Storage option: an analytic approach

Dmitry Lesnik
RWE Supply & Trading
dmitry.lesnik@rwe.com
Storage option

- Problem definition, solution outline
- Price processes, correlation function
- Example calculation: simple storage and swing contracts
- Applications and range of applicability
Storage option

• Conclusions
  • Closed form solution can be used in semi-analytic valuation methods.
  • Greeks:
    • Delta and Gamma for hedging
    • Dependency on the price model parameters (volatility, mean reversion,...).

• Range of applicability of the approximation
  • Simplified constraints
  • No operating and Bid-Offer costs
  • Continuous time
Storage Problem formulation

• Storage parameters
  volume \( q(t); \ 0 \leq t \leq T; \)
  constraints: \( 0 \leq q(t) \leq Q_{\text{max}} \)
  \( r_{\text{min}} \leq \Delta q(t) \leq r_{\text{max}} \)

• Price curve
  o Intrinsic: \( F_i = F(t_i) \)
  o Stochastic: \( F = F(\tau, t) \)
    \( F(0, t) = F_0(t) \)
    \( \frac{dF(\tau, t)}{F(\tau, t)} = \sigma(\tau, t) \, dZ \)
Storage option: intrinsic problem

- Optimal dispatch => maximised profit
  \[ S = -\sum_i \Delta q_i F_i \quad S = -\int q(t) F(t) \, dt \]
- Variational approach
  \[ \frac{\delta S}{\delta q} = 0 \quad \text{conditional on} \quad Q_{\text{end}} = Q_{\text{start}} + \int q(t) \, dt \]
- Trigger price level \( C \):
  \[ \Delta q(t) = r_{\text{max}}, \quad \text{if } F(t) < C \]
  \[ \Delta q(t) = r_{\text{min}}, \quad \text{if } F(t) > C \]
Storage option: intrinsic problem

- Deterministic solution

\[ \Delta q(t) = r_{\text{min}} + (r_{\text{max}} - r_{\text{min}}) \Theta(C - F(t)) \]

- Possible deviations:
  - Time discretisation
  - Constraints (upper and lower boundaries)
  - Other constraint types
    - injection/release costs
    - carry costs
    - cycle constraint
    - volume adjusted injection/release rates
Storage option: stochastic problem

- Forward price model
  \[ dF(\tau, t) = \sigma(\tau, t, F) dW_\tau(t) \]
  \[ L(\tau, t_1, t_2) = \frac{1}{dt} \langle dF(\tau, t_1) dF(\tau, t_2) \rangle \]

- One-factor model
  \[ \frac{dF(\tau, t)}{F(\tau, t)} = \sigma_0 e^{-\alpha(t-\tau)} dW_\tau \]
  \[ L(\tau, t_1, t_2) = \frac{1}{dt} \langle dF(\tau, t_1) dF(\tau, t_2) \rangle = \langle F(\tau, t_1) F(\tau, t_2) \rangle \sigma_0^2 e^{-\alpha(t_1-\tau)-\alpha(t_2-\tau)} \]
Storage option: stochastic problem

- Rolling intrinsic strategy
- Functions of the observation time:
  Forward curve, trigger level, optimal trajectory, target function

\[ S(\tau) = -\int q(\tau, t) F(\tau, t) \, dt; \quad q(\tau, t) = r_{\text{min}} + (r_{\text{max}} - r_{\text{min}}) \Theta(C(\tau) - F(\tau, t)) \]

- Variation of the target function

\[ \delta S = \frac{\delta S}{\delta F} \delta F + \frac{\delta S}{\delta C} \delta C + \frac{1}{2} \left( \frac{\delta^2 S}{\delta F^2} \delta F^2 + 2 \frac{\delta^2 S}{\delta F \delta C} \delta F \delta C + \frac{\delta^2 S}{\delta C^2} \delta C^2 \right) \]

- Relation between \( \delta C \) and \( \delta F \) is different for storage and swing options
Storage option: stochastic problem

- Option Gamma

\[ \delta S = \Delta dF + \frac{1}{2} \Gamma dF^2, \quad \text{where} \quad \Delta = \frac{\partial S}{\partial F}; \quad \Gamma = \frac{\partial^2 S}{\partial F^2}; \]

- Option time value \( V(t) \)

\[ \gamma(\tau) = \frac{1}{d\tau} \langle \delta S \rangle \quad V(t) = \int_{0}^{t} \gamma(\tau) d\tau \]
Storage option: stochastic problem

- Gamma of the storage option

\[ \gamma(\tau) = \frac{(r_{\text{max}} - r_{\text{min}})}{2} \sum_i \frac{L_i}{|F_i|} - \frac{(r_{\text{max}} - r_{\text{min}})}{2} \sum_{ij} \frac{L_{ij}}{|F_i| |F_j|} \]

- Gamma of the swing option

\[ \gamma(\tau) = \frac{(r_{\text{max}} - r_{\text{min}})}{2} \sum_i \frac{L_i}{|F_i|} \]

where

\[ \{ t_i : F(t_i) = C \} \quad \text{and} \quad L_{ij} = L(\tau, t_i, t_j) = \frac{1}{dt} \left\langle dF(\tau, t_i) dF(\tau, t_j) \right\rangle \quad \text{and} \quad \dot{F}_i = \frac{\partial}{\partial t} F(\tau, t) \bigg|_{t=t_i} \]
Storage option time value

\[ V \approx k \sigma_0^2 T^4 \alpha^2, \quad \text{as } \alpha \to 0 \]

\[ V \approx k \frac{\sigma_0^2 T}{\alpha}, \quad \text{as } \alpha \to \infty \]

- prices positive
- zero-sum game
- time value is due to the forward curve distortion

\[ \frac{dF(\tau,t)}{F(\tau,t)} = \sigma_0 e^{-\alpha(t-\tau)} dW_\tau \]
Swing option time value

\[ V \approx 2k, \quad \text{as} \quad \alpha \rightarrow 0 \]

\[ V \approx k \frac{\sigma_0^2 T}{\alpha}, \quad \text{as} \quad \alpha \rightarrow \infty \]

- prices are sign indefinite
- time value is due to the absolute forward curve volatility
Contract specific price model calibration

- Split of the time value
  - Two factor price process
    \[ \frac{dF(\tau,t)}{F(\tau,t)} = \sigma_1 e^{-\alpha_1(t-\tau)} dW^{(1)}_\tau + \sigma_2 e^{-\alpha_2(t-\tau)} dW^{(2)}_\tau; \quad \left\langle dW^{(1)} dW^{(2)} \right\rangle = 0; \]
  - Split of correlation function and of the time value
    \[ L = L^{(1)} + L^{(2)} \]
    \[ V = V^{(1)} + V^{(2)} \]
  - One can neglect the long term volatility for the storage

- Calibration accuracy
  - mind the sensitivity of the time value to the model parameters
Conclusions

• Analytic expressions for
  – option time value, option gamma, greeks
• Better understanding of the mechanisms driving the time value
• Exposure to the price model calibration
• Mind the gap:
  – the obtained results are only an approximation. The time value can only be trusted within a small range of parameters.
  – However most of qualitative results are universal.
  – Greeks are more robust and less sensitive to the approximation