Storage option: an analytic approach

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Storage option

- Problem definition, solution outline
- Price processes, correlation function
- Example calculation: simple storage and swing contracts
- Applications and range of applicability

Storage option

- Conclusions
  - Closed form solution can be used in semi-analytic valuation methods.
  - Greeks:
    - Delta and Gamma for hedging
    - Dependency on the price model parameters (volatility, mean reversion,...).
- Range of applicability of the approximation
  - Simplified constraints
  - No operating and Bid-Offer costs
  - Continuous time

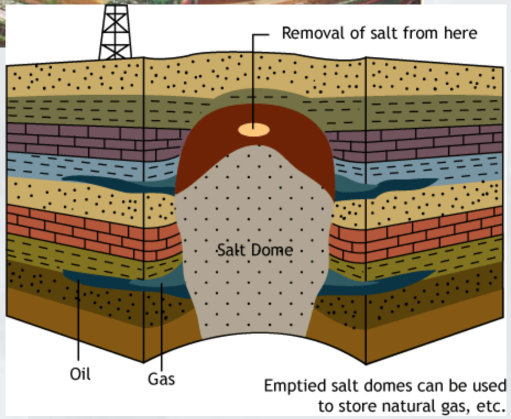
Storage Problem formulation

- Storage parameters

  volume $q(t); \quad 0 \le t \le T;$

  constraints: $0 \le q(t) \le Q_{\max}$

  $r_{\min} \le \Delta q(t) \le r_{\max}$



- Price curve
  - Intrinsic: $F_i = F(t_i)$
  - Stochastic: $F = F(\tau, t)$

$$F(0,t) = F_0(t)$$

$$\frac{dF(\tau,t)}{F(\tau,t)} = \sigma(\tau,t)\,dZ$$

Storage option: intrinsic problem

- Optimal dispatch => maximised profit

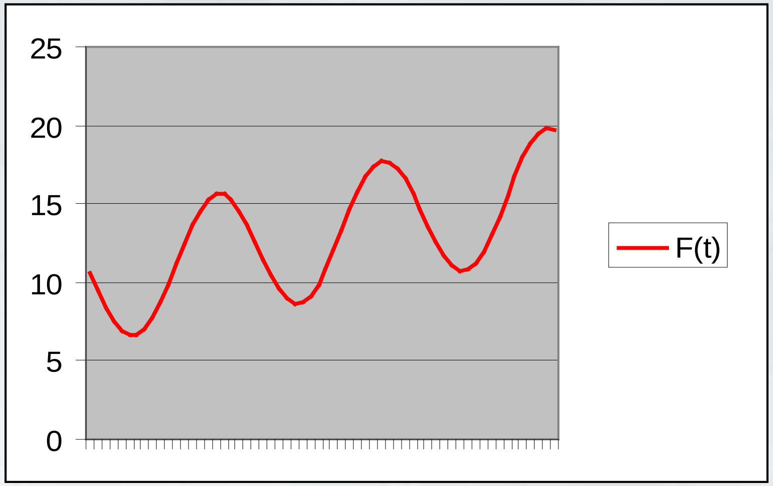
$$S = -\sum_i \Delta q_i F_i \qquad S = -\int \dot{q}(t) F(t)\, dt$$

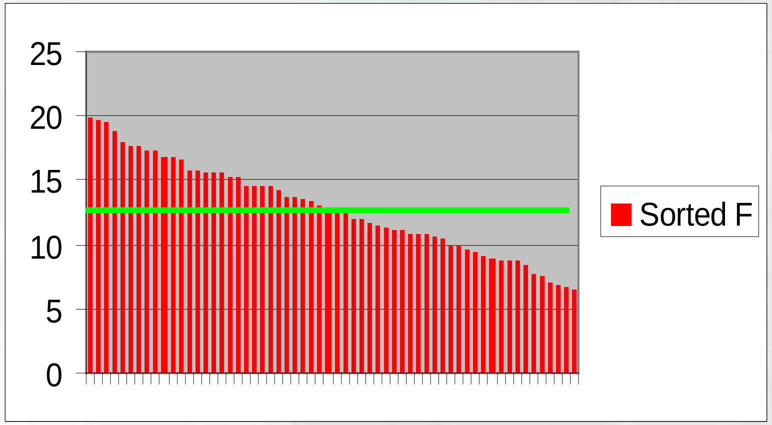
- Variational approach

$$\frac{\delta S}{\delta q} = 0 \quad \text{conditional on} \quad Q_{end} = Q_{start} + \int \dot{q}(t)\, dt$$

- Trigger price level *C:*

$$\Delta q(t) = r_{\max}, \quad if\ F(t) < C$$
$$\Delta q(t) = r_{\min}, \quad if\ F(t) > C$$

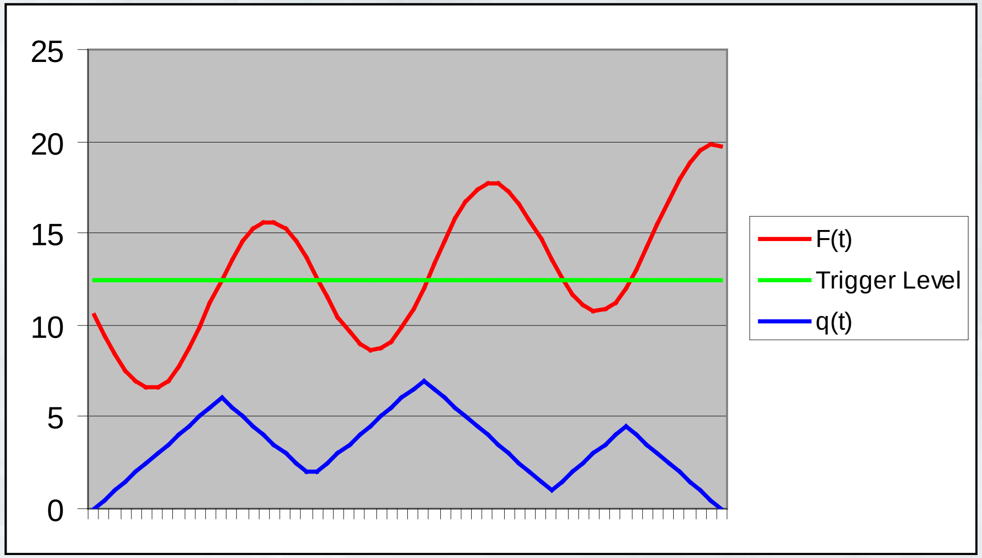




Storage option: intrinsic problem

- Deterministic solution

$$\Delta q(t) = r_{\min} + (r_{\max} - r_{\min})\Theta(C - F(t))$$



- Possible deviations:
  - Time discretisation
  - Constraints (upper and lower boundaries)
  - Other constraint types
    - injection/release costs
    - carry costs
    - cycle constraint
    - volume adjusted injection/release rates

Storage option: stochastic problem

- Forward price model

$$dF(\tau,t)=\sigma(\tau,t,F)\,dW_\tau(t)$$

$$L(\tau,t_1,t_2)=\frac{1}{dt}\left\langle dF(\tau,t_1)\,dF(\tau,t_2)\right\rangle$$

- One-factor model

$$\frac{dF(\tau,t)}{F(\tau,t)}=\sigma_0\,e^{-\alpha(t-\tau)}\,dW_\tau$$

$$L(\tau,t_1,t_2)=\frac{1}{dt}\left\langle dF(\tau,t_1)\,dF(\tau,t_2)\right\rangle=\left\langle F(\tau,t_1)\,F(\tau,t_2)\right\rangle\sigma_0^2\,e^{-\alpha(t_1-\tau)-\alpha(t_2-\tau)}$$

Storage option: stochastic problem

- Rolling intrinsic strategy
- Functions of the observation time:

  Forward curve, trigger level, optimal trajectory, target function

$$S(\tau) = -\int \dot{q}(\tau,t)\, F(\tau,t)\, dt\, ; \quad \dot{q}(\tau,t) = r_{\min} + (r_{\max} - r_{\min})\ \Theta\big(C(\tau) - F(\tau,t)\big)$$

- Variation of the target function

$$\delta S = \frac{\delta S}{\delta F}\delta F + \frac{\delta S}{\delta C}\delta C + \frac{1}{2}\left(\frac{\delta^2 S}{\delta F^2}\delta F^2 + 2\frac{\delta^2 S}{\delta F\,\delta C}\delta F\,\delta C + \frac{\delta^2 S}{\delta C^2}\delta C^2\right)$$

- Relation between δC and δF is different for storage and swing options

Storage option: stochastic problem

- Option Gamma

$$\delta S = \Delta\, dF + \frac{1}{2}\Gamma\, dF^2, \qquad where \quad \Delta = \frac{\partial S}{\partial F}; \quad \Gamma = \frac{\partial^2 S}{\partial F^2};$$

- Option time value *V(t)*

$$\gamma(\tau) = \frac{1}{d\tau}\left\langle \delta S \right\rangle \qquad\qquad V(t) = \int_0^t \gamma(\tau)\, d\tau$$

Storage option: stochastic problem

- Gamma of the storage option

$$\gamma(\tau) = \frac{(r_{\max} - r_{\min})}{2} \sum_i \frac{L_i}{|\dot{F}_i|} - \frac{(r_{\max} - r_{\min})}{2\sum_i 1/|\dot{F}_i|} \sum_{ij} \frac{L_{ij}}{|\dot{F}_i||\dot{F}_j|}$$

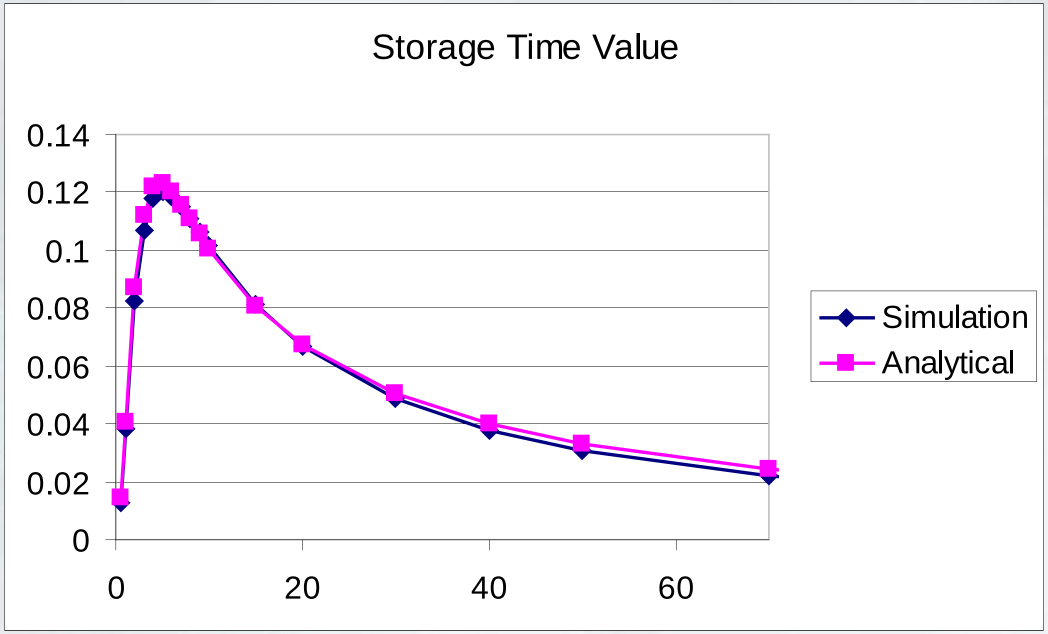
- Gamma of the swing option

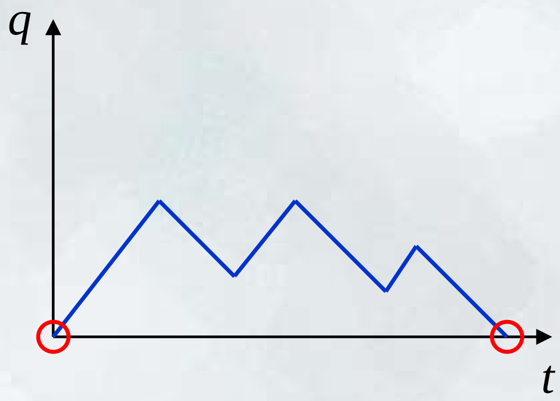
$$\gamma(\tau) = \frac{(r_{\max} - r_{\min})}{2} \sum_i \frac{L_i}{|\dot{F}_i|}$$

where

$$\{t_i : F(t_i) = C\} \qquad L_{ij} = L(\tau, t_i, t_j) = \frac{1}{dt}\left\langle dF(\tau, t_i)\, dF(\tau, t_j)\right\rangle \qquad \dot{F}_i = \frac{\partial}{\partial t} F(\tau, t)\Big|_{t=t_i}$$

Storage option time value





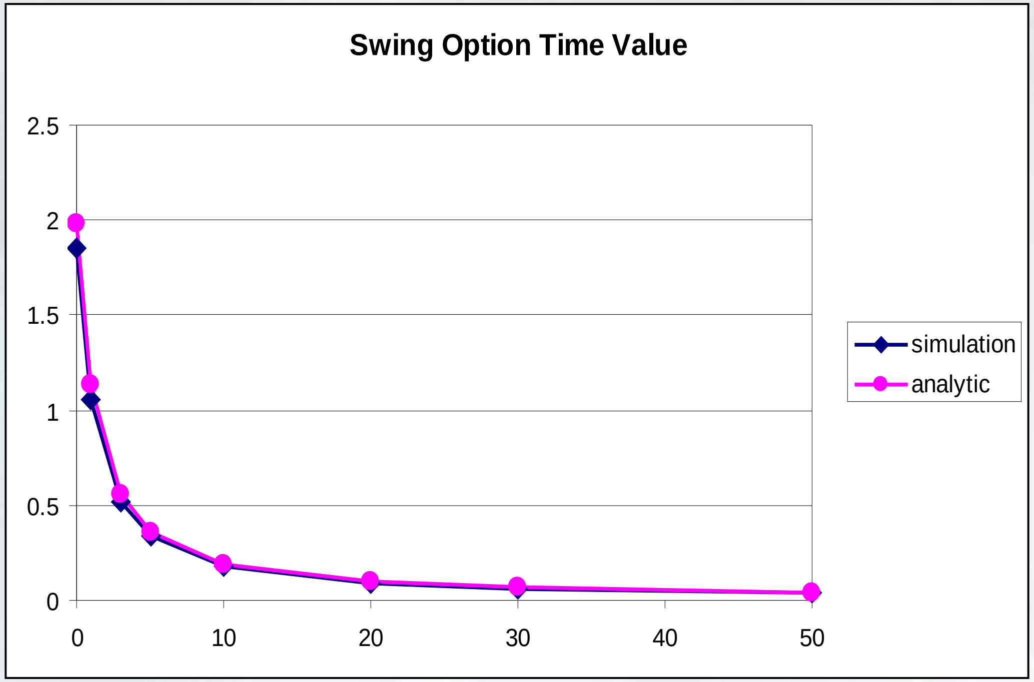
- prices positive
- zero-sum game
- time value is due to the forward curve distortion

$$V \approx k\,\sigma_0^2\,T^4\,\alpha^2, \qquad as \quad \alpha \to 0$$

$$V \approx k\,\frac{\sigma_0^2\,T}{\alpha}, \qquad as \quad \alpha \to \infty$$

$$\frac{dF(\tau,t)}{F(\tau,t)} = \sigma_0\, e^{-\alpha(t-\tau)}\, dW_\tau$$

Swing option time value



$$V \approx 2k \ , \qquad as \quad \alpha \to 0$$

$$V \approx k \frac{\sigma_0^2 T}{\alpha} \ , \qquad as \quad \alpha \to \infty$$

- prices are sign indefinite
- time value is due to the absolute forward curve volatility

Contract specific price model calibration

- Split of the time value
  - Two factor price process

$$\frac{dF(\tau,t)}{F(\tau,t)} = \sigma_1 e^{-\alpha_1 (t-\tau)} dW_\tau^{(1)} + \sigma_2 e^{-\alpha_2 (t-\tau)} dW_\tau^{(2)}; \quad \left\langle dW^{(1)} dW^{(2)} \right\rangle = 0;$$

  - Split of correlation function and of the time value

$$L = L^{(1)} + L^{(2)}$$

$$V = V^{(1)} + V^{(2)}$$

  - One can neglect the long term volatility for the storage
- Calibration accuracy
  - mind the sensitivity of the time value to the model parameters

Conclusions

- Analytic expressions for
  - option time value, option gamma, greeks
- Better understanding of the mechanisms driving the time value
- Exposure to the price model calibration
- Mind the gap:
  - the obtained results are only an approximation. The time value can only be trusted within a small range of parameters.
  - However most of qualitative results are universal.
  - Greeks are more robust and less sensitive to the approximation