On Optimal Exercise of Swing Options in Electricity Markets

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Swing Option

- Swing options are sold on deregulated electricity (and other commodity) markets. These contracts appear in various forms
- In this talk, we consider a specific contract, a flexible load contract, which is a particular version of a swing option
- Holder has a right to buy a volume of electricity over a time interval
- As an example, assume that the time horizon T of the contract is 1 year and the that it is possible to exercise every hour – that is, there is 8760 possible exercise times
- Furthermore, assume that the holder has 4380 exercise rights
- Given the high number of possible exercise times and exercise rights, we model the multiple strike option as a continuous time control problem with a total volume constraint M
- Now, Z can be regarded as a nondecreasing process with continuous paths

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The Model

- To propose a model, assume that $(\Omega, \mathcal{F}, \mathbf{F}, \mathbf{P})$ is a complete probability space
- We assume that the price of electricity evolves according to the strongly unique solution of the Itô equation

$$dP_t = \mu(t, P_t)dt + \sigma(t, P_t)dW_t$$

  with $P_0 = p$, where the functions μ and σ are sufficiently well behaving Lipschitz-continuous functions
- The class of admissible exercise policies consists of processes

$$Z_t = \int_0^t u_s ds,$$

  where u is progressively $\mathbb{F}$-measurable and satisfies the constraints $u_s \in [0, \bar{u}]$ for all $s \in [0, T]$ and $Z_T \leq M$ for some $M > 0$

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- *The optimization problem*: Find the admissible exercise rate u^* which gives

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  with the final value $V(T, Z_T, P_T) = 0$. Here, $r > 0$ is the constant rate of discounting and $K > 0$ is the strike price.

- By using Itô formula to process $t \mapsto e^{-rt}(P_t - K)Z_t$, the optimization problem can be rewritten as

$$V(t, Z_t, P_t) = (K - P_t)Z_t + \sup_{u \in \mathcal{U}} \mathbf{E}\left[e^{-r(T-t)}(P_T - K)Z_T + \int_t^T e^{-r(s-t)}\theta(s, P_s)Z_s ds \middle| \mathcal{F}_t \right],$$

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Some Properties of the Model

- Consider first the limiting case $M \geq \bar{u}T$, that is, the case when there is no effective total volume constraint
- In this case, the optimal exercise rate is

$$u_t^* = \begin{cases} \bar{u}, & P_t > K, \\ 0, & P_t \leq K, \end{cases}$$

  for all $t \in [0, T]$
- Moreover, the marginal value $V_z \equiv 0$ (marginal lost option value)
- In other words, optimal exercise does not depend on the current state of Z
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- Consider now the case $M < \bar{u}T$, that is, the case when there is an effective total volume constraint
- In this case, it can be shown that the marginal value $V_z < 0$ (the option loses value when used)
- The result is proved by studying the difference quotient

$$\lim_{\varepsilon \to 0} \frac{V(t, Z_t + \varepsilon, P_t) - V(t, Z_t, P_t)}{\varepsilon}$$

- So the optimal exercise depends on the current state of Z
- Moreover, it can be proved that the value V is *concave* in z
- In other words, the marginal lost option value is larger close to the constraint M than away from it
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Optimal Exercise: Necessary Conditions

- Bellman principle: the optimal value V should satisfy

$$V(t, Z_t, P_t) =$$
$$\sup_{u \in \mathcal{U}} \mathbf{E}\left[\int_t^w e^{-r(s-t)}(P_s - K)u_s ds + e^{-r(w-t)}V(w, Z_w, P_w)\middle|\mathcal{F}_t\right]$$

- By a standard argument (that is, by assuming that V is smooth enough and using Itô formula to the process $t \mapsto e^{-rt}V(t, Z_t, P_t)$), this gives rise to the HJB-equation

$$V_t(t,z,p) + \frac{1}{2}\sigma^2(t,p)V_{pp}(t,z,p) + \mu(t,p)V_p(t,z,p) - rV(t,z,p) + \sup_u \{u(t)(p - K + V_z(t,z,p))\} = 0,$$

where u varies over the set of non-decreasing functions defined on $[0, T]$ satisfying the conditions $0 \le u(t) \le \bar{u}$ and $\int_0^T u(t)dt \le M$

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- Consider the term $\sup_u \{u(t)(p - K + V_z(t,z,p))\}$
- Since u is non-negative, we observe that the supremum is given by

$$\hat{u}_t = \begin{cases} \bar{u}, & P_t - K > -V_z(t, Z_t, P_t), \\ 0, & P_t - K \leq -V_z(t, Z_t, P_t), \end{cases}$$

- Interpretation: Exercise the option when ever the instantaneous exercise payoff dominates the marginal lost option value
- We recall that when $M \geq \bar{u}T$, then $V_z \equiv 0$ – we find that the same interpretation holds also in this limiting case
- It is also possible to prove a verification result, i.e. to pin down a set of sufficient conditions for a given function to coincide with the value V

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Numerical solution of the HJB-equation

- We studied the model numerically using a first order backward time stepping scheme
- The grid was set to be uniform in t- and z-direction and, having mean reverting prices in mind, adaptive in p-direction
- The scheme is both stable and consistent under the condition $\bar{u}\Delta t \leq \Delta z$ – this is due to the local growth constraint on Z
- For illustration, consider the case when prices are given by the geometric Ornstein-Uhlenbeck process

$$P_t = \exp(X_t)\,, \qquad dX_t = \kappa(\mu - X_t)dt + \sigma dW_t$$

- Fix the parameter configuration $\mu = \ln 40$, $r = 0$, $\sigma = 5$, $\kappa = 1$, $T = 1$, $M = 0.5$, $\bar{u} = 1$ and $K = 0$

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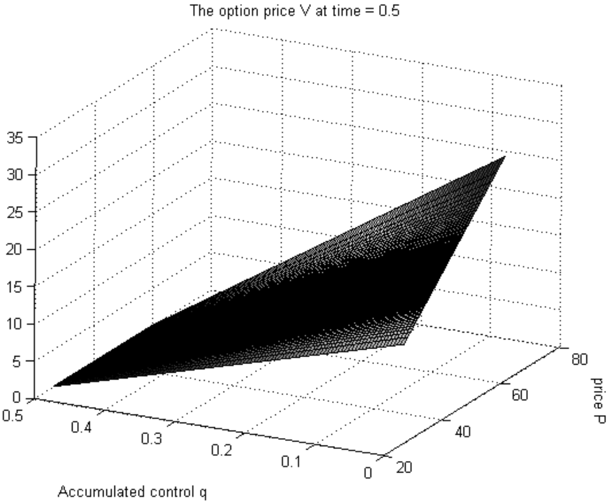

Figure. Option price

Numerical solution of the HJB-equation

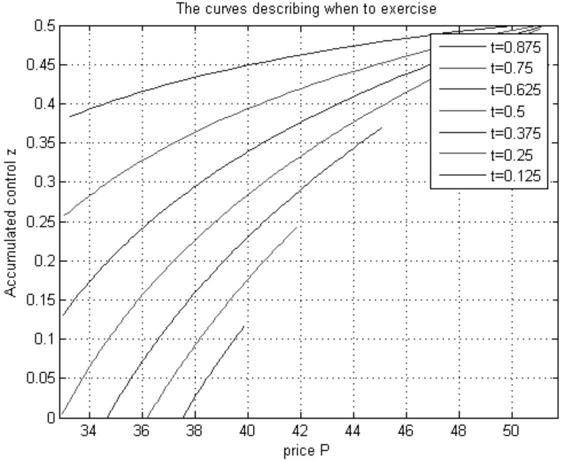

Figure. Exercise boundaries

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