A SETARX model for spikes and antispikes in electricity markets

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Electricity Data and Mathematical Price Models

What model for such a behavior?

6 years of data: AESO SP prices (Alberta, Canada), Apr. 7 2001 - Apr. 6 2007, including electricity demand.

Figure 1: Prices and demand in the Alberta power market: 5 years from Apr-7-2001 to Apr-6-2007, time in hours; a) prices, b) demand.
Simple Model, Complex Behaviour

A model that:

- includes energy demand
- in continuous time supports **spikes without jumps**
- has a **rich phenomenology**, as that seen in data
- has some **microeconomic foundations**
- in discrete time belongs to a standard class of models (SETARX) and preserves its continuous time properties
- is analyzed in a nonstandard way
Data

Alberta (Canada) wholesale power market (now also retail), a voluntary pool (not all energy is traded here).

Data from AESO site:
1. energy prices in Canadian dollars
2. demand (load) in Megawatt-hour (MWh)

Players:
1. gencos
2. grid (Transmission System Operator, TSO): AESO
3. Independent System Operator (ISO) and Exchange: AESO
Electricity Prices: Spikes Have a Structure

Discrete i-time prices $\hat{p}_i$ available for each hour.

Hourly demand data is available.

Spikes appear only during demand crests, but not always.

Figure 2: Alberta power market: one week from Mon Jan-08-2007 to Sun Jan-14-2007, time in hours; a) system prices in C$, b) demand in MW.
Spikes and Baseline

Spikes originate from a first mean reversion mechanism.

In their baselines, prices follow smoothly the demand daily seasonality - a second mean reversion type.

Figure 3: Alberta power market: one month from Jan-1-2007 to Jan-31-2007, time in hours; a) prices, b) demand
Logprices and Antispikes

Prices $p$ show spikes, \textbf{logprices} $x = \log p$ show spikes and \textbf{antispikes}, with a mean reversion similar to the spikes mean reversion.

Logprices baseline has yearly periodicity. \textbf{Third mean reversion type.}

\textbf{Figure 4:} Alberta power market; a) one year of prices, from hour 1 of Apr-07-2006 to hour 24 of Apr-07-2007; b) logprices, same scale; c) detail of b): 3 weeks of logprices from hour 1 of Aug-14 to hour 24 of Sep-3.
Periodic Parossistic Phases

There are seasons in which demand triggers higher frequency spiking (e.g. winter).

This is a second type of seasonality: spikes don’t change in structure but become more frequent.

Fourth mean reversion type.

Figure 5: Prices and demand in the Alberta power market: 5 years from Apr-7-2001 to Apr-6-2007, time in hours; a) prices, b) demand.
Modelling Approaches

- **Top-down**: the standard financial way.
- **Bottom-up**: agents on networks.
- **Hybrid**: take into consideration microeconomics, use few degrees of freedom.
Power Markets as Tight Markets

Features that make power markets different from stock markets:

- A Rigid Periodic demand $d(t)$ drives prices $p$: $p = p(d(t))$
- Capacity constraints affect prices
- Grid constraints affect prices
- Anticipated constrained times are the best times to game the system...

Being potentially constrained, the power market sometimes becomes a tight market
The Threshold

Technical constraints introduce a **threshold** in the price formation mechanism.

- **Below** the threshold, prices react smoothly to demand variations.
- **Above** the threshold, prices can react in a non-smooth way.

![Figure 6: Threshold effects](image)
Some Stochastic Financial Models

Continuous-Time (from high frequency finance):

- **Jump-diffusion**: compound Poisson, Lévy.

Discrete-Time (more natural, since data are discrete):

- **ARX**: driven linear autoregressions.
- **Switching ARX**.
- **TARX**: *SETARX models for spikes and antispikes* Lucheroni 2010.
SETARX Models

Discrete-Time

- **AR(M)X:** $L^h x_i = x_{i-h}, \quad \phi^M(L) = 1 - \ldots - \phi_M L^M$

  \[
  \begin{cases}
  \phi^M(L) x_i = \sigma \varepsilon_i + d_{i-1} \\
  \end{cases}
  \]

- **Switching ARX:** 2 regimes $R$, 1 threshold $T$

  \[
  \begin{cases}
  \phi^I_2(L) x_i = \sigma \varepsilon_i + d_{i-1}, \quad u(i) \geq T \quad (R = I) \\
  \phi^II_2(L) x_i = \sigma \varepsilon_i + d_{i-1}, \quad u(i) < T \quad (R = II) \\
  \end{cases}
  \]

- $u(i) = U(x_i) \to \text{SETARX}$
The McKean Oscillator

Discrete or continuous-time. Two coupled nonlinear diffusions $x$ and $y$. 2 thresholds $D_L$ and $D_R$, then 3 regimes. Forcing $f$. Noise $\xi = dW/dt$.

$$\epsilon \dot{x} = g_R(x; a) - y$$
$$\dot{y} = x - \gamma b y + b + f(t) + \sigma(d) \xi(t)$$

where $g_R(x; D_L, D_R) =$

$$\begin{cases} 
-\beta_L(x + D_L) - \gamma_0 D_L, & -\infty < x \leq -D_L \\
\gamma_0 x, & -D_L < x < D_R \\
-\beta_R(x - D_R) + \gamma_0 D_R, & D_R \leq x < +\infty 
\end{cases}$$

(C. Lucheroni (Unicam))
$f = 0$: No Forcing

For forcing $f = 0$, no seasonality is included.

Logprice $x(t)$.

Price $p(t) = e^{x(t)}$.

Spikes with different heights and widths, springing from a constant baseline mean reversion level.

Figure 7: McKean model (from new form). Parameters: $\epsilon = 0.3$, $d = 0.1$, $a = 1$, $b = -0.5$, $\gamma_b = 1$. a) logprice $x(t)$ dynamics, b) price $p(t) = \exp x(t)$ dynamics.
McKean Oscillator at $f = 0$

Phase space $\{x(t), y(t)\}$.

Nullclines:
$x$ for $\dot{x} = 0$, $y$ for $\dot{y} = 0$.

Logprice $x(t)$.

Auxiliary coordinate $y(t)$.

$0 << \epsilon < 1$ (soft regime)
Dynamically-Critical Points

The McKean model undergoes a subcritical Hopf transition as $b$ changes. Spikes can be formed close to $b_2$, subthreshold, as stochastic orbits.

Figure 8: FNS model, $x$ dynamic range
Forcing $f(t) = A \sin(\omega t)$, to include **daily baseline seasonality.**

Stochastically resonating spiking - SRS (second mean reversion type).

Figure 9: McKean model. $\epsilon = 0.3, d = 0.1, b = 0, \gamma_b = 1, A = 0.5, \omega = \frac{\pi}{2}$. a)
Movie: Resonating Power Market

Stable, unstable, metastable regions.
Two Frequencies: Paroxysmatic Phases

- periodic ‘fountains’ of spikes
- demand: only 2 frequencies (more can be added), no trend
- fourth mean reversion type

\[ f(t) = u \left( v \sin\left(\frac{\omega_f}{365} t\right) + \sin(\omega_f t) \right) \]
Real and Simulated Series

Figure 10: Prices and demand in the Alberta power market: 5 years from Apr-7-2001 to Apr-6-2007, time in hours; a) prices, b) demand.

Figure 11: Simulation of 6 years of power market prices, extended model for $\epsilon = 0.15$, $\kappa = 1$, $\lambda = 1$, $b = 0$, $\gamma = 1$, $\xi = 2$, $c = -1$, $f$ with $u = 2$, $v = 0.06$, $\omega_f = 4$, $d = 0.1$, $\Delta t = 0.035$; a) price process; b) forcing: the smaller yearly $\omega_f/365$ frequency modulates the much higher daily $\omega_f$ frequency, which cannot be resolved in the picture.
Antispikes: SAS McKean

Spike-AntiSpike McKean Model (SAS): 5 regimes plus forcing ($L$ for left, $R$ for right).

\[
\begin{align*}
\epsilon \dot{x} &= g^\text{SAS}_R(x; C_L, C_R) - y \\
\dot{y} &= x - \gamma_b y + b + f(t) + \sigma(d) \xi(t)
\end{align*}
\]

where \( g^\text{SAS}_R(x; C_L, C_R) = \)

\[
\begin{cases}
-\alpha_L (x + C_L), & -\infty < x \leq -C_L \\
\beta_L (x + C_L), & -C_L < x < -D_L = -\frac{\beta_L}{\gamma_0 + \beta_L} C_L \\
-\gamma_0 x, & -D_L \leq x \leq D_R = \frac{\beta_R}{\gamma_0 + \beta_R} C_R \\
\beta_R (x - D_R), & D_R < x < C_R \\
-\alpha_R (x - D_R), & C_R \leq x < +\infty
\end{cases}
\]
SAS McKean Dynamics

Figure 12: SAS McKean model phase-space for $f = 0$. Other parameters: $\alpha_L = \alpha_R = 1$, $\beta_L = \beta_R = 1$, $\gamma_0 = 1$, $C_L = 1/2$, $C_R = 3/2$, $b = -1/2$, $\gamma_b = 1$.

Figure 13: SAS McKean model for $f \neq 0$. Parameters: $\epsilon = 0.3$, $s = 0.1$, $\alpha_L = \alpha_R = 1$, $\beta_L = \beta_R = 1$, $\gamma_0 = 1$, $C_L = 1/2$, $C_R = 3/2$, $b = -1/2$, $\gamma_b = 1$, $A = 0.5$, $\omega_0 = \pi/2$. 
Changes of Baseline

\[-\beta_L(x + D_L) - \gamma_0 D_L + \beta_L \Sigma(t), \quad -\infty < x \leq -D_L + \Sigma(t), \quad R = R_1\]

\[\gamma_0 x - \gamma_0 \Sigma(t), \quad -D_L + \Sigma(t) < x < D_R + \Sigma(t), \quad R = R_2\]

\[-\beta_R(x - D_R) + \gamma_0 D_R + \beta_R \Sigma(t) \quad \text{if} \quad D_R + \Sigma(t) \leq x < +\infty, \quad R = R_3.\]

Third mean reversion type.

Figure 14: Floating McKean model for \( f = 0 \).
Parameters: \( \epsilon = 0.5, s = 0.4, \beta_L = 1, \beta_R = 1, \gamma_0 = 1, D_L = D_R = \Delta = 1, b = 1, \gamma_b = 1, \]
\( A_s = 1, \omega_s = 2\pi/250, dt = 0.01 \). a) logprice \( x(t) \) dynamics, b) regime dynamics, d) price dynamics.
Calibration for $f = 0$

\[
L = -\frac{N-1}{2} \ln(2\pi \sigma^2/\Delta t) - \frac{\Delta t}{2\sigma^2} \sum_{n=2}^{N} (\Omega_R(n+1,n))^2;
\]

\[
\Omega_R(n+1,n) = \sum_{i=1}^{3} \Omega_{Ri}(n+1,n) \mathbb{1}[\hat{x}_n \in R_i];
\]

\[
\Omega_{Ri}(n+1,n) = \epsilon \hat{Z}_{n+1} + \epsilon \hat{Z}_n (1 - 1/\Delta t) + A_R^1(n) \beta_L + A_R^2(n) \gamma_0 + A_R^3(n) \beta_R + A_R^4(n) \gamma_b + A_R^5(n) b;
\]

\[
C \Theta = -V
\]

\[
\Theta = \{ \beta_L, \gamma_0, \beta_R, \gamma_b, b \}; \quad C_k^i = \left( \sum_{n=2}^{N} A_R^k A_R^i \right);
\]

\[
V^i = (\epsilon \hat{Z}_{n+1} + \epsilon \hat{Z}_n (1 - 1/\Delta t)) \sum_{n=2}^{N} A_R^i(n)
\]
Summary 1

The model set includes **complex mean reversion** for:

1. spikes and antispikes (Hopf bifurcations)
2. baseline prices daily periodicity (forcing at daily frequency)
3. baseline prices yearly seasonality (rigid displacement)
4. frequency seasonality (second lower frequency in forcing)
A model for prices of spot power markets is discussed, that

- takes into account fundamental market microeconomic features
- incorporates naturally an intrinsic threshold
- uses a single mechanism to model short and long term mean reversion
- exploits critical point analysis
- uses only one source of noise
- can be easily calibrated at hour scales
- consists actually of a set of models
Thank you

The most exciting phrase to hear in Science is not “Eureka”, but “That’s funny”!

Isaac Asimov
C. Lucheroni, *SETARX models for spikes and antispikes in electricity prices*, 2010

C. Lucheroni, *TARX models for spikes in electricity markets*, 2010
