)$ defines the size of the jump and has density:

$$p(x, \theta_3, \psi) = \frac{\theta_3 e^{-\theta_3 x}}{1 - e^{-\theta_3 \psi}}, \quad 0 \leq x \leq \psi. \quad (3)$$

- $\theta_3$ is a parameter ensuring that $p$ is a probability density function
- $\psi$ is the maximum jump size
Mean reversion and spikes in the Threshold Model
The solution of the model under $Q$

$$E(T) = D(t, T) + J(t, T)$$  \hspace{1cm} (4)

where

$$D(t, T) = \mu(T) + \left( E(t) - \mu(t) \right) e^{-\theta_1 (T-t)} + \int_t^T \sigma(y)e^{-\theta_1 (T-y)} \, dW(y)$$  \hspace{1cm} (5)

and

$$J(t, T) = e^{-\theta_1 T} \sum_{i=1}^{N(T-t)} e^{\theta_1 \tau_i} h(\tau_i^{-}) \left[ \ln J \right]_i$$  \hspace{1cm} (6)

- Choose a particular measure derived from the market prices of futures contracts.
Approximation of the continuous-time process

• The time interval \([t, T]\) is partitioned into \(n\) distinct subintervals using \(n + 1\) knots \(t_i\)

\[ t =: t_0 < t_1 < \cdots < t_{n-1} < t_n := T \]

• \(t_{i+1} - t_i = \delta t\), for all \(i\)

• Start by \(\tilde{E}(t_0) := E(t_0)\)

• Construct an approximating process that tracks the original process in each sub-interval
The approximating jump process: properties

- At most one jump allowed in each time interval
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• Size of the jump: the same as the size of the first jump of the continuous-time process
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- Direction of the jump: Depends on the value of the underlying at the CENTER of the interval, if it moves SOLELY by mean-reversion from the beginning of the interval.
The approximating jump process: properties

• **At most one jump** allowed in each time interval

• **Size of the jump:** the same as the size of the first jump of the continuous-time process

• **Direction of the jump:** Depends on the value of the underlying at the CENTER of the interval, if it moves SOLELY by mean-reversion from the beginning of the interval.

• **Direction of jump in the original process:** Depends on the value of the underlying at a RANDOM time within the interval, if it moves SOLELY by mean-reversion + noise from the beginning of the interval.
The jump part of the approximating process

- Jump part of the original process

\[ J(t_{u-\kappa}, t_{u-\kappa+1}) = e^{-\theta_1 t_{u-\kappa+1}} \sum_{i=1}^{N[\Delta t(u-\kappa)]} e^{\theta_1 \tau_i} h(\tau_i^-) [\ln J]_i \]  (7)

- Jump part of the approximating process

\[ \tilde{J}(t_{m-\kappa}, t_{m-\kappa+1}) := e^{-\theta_1 t_{m-\kappa+1}} e^{\theta_1 (t_{m-\kappa} + (\delta t/2))} h'(t_{m-\kappa} + \frac{\delta t}{2}) \times [\ln J]_1 1\{N[\Delta t(m-\kappa)] \geq 1\} \]  (8)

- The function \( h'(\alpha) \), for any \( \alpha \in (t_{m-\kappa}, t_{m-\kappa+1}] \), is defined as:

\[ h'(\alpha) := 1\{D_c(t_{m-\kappa}, \alpha) < T(\alpha)\} - 1\{D_c(t_{m-\kappa}, \alpha) \geq T(\alpha)\} \]  (9)

where \( D_c(t_{m-\kappa}, \alpha) \) is defined as:

\[ D_c(t_{m-\kappa}, \alpha) = \mu(\alpha) + (\tilde{E}(t_{m-\kappa}) - \mu(t_{m-\kappa})) e^{-\theta_1 (\alpha - t_{m-\kappa})} \]  (10)
The approximating process under $Q$

\[ \tilde{E}[t_i + \delta t | \tilde{E}(t_i)] = \tilde{D}[t_i, t_i + \delta t | \tilde{E}(t_i)] + \tilde{J}[t_i, t_i + \delta t | \tilde{E}(t_i)] \quad (11) \]

where

\[ \tilde{D}[t_i, t_i + \delta t | \tilde{E}(t_i)] = \mu(t_i + \delta t) + (\tilde{E}(t_i) - \mu(t_i)) e^{-\theta_1 \delta t} \]

\[ + \sigma(t_i + \delta t) e^{-\theta_1 (t_i + \delta t)} \int_{t_i}^{t_i + \delta t} e^{\theta_1 y} dW(y) \quad (12) \]

and

\[ \tilde{J}[t_i, t_i + \delta t | \tilde{E}(t_i)] = e^{-\theta_1 \frac{\delta t}{2}} h'(t_i + \frac{\delta t}{2}) [\ln J]_1 1\{N[\Delta t(i)] \geq 1\} \quad (13) \]
Density of the components of the approximating process

• normal distribution with calculable mean and variance for the process

\[
\tilde{D}(t_i, t_i + \delta t) \bigg| \tilde{E}(t_i) = \mu(t_i + \delta t) + \left( \tilde{E}(t_i) - \mu(t_i) \right) e^{-\theta_1 \delta t} \\
+ \sigma(t_i + \delta t) e^{-\theta_1 (t_i + \delta t)} \int_{t_i}^{t_i + \delta t} e^{\theta_1 y} dW(y)
\]

• Conditional on the occurrence of at least one jump, the approximating jump process

\[
\tilde{J}(t_i, t_i + \delta t) \bigg| \tilde{E}(t_i) = e^{-\theta_1 \delta t} h'(t_i + \frac{\delta t}{2}) \ln J \mid \{N[\Delta t(i)] \geq 1\} (14)
\]

has a density given by

\[
f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|
\]

where \( g(x) = h'(t_i + \frac{\delta t}{2}) e^{-\theta_1 \frac{\delta t}{2}} x \), and \( f_X \) is the density of the jump size.
Density of the approximating process

- Conditioning on an initial value \( \widetilde{E}(t_i) \):

\[
\widetilde{E}(t_i + \delta t | \widetilde{E}(t_i)) = \widetilde{D}(t_i, t_i + \delta t | \widetilde{E}(t_i)) + \widetilde{J}(t_i, t_i + \delta t | \widetilde{E}(t_i))
\]

- If no jump occurs then its density is defined from the density of

\[
\widetilde{D}(t_i, t_i + \delta t | \widetilde{E}(t_i))
\]

- If at least one jump occurs its density is defined by the convolution of the densities of

\[
\widetilde{D}(t_i, t_i + \delta t | \widetilde{E}(t_i))
\]

and

\[
\widetilde{J}(t_i, t_i + \delta t | \widetilde{E}(t_i))
\]
Discretization of the density of a stochastic process one time-step ahead

- The density is divided into sections
- The probability mass within a section is assigned to the transition probability from the starting node to the node in the middle of the section.
- A probability threshold $\Pi$ prevents movements to sections with very low probability mass.
First step on the tree
Second step: A different conditional probability distribution
Third step: Mean reversion starts influencing the conditional distribution
Fourth step: Strong mean reversion pull
Some of the up movements have very low probability
Mean reversion: Only downward movements
Arrival probability

The probability of arriving at this node is called "arrival probability"
A full one-year grid, time changing parameters

movements during this period of very high jump intensity have been calculated in two sub-steps for increased accuracy. This results in additional nodes of low arrival probability.
A full one-year grid, time changing parameters, “filtering” on
# Grid applications: European style options, time changing parameters

<table>
<thead>
<tr>
<th>Option matures on</th>
<th>Parameter values at maturity</th>
<th>method</th>
<th>running time (sec)</th>
<th>Strike = $e^2$</th>
<th>Strike = $e^3$</th>
<th>Strike = $e^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 Jan 2009</td>
<td>$\mu = 2.99$</td>
<td>Monte Carlo</td>
<td>40</td>
<td>13.77</td>
<td>1.93</td>
<td>0</td>
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<tr>
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<td>Grid, all nodes included</td>
<td>0.8</td>
<td>13.76</td>
<td>1.95</td>
<td>0</td>
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<td>$\sigma = 1.3821$</td>
<td>Grid, filtering on</td>
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<td>13.76</td>
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<td>0</td>
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<td>8.51</td>
<td>2.06</td>
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<td>Grid, all nodes included</td>
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<td>81.49</td>
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<td>$\sigma = 1.5$</td>
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<td>81.43</td>
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<td>$\lambda_J = 12.52$</td>
<td>Grid, all nodes included</td>
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<td>23.02</td>
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<td>31 Dec 2009</td>
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<td>430</td>
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<td>37</td>
<td>12.31</td>
<td>1.35</td>
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<tr>
<td></td>
<td>$\sigma = 1.3827$</td>
<td>Grid, filtering on</td>
<td>23</td>
<td>12.29</td>
<td>1.35</td>
<td>0</td>
</tr>
</tbody>
</table>
Swing option pricing on the tree
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All possible nodes

All possible values of cumulative volume transacted up to that node

Compare with expected future value
Swing option pricing on the tree

All possible nodes

All possible values of cumulative volume transacted up to that node

Compare with expected future value
Take optimal transaction decisions
Swing option pricing on the tree

All possible nodes

All possible values of cumulative volume transacted up to that node

Move backwards and repeat for all nodes and all time-steps

Compare with expected future value
Take optimal transaction decisions
Grid and Monte Carlo methods for pricing swing options
Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
Grid and Monte Carlo methods for pricing swing options

• Both methods: Optimal transaction decisions and prices needed for each combination of:
  – admissible cumulative volume,
Grid and Monte Carlo methods for pricing swing options

- Both methods: Optimal transaction decisions and prices needed for each combination of:
  - admissible cumulative volume,
  - value of the underlying,
Grid and Monte Carlo methods for pricing swing options

• Both methods: Optimal transaction decisions and prices needed for each combination of:
  
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  – value of the underlying,
  – time-step
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• Monte Carlo method (Longstaff - Schwartz)
Grid and Monte Carlo methods for pricing swing options

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  - possible values of the underlying are generated from 1,000 paths
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• Grid method
**Grid and Monte Carlo methods for pricing swing options**

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  - admissible cumulative **volume**,  
  - **value** of the underlying,  
  - time-step  

- Monte Carlo method (Longstaff - Schwartz)  
  - possible values of the underlying are generated from 1,000 paths  

- Grid method  
  - Possible values of the underlying are represented by the nodes of the grid at each time-step (about 200 nodes)
## Swing option prices: Time varying parameters

<table>
<thead>
<tr>
<th>Storage contract</th>
<th>parameters</th>
<th>Price</th>
<th>Grid</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valuation date</td>
<td>Start date</td>
<td>End date</td>
<td>min</td>
<td>max</td>
</tr>
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<td>$\mu = 3.11$</td>
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<tr>
<td>01-Jan-09</td>
<td>01-Jan-09</td>
<td>31-Mar-09</td>
<td>$\sigma = 1.38$</td>
<td>$\sigma = 1.43$</td>
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<td>$\lambda_J = 10^{-4}$</td>
<td>$\lambda_J = 1.60$</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$\mu = 3.18$</td>
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<tr>
<td>01-May-09</td>
<td>01-Jun-09</td>
<td>31-Jul-09</td>
<td>$\sigma = 1.46$</td>
<td>$\sigma = 1.5$</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$\lambda_J = 6.70$</td>
<td>$\lambda_J = 56$</td>
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- A lot more paths are needed for the Monte Carlo method to produce smaller confidence intervals
Swing option prices: Time varying parameters

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<td>Valuation date</td>
<td>Start date</td>
<td>End date</td>
<td>$\mu$</td>
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<td>01-Jan-09</td>
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<td>01-Jun-09</td>
<td>31-Jul-09</td>
<td>3.18</td>
</tr>
</tbody>
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- A lot more paths are needed for the Monte Carlo method to produce smaller confidence intervals.
- For European options, 50,000 paths were needed in order to achieve narrow confidence intervals.
Swing option prices: Time varying parameters

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<td>min</td>
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<tr>
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<td>31-Mar-09</td>
<td>$\mu = 2.99$</td>
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<td>$\sigma = 1.38$</td>
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- The grid presents a very promising approach, achieving a good balance between accuracy and calculation time.
Thank you for your attention.