A SETARX model for spikes and antispikes in electricity markets

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Electricity Data and Mathematical Price Models

What model for such a behavior ?

6 years of data: **AESO SP prices** (Alberta, Canada), Apr. 7 2001 - Apr. 6 2007, including **electricity demand**.

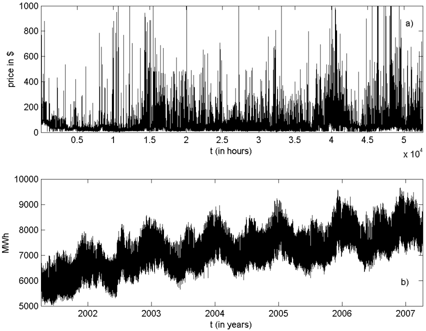


Figure 1: Prices and demand in the Alberta power market: 5 years from Apr-7-2001 to Apr-6-2007, time in hours; a) prices, b) demand

Simple Model, Complex Behaviour

A model that:

- includes energy demand
- in continuous time supports **spikes without jumps**
- has a **rich phenomenology**, as that seen in data
- has some **microeconomic foundations**
- in discrete time belongs to a standard class of models (SETARX) and preserves its continuous time properties
- is analyzed in a nonstandard way

Data

Alberta (Canada) **wholesale power market** (now also retail), a voluntary **pool** (not all energy is traded here).

Data from AESO site:

1. energy prices in Canadian dollars
2. demand (load) in Megawatt-hour (MWh)

Players:

1. gencos
2. grid (Transmission System Operator, TSO): AESO
3. Independent System Operator (ISO) and Exchange: AESO

Electricity Prices: Spikes Have a Structure

Discrete i-time prices $\hat{p}_i$ available for **each hour**.

Hourly demand data is available.

Spikes appear only during demand crests, **but not always**.

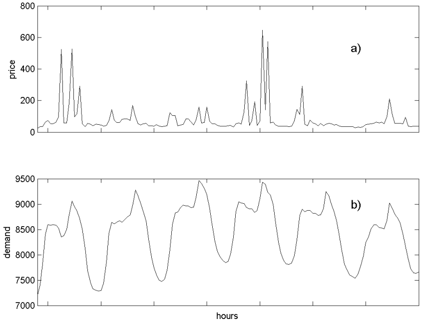


Figure 2: Alberta power market: one week from Mon Jan-08-2007 to Sun Jan-14-2007, time in hours; a) system prices in C$, b) demand in MW!

Spikes and Baseline

Spikes originate from a first mean reversion mechanism.

In their **baselines**, prices follow smoothly the demand daily seasonality - a second mean reversion type.

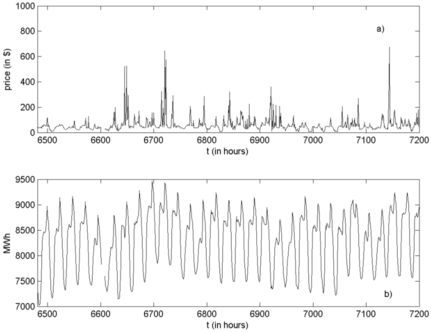


Figure 3: Alberta power market: one month from Jan-1-2007 to Jan-31-2007, time in hours; a) prices, b) demand

Logprices and Antispikes

Prices p show spikes, **logprices** $x = \log p$ show spikes and **antispikes**, with a mean reversion similar to the spikes mean reversion.

Logprices baseline has yearly periodicity. Third mean reversion type.

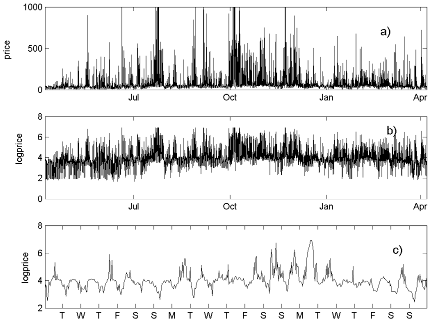


Figure 4: Alberta power market; a) one year of prices, from hour 1 of Apr-07-2006 to hour 24 of Apr-07-2007; b) logprices, same scale; c) detail of b): 3 weeks of logprices from hour 1 of Aug-14 to hour 24 of Sep-3.

Periodic Parossistic Phases

There are seasons in which demand triggers higher frequency spiking (e.g. winter)

This is a second type of seasonality: **spikes don't change in structure but become more frequent**

Fourth mean reversion type.

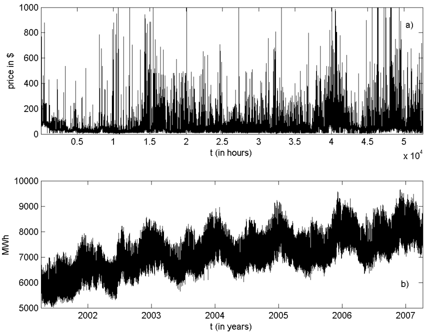


Figure 5: Prices and demand in the Alberta power market: 5 years from Apr-7-2001 to Apr-6-2007, time in hours; a) prices, b) demand

Modelling Approaches

- Top-down: the standard financial way.
- Bottom-up: agents on networks.
- **Hybrid**: take in consideration microeconomics, use few degrees of freedom.

Power Markets as Tight Markets

Features that make power markets different from stock markets:

- **A Rigid Periodic demand** $d(t)$ **drives prices** p: $p = p(d(t))$
- **Capacity constraints** affect prices
- **Grid constraints** affect prices
- Anticipated constrained times are the best times to game the system...

Being **potentially constrained**, the power market sometimes becomes a **tight market**

The Threshold

Technical constraints introduce a **threshold** in the price formation mechanism

- **Below** the threshold prices react smoothly to demand variations
- **Above** the threshold prices **can** react in a non-smooth way.

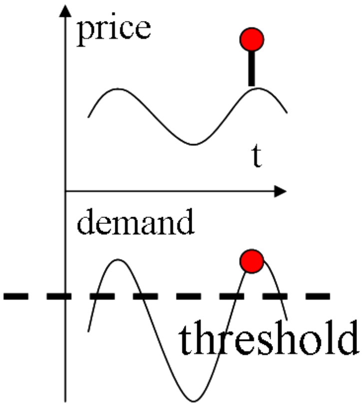


Figure 6: Threshold effects

Some Stochastic Financial Models

Continuous-Time (from high frequency finance):

- **Jump-diffusion**: compound Poisson, Lévy.
- **Nonlinear pure diffusion**: *Resonating Market Models*, Lucheroni 2007.

Discrete-Time (more natural, since data are discrete):

- **ARX**: driven linear autoregressions.
- **Switching ARX**.
- **TARX**: *SETARX models for spikes and antispikes* Lucheroni 2010.

SETARX Models

Discrete-Time

- **AR(M)X**: $L^h x_i = x_{i-h}, \quad \phi^M(L) = 1 - \ldots - \phi_M L^M$

$$\left\{ \phi^M(L)\, x_i = \sigma\, \varepsilon_i + d_{i-1} \right.$$

- **Switching ARX**: 2 **regimes** R, 1 threshold T

$$\left\{ \begin{array}{ll} \phi^2_I(L)\, x_i = \sigma\, \varepsilon_i + d_{i-1}, & u(i) \geq T \quad (R = I) \\ \phi^2_{II}(L)\, x_i = \sigma\, \varepsilon_i + d_{i-1}, & u(i) < T \quad (R = II) \end{array} \right.$$

- $u(i) = U(x_i) \rightarrow$ **SETARX**

The McKean Oscillator

Discrete or continous-time. Two coupled nonlinear diffusions x and y. 2 **thresholds** D_L and D_R, then 3 **regimes**. **Forcing** f. Noise $\xi = dW/dt$.

$$
\begin{aligned}
\epsilon \dot{x} &= g_R(x; a) - y \\
\dot{y} &= x - \gamma_b y + b + f(t) + \sigma(d)\,\xi(t)
\end{aligned}
$$

where $g_R(x; D_L, D_R) =$

$$
\begin{cases}
-\beta_L(x + D_L) - \gamma_0 D_L, & -\infty < x \leq -D_L \quad (I) \\
\gamma_0 x, & -D_L < x < D_R \quad (II) \\
-\beta_R(x - D_R) + \gamma_0 D_R, & D_R \leq x < +\infty \quad (III)
\end{cases}
$$

$f = 0$: No Forcing

For forcing $f = 0$, **no seasonality is included**.

Logprice $x(t)$.

Price $p(t) = e^{x(t)}$.

Spikes with different heights and widths, springing from a **constant baseline mean reversion level**.

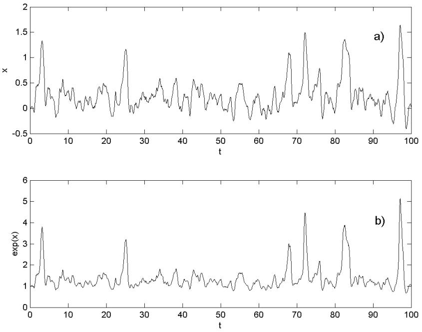


Figure 7: McKean model (from new form). Parameters: $\epsilon = 0.3$, $d = 0.1$, $a = 1$, $b = -0.5$, $\gamma_b = 1$. a) logprice $x(t)$ dynamics, b) price $p(t) = \exp\ x(t)$ dynamics.

McKean Oscillator at $f = 0$

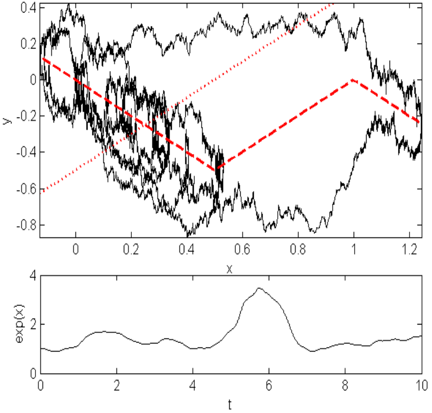
Phase space $\{x(t), y(t)\}$.

Nullclines: x for $\dot{x} = 0$, y for $\dot{y} = 0$.

Logprice $x(t)$.

Auxiliary coordinate $y(t)$.

$0 << \epsilon < 1$ (soft regime)



Dynamically-Critical Points

The McKean model undergoes a **subcritical** Hopf transition as *b* changes. Spikes can be formed close to b_2, **subthreshold**, as **stochastic orbits**.

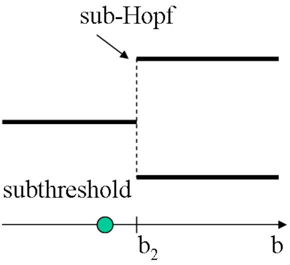


Figure 8: FNS model, *x* dynamic range

$f \neq 0$: SRS

Forcing $f(t) = A \sin(\omega t)$, to include **daily baseline seasonality**.

Stochastically resonating spiking - SRS (second mean reversion type).

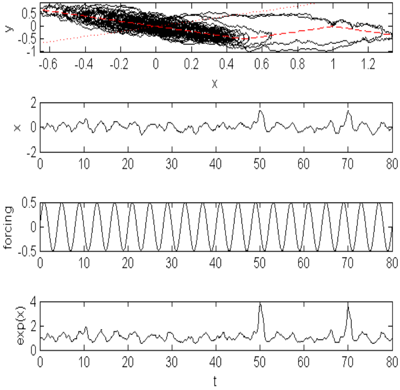
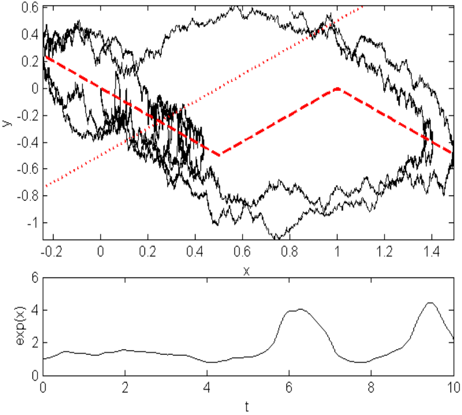


Figure 9: McKean model. $\epsilon = 0.3$, $d = 0.1$, $b = 0$, $\gamma_b = 1$, $A = 0.5$, $\omega = \pi/2$. a)

Movie: Resonating Power Market

Stable, unstable, metastable regions.



Two Frequencies: Parossistic Phases

- periodic ‘fountains’ of spikes
- demand: only 2 frequencies (more can be added), **no trend**
- fourth mean reversion type

$$f(t) = u\left(v\sin(\frac{\omega_f}{365}t) + \sin(\omega_f t)\right)$$

Real and Simulated Series

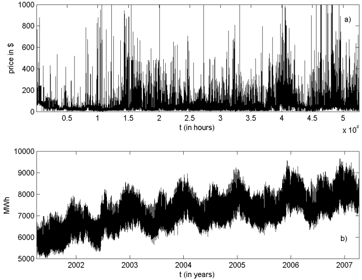


Figure 10: Prices and demand in the Alberta power market: 5 years from Apr-7-2001 to Apr-6-2007, time in hours; a) prices, b) demand.

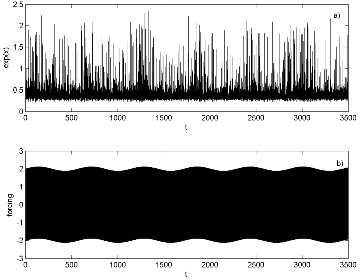


Figure 11: Simulation of 6 years of power market prices, extended model for $\epsilon = 0.15$, $\kappa = 1$, $\lambda = 1$, $b = 0$, $\gamma = 1$, $\xi = 2$, $c = -1$, f with $u = 2$, $v = 0.06$, $\omega_f = 4$, $d = 0.1$, $\Delta t = 0.035$; a) price process ; b) forcing: the smaller yearly $\omega_f/365$ frequency modulates the much higher daily ω_f frequency, which cannot be resolved in the picture.

Antispikes: SAS McKean

Spike-AntiSpike McKean Model (SAS): 5 regimes plus forcing (L for left, R for right).

$$\begin{aligned} \epsilon\dot{x} &= g_R^{SAS}(x; C_L, C_R) - y \\ \dot{y} &= x - \gamma_b y + b + f(t) + \sigma(d)\,\xi(t) \end{aligned}$$

where $g_R^{SAS}(x; C_L, C_R) =$

$$\left\{ \begin{array}{rrcll} -\alpha_L(x + C_L), & -\infty & < x \leq & -C_L & (I) \\ \beta_L(x + C_L), & -C_L & < x < & -D_L = -\frac{\beta_L}{\gamma_0+\beta_L} C_L & (II) \\ -\gamma_0 x, & -D_L & \leq x \leq & D_R = \frac{\beta_R}{\gamma_0+\beta_R} C_R & (III) \\ \beta_R(x - D_R), & D_R & < x < & C_R & (IV) \\ -\alpha_R(x - D_R), & C_R & \leq x < & +\infty & (V) \end{array} \right.$$

SAS McKean Dynamics

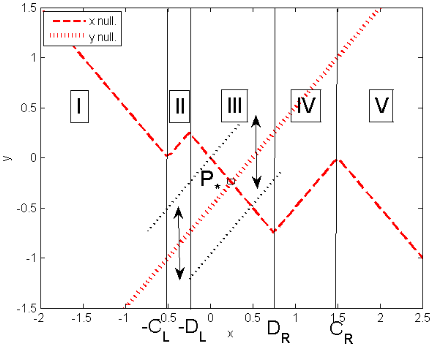


Figure 12: SAS McKean model phase-space for $f = 0$. Other parameters: $\alpha_L = \alpha_R = 1$, $\beta_L = \beta_R = 1$, $\gamma_0 = 1$, $C_L = 1/2$, $C_R = 3/2$, $b = -1/2$, $\gamma_b = 1$.

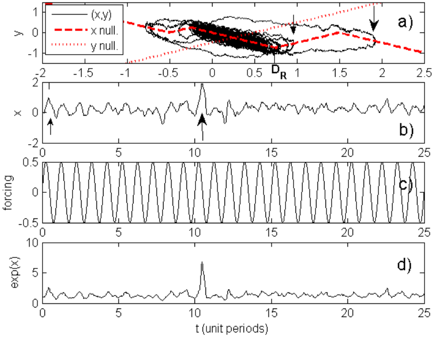


Figure 13: SAS McKean model for $f \neq 0$. Parameters: $\epsilon = 0.3$, $s = 0.1$, $\alpha_L = \alpha_R = 1$, $\beta_L = \beta_R = 1$, $\gamma_0 = 1$, $C_L = 1/2$, $C_R = 3/2$, $b = -1/2$, $\gamma_b = 1$, $A = 0.5$, $\omega_0 = \pi/2$.

Changes of Baseline

$$
\begin{cases}
-\beta_L(x + D_L) - \gamma_0 D_L + \beta_L \Sigma(t), \\
-\infty < x \leq -D_L + \Sigma(t), \\
R = R_1 \\
\gamma_0 x - \gamma_0 \Sigma(t), \\
-D_L + \Sigma(t) < x < D_R + \Sigma(t), \\
R = R_2 \\
-\beta_R(x - D_R) + \gamma_0 D_R + \beta_R \Sigma(t) \\
D_R + \Sigma(t) \leq x < +\infty, \\
R = R_3 .
\end{cases}
$$

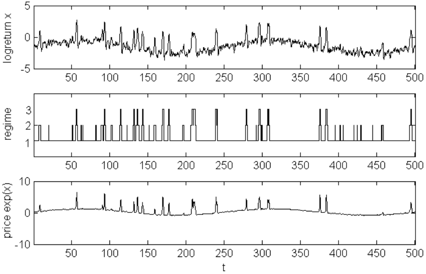


Figure 14: Floating McKean model for $f = 0$. Parameters: $\epsilon = 0.5$, $s = 0.4$, $\beta_L = 1$, $\beta_R = 1$, $\gamma_0 = 1$, $D_L = D_R = \Delta = 1$, $b = 1$, $\gamma_b = 1$, $A_s = 1$, $\omega_s = 2\pi/250$, $dt = 0.01$. a) logprice $x(t)$ dynamics, b) regime dynamics, d) price dynamics.

Third mean reversion type.

Calibration for $f = 0$

$$L = -\frac{N-1}{2}\ln(2\pi\sigma^2/\Delta t) - \frac{\Delta t}{2\sigma^2}\sum_{n=2}^{N}\left(\Omega_R(n+1,n)\right)^2;$$

$$\Omega_R(n+1,n) = \sum_{i=1}^{3}\Omega_{R_i}(n+1,n)\mathbb{1}[\hat{x}_n \in R_i];$$

$$\Omega_{R_i}(n+1,n) = \epsilon\hat{z}_{n+1} + \epsilon\hat{z}_n(1-1/\Delta t)+$$
$$A_R^1(n)\,\beta_L + A_R^2(n)\,\gamma_0 + A_R^3(n)\,\beta_R + A_R^4(n)\,\gamma_b + A_R^5(n)\,b;$$

$$C\Theta = -V$$

$$\Theta = \{\beta_L, \gamma_0, \beta_R, \gamma_b, b\}; \quad C_k^j = (\sum_{n=2}^{N} A_R^k A_R^j);$$

$$V^j = (\epsilon\hat{z}_{n+1} + \epsilon\hat{z}_n(1-1/\Delta t))\sum_{n=2}^{N} A_R^j(n)$$

Summary 1

The model set includes **complex mean reversion** for:

1. spikes and antispikes (Hopf bifurcations)
2. baseline prices daily periodicity (forcing at daily frequency)
3. baseline prices yearly seasonality (rigid displacement)
4. frequency seasonality (second lower frequency in forcing)

Summary 2

A model for prices of spot power markets is discussed, that

- takes into account fundamental market microeconomic features
- incorporates naturally an **intrinsic threshold**
- uses a **single mechanism** to model short and long term mean reversion
- exploits **critical point analysis**
- uses only **one** source of noise
- can be easily calibrated at hour scales
- consists actually of a set of models

Thank you

The most exciting phrase to hear in Science is not "Eureka", but "That's funny" !

Isaac Asimov

References

- C.Lucheroni, *SETARX models for spikes and antispikes in electricity prices*, 2010
- C.Lucheroni, *TARX models for spikes in electricity markets*, 2010
- C.Lucheroni, *Resonating Models for the Electric Power Market*, Phys. Rev. E **76**, p. 056116, 2007
- C. Lucheroni, *Stochastic Models of Resonating Markets*, J. of Econ. Int. and Coord., vol. 5, 1, p. 77 2010.