Hedging with CO$_2$ allowances: the ECX market

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Motivation

1. The trading scheme for carbon dioxide (CO₂) emission allowances is one of the major steps towards reducing the environmental burden.


3. CO₂ spot and futures contracts for ECX: lack of hedging analysis.

4. Hedging is analyzed for several commodities markets.

5. What about hedging using CO₂ contracts? How it behaves?


7. Utility gains are considered in order to take into account risk-return considerations.
1. Calculate for the first time hedge ratios for the CO$_2$ allowances market.

2. Extend the data span considered by previous authors that mostly covered the Phase I period (2005-2007).

3. Use both static and dynamic hedging strategies which allows us to compare different specifications.

4. Help to identify the internal dynamics of widely traded CO$_2$ emission allowances, essential in pricing of the contracts, while the implications of the study are expected to be functional for risk managers, individual investors and hedgers dealing with the carbon allowances trading markets.
Overview

- Methodology
- Data and Summary Statistics
- Empirical Results
- Conclusions
Methodology: Optimal hedge ratio

- \( h_t = \frac{\text{Cov}(s_{t+1}, f_{t+1})}{\text{Var}(f_{t+1})} \)

- Effectiveness of the hedging strategy measured by: the degree of hedging effectiveness (EH) - variance reduction - and utility maximization - from a utility gains standpoint

- Variance Reduction:
  \[
  EH = \frac{\text{Var}(\Delta S_t) - \text{Var}(\Delta h_t)}{\text{Var}(\Delta S_t)} = 1 - \left[ \frac{\text{Variance}_{\text{hedged Portfolio}}}{\text{Variance}_{\text{unhedged Portfolio}}} \right]
  \]

- Expected Utility gains:
  \[
  \max_h U \left[ E(r_{p,t}), \sigma_{p,t}; \eta (r_{p,t}) \right] = \max_h E(r_{p,t}) - 0.5 \eta (r_{p,t}) \sigma_{p,t}^2 ;
  \]
  \[
  E[U(r_{p,t} \mid \psi_{t-1})] = E[r_{p,t} \mid \psi_{t-1}] - \lambda \text{Var}[r_{p,t} \mid \psi_{t-1}]
  \]
When a hedge where the futures position have the same size but the opposite sign than the position held in the spot market is considered, we have what is called a naïve hedge ratio ($h_t = 1, \forall t$).

Empirically, the one period hedge ratio is estimated by the slope from the following ordinary least squared (OLS) regression equation:

$$s_{t+1} = \alpha + h^* f_{t+1} + \varepsilon_t$$

where $\varepsilon_t$ is the error term from OLS estimation, $s_{t+1}$ and $f_{t+1}$ are the changes in the spot and futures prices, respectively, between time $t$ and $t + 1$, and $h^*$ is the minimum hedge ratio.
Methodology: Time varying hedge ratio

- Multivariate GARCH models
- 1) BEKK

\[
\begin{bmatrix}
\sigma_{ss,t}^2 & \sigma_{sf,t}^2 \\
\sigma_{fs,t}^2 & \sigma_{ff,t}^2
\end{bmatrix}
= \begin{bmatrix}
c_{ss} & c_{sf} \\
0 & c_{ff}
\end{bmatrix}'
\begin{bmatrix}
c_{ss} & c_{sf} \\
0 & c_{ff}
\end{bmatrix} +
\begin{bmatrix}
a_{ss} & a_{sf} \\
0 & c_{ff}
\end{bmatrix}'
\begin{bmatrix}
\varepsilon_{s,t-1}^2 & \varepsilon_{s,t-1}\varepsilon_{f,t-1} \\
\varepsilon_{f,t-1}\varepsilon_{s,t-1} & \varepsilon_{f,t-1}^2
\end{bmatrix}
\begin{bmatrix}
a_{ss} & a_{sf} \\
a_{fs} & a_{ff}
\end{bmatrix} +
\begin{bmatrix}
b_{ss} & b_{sf} \\
b_{fs} & b_{ff}
\end{bmatrix}'
\begin{bmatrix}
\sigma_{ss,t-1}^2 & \sigma_{sf,t-1}^2 \\
\sigma_{fs,t-1}^2 & \sigma_{ff,t-1}^2
\end{bmatrix}
\begin{bmatrix}
b_{ss} & b_{sf} \\
b_{fs} & b_{ff}
\end{bmatrix}
\]

- \(\varepsilon_t | \phi_{t-1} \sim BN(0, H_t)\) with \(\varepsilon_t = [\varepsilon_{st}]\) and \(H_t = \begin{bmatrix}
\sigma_{ss,t}^2 & \sigma_{sf,t}^2 \\
\sigma_{fs,t}^2 & \sigma_{ff,t}^2
\end{bmatrix}\)
- \(\varepsilon_t | \phi_{t-1} \sim t(0, H_t, \nu)\); where \(\nu\) is the degrees of freedom parameter of a conditional bivariate t-student distribution.
Methodology: Time varying hedge ratio

2) Diagonal BEKK

\[
\begin{align*}
\sigma_{ss,t}^2 &= c_{ss} + a_{ss} \varepsilon_{s,t-1}^2 + b_{ss} \sigma_{ss,t-1}^2 \\
\sigma_{sf,t}^2 &= c_{sf} + a_{sf} \varepsilon_{s,t-1} \varepsilon_{f,t-1} + b_{sf} \sigma_{sf,t-1}^2 \\
\sigma_{ff,t}^2 &= c_{ff} + a_{ff} \varepsilon_{f,t-1}^2 + b_{ff} \sigma_{ff,t-1}^2
\end{align*}
\]

\[\varepsilon_t | \phi_{t-1} \sim BN(0, H_t)\]

\[\varepsilon_t | \phi_{t-1} \sim t(0, H_t, \nu) ; \text{ where } \nu \text{ is the degrees of freedom parameter of a conditional bivariate t-student distribution.}\]
3) Bollerslev’s CCC

In the Bollerslev’s (1990) model, covariances between $i$ and $j$ are allowed to vary only through the product of standard deviations with a correlation coefficient which is constant through time (constant correlation model or CCC). The dynamic of standard deviations is governed by the GARCH(1,1) variances’ dynamic or any univariate GARCH model. Keeping the covariance matrix $\Sigma_t = [\sigma_{ij,t}]$, we have

$$\sigma_{ii,t} = \omega_{ii} + \beta_{ii}\sigma_{ii,t-1} + \alpha_{ii}\eta_{i,t}^2$$

and

$$\sigma_{ij,t} = \rho_{ij}\sqrt{\sigma_{ii,t}\sigma_{jj,t}}$$
Methodology: Time varying hedge ratio

- 4) DCC model of Engle
- Correlations between returns may not be constant trough time
- The general form of the dynamic conditional correlation (DCC) model introduced by Engle (2002) is defined by

\[
\Sigma_t = D_t R_t D_t
\]

where

\[
R_t = Q_{t^*}^{-1} Q_t Q_{t^*}^{-1}
\]

and

\[
Q_t = \left(1 - \sum_{p=1}^{P} \alpha_p - \sum_{q=1}^{Q} \beta_q\right) \bar{Q} + \sum_{p=1}^{P} \alpha_p \left(\eta_t \eta_t'_{-p}\right) + \sum_{q=1}^{Q} \beta_q Q_{t-q}
\]

where \(D_t\) is a \(n \times n\) diagonal matrix of time varying standard deviations, \(D_t\) is a \(n \times n\) time varying correlation matrix, \(\bar{Q}\) is the unconditional covariance matrix using standardized residuals from the univariate estimates, and \(Q_{t^*}\) is a diagonal matrix of the square root of the diagonal elements of \(Q_t\). Time varying correlation matrix defined as \(R_t = \left[\rho_{ij,t}\right]\) with \(\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}\). DCC differs from CCC mainly in that it allows the correlation matrix to be changed over time.
June 24, 2005 to October 9, 2009 for futures ECX and Bluenext spot

Daily returns

One future contract to hedge the spot price variation: Future Contract Maturing in December 2009 (FutDec09) to cover the all period

<table>
<thead>
<tr>
<th>ECX Series</th>
<th>mean</th>
<th>variance</th>
<th>skewness</th>
<th>kurtosis</th>
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<tbody>
<tr>
<td>Spot CO2</td>
<td>0.044</td>
<td>4.045</td>
<td>0.671</td>
<td>45.072</td>
</tr>
<tr>
<td>FutDec05</td>
<td>0.132</td>
<td>2.831</td>
<td>-1.811</td>
<td>12.494</td>
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<tr>
<td>FutDec06</td>
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<td>4.864</td>
<td>-0.292</td>
<td>44.226</td>
</tr>
<tr>
<td>FutDec07</td>
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<td>7.423</td>
<td>-0.821</td>
<td>18.152</td>
</tr>
<tr>
<td>FutDec08</td>
<td>0.110</td>
<td>2.944</td>
<td>-1.558</td>
<td>10.310</td>
</tr>
<tr>
<td>FutDec09</td>
<td>-0.009</td>
<td>3.353</td>
<td>-1.718</td>
<td>20.844</td>
</tr>
<tr>
<td>FutDec10</td>
<td>-0.002</td>
<td>3.322</td>
<td>-1.660</td>
<td>20.104</td>
</tr>
<tr>
<td>FutDec11</td>
<td>0.005</td>
<td>3.332</td>
<td>-1.600</td>
<td>18.576</td>
</tr>
<tr>
<td>FutDec12</td>
<td>0.011</td>
<td>3.404</td>
<td>-1.564</td>
<td>16.965</td>
</tr>
</tbody>
</table>
Estimated spot and futures volatility for each multivariate model

Figure 1: Conditional volatility for the spot CO₂ allowances in the ECX market

Figure 2: Conditional volatility for the Futures December 2009 CO₂ allowances in the ECX market
Estimated Covariance and Conditional Correlation

The diagrams illustrate the estimated covariance and conditional correlations for various models:

- Covariance Spot Futures DBEKK
- Covariance Spot Futures T-DBEKK
- Conditional Correlations implied by the DBEKK model
- Conditional Correlations implied by the T-DBEKK model
- Covariance Spot Futures CCC
- Covariance Spot Futures DCC
- Conditional Correlations implied by the CCC model
- Conditional Correlations implied by the DCC model
- Covariance Spot Futures BBK
- Covariance Spot Futures T-BBK
- Conditional Correlations implied by the BBK model
- Conditional Correlations implied by the T-BBK model
Conditional Hedge Ratio

Hedge Ratios: DBEKK, OLS and NAIVE

Hedge Ratios: T-DBEKK, OLS and NAIVE

Hedge Ratios: CCC, OLS and NAIVE

Hedge Ratios: DCC, OLS and NAIVE

Hedge Ratios: BEKK, OLS and NAIVE

Hedge Ratios: T-BEKK, OLS and NAIVE
## Variance Reduction measure: Hedging effectiveness

<table>
<thead>
<tr>
<th>Hedging</th>
<th>In the Sample</th>
<th>Out of Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot variance (no hedging) ($h = 0$)</td>
<td>13.61</td>
<td>4.53</td>
</tr>
<tr>
<td>Naive ($h = 1$)</td>
<td>74.32</td>
<td>98.62</td>
</tr>
<tr>
<td>OLS ($h = \frac{\sigma_{FS}}{\sigma_F^2}$)</td>
<td>74.04</td>
<td>99.04</td>
</tr>
<tr>
<td>Diag-BEKK ($h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}$)</td>
<td>85.53</td>
<td>99.04</td>
</tr>
<tr>
<td>T-Diag-BEKK ($h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}$)</td>
<td>86.63 +</td>
<td>99.06</td>
</tr>
<tr>
<td>CCC ($h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}$)</td>
<td>83.72</td>
<td>97.72</td>
</tr>
<tr>
<td>DCC ($h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}$)</td>
<td>85.64</td>
<td>99.01</td>
</tr>
<tr>
<td>BEKK ($h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}$)</td>
<td>85.34</td>
<td>99.00</td>
</tr>
<tr>
<td>T-BEKK ($h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}$)</td>
<td>85.03</td>
<td>99.07 +</td>
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Utility Gains for alternative risk aversion levels and different models

$\eta = 1$; Risk Averse

<table>
<thead>
<tr>
<th></th>
<th>In Sample</th>
<th>Out Sample</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>Return</td>
</tr>
<tr>
<td>Unhedge</td>
<td>13.61</td>
<td>-0.01</td>
</tr>
<tr>
<td>Naive</td>
<td>3.50</td>
<td>0.00</td>
</tr>
<tr>
<td>OLS</td>
<td>3.53</td>
<td>0.00</td>
</tr>
<tr>
<td>Diag-BEKK</td>
<td>1.97</td>
<td>0.01</td>
</tr>
<tr>
<td>T-Diag-BEKK</td>
<td>1.82</td>
<td>0.00</td>
</tr>
<tr>
<td>CCC</td>
<td>2.22</td>
<td>-0.01</td>
</tr>
<tr>
<td>DCC</td>
<td>1.96</td>
<td>-0.00</td>
</tr>
<tr>
<td>BEKK</td>
<td>2.00</td>
<td>0.01</td>
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<tr>
<td>T-BEKK</td>
<td>2.04</td>
<td>0.00</td>
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Utility Gains for alternative risk aversion levels and different models

\[ \eta = 2; \text{ Risk Neutral} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>In Sample</th>
<th></th>
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<th>Out Sample</th>
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<tr>
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<td>Variance</td>
<td>Return</td>
<td>Exp Util</td>
<td>Gain</td>
<td>Variance</td>
<td>Return</td>
<td>Exp Util</td>
<td>Gain</td>
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<td>-0.01</td>
<td>-27.24</td>
<td>-</td>
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<td>-0.13</td>
<td>-9.20</td>
<td>-</td>
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<tr>
<td>Naive</td>
<td>3.50</td>
<td>0.00</td>
<td>-6.99</td>
<td>20.25</td>
<td>0.06</td>
<td>0.01</td>
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<td>OLS</td>
<td>3.53</td>
<td>0.00</td>
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<td>20.17</td>
<td>0.04</td>
<td>0.00</td>
<td>-0.09</td>
<td>9.12</td>
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<tr>
<td>Diag-BEKK</td>
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<td>0.01</td>
<td>-3.93</td>
<td>23.31</td>
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<td>-0.00</td>
<td>-0.09</td>
<td>9.12</td>
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<td>T-Diag-BEKK</td>
<td>1.82</td>
<td>0.00</td>
<td>-3.64</td>
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<td>CCC</td>
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<td>9.11</td>
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<td>9.11</td>
</tr>
<tr>
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<td>2.04</td>
<td>0.00</td>
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<td>23.17</td>
<td>0.04</td>
<td>-0.00</td>
<td>-0.08</td>
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Utility Gains for alternative risk aversion levels and different models

$\eta = 4$; Risk Lover

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<th>In Sample</th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>Return</td>
<td>Exp Util$^a$</td>
<td>Gain$^b$</td>
<td>Variance</td>
<td>Return</td>
<td>Exp Util$^a$</td>
<td>Gain$^b$</td>
</tr>
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<td>-</td>
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<td>-0.00</td>
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</tr>
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$^a$ Expected Utility: $E[U(r_{p,t} | \psi_{t-1})] = E[r_{p,t} | \psi_{t-1}] - \lambda Var(r_{p,t} | \psi_{t-1})$

$^b$ Utility Gain of Hedging Models over Unhedged Position
Conclusions

- Empirically estimate optimal hedge ratios in the EU ETS CO$_2$ allowances markets.
- Results indicate that taking into account transaction costs of rebalancing daily the hedged portfolio in dynamic MGARCH models will imply that their better statistical performance in the EU ETS market becomes seriously questioned.
- Taking into account the data leptokurtosis through the error distribution assumption indicates superior gains, measured by variance reduction, obtained from the multivariate model BEKK (Diagonal), for both in sample and out of sample results (BEKK). Moreover, utility gains increase with the investor’s preference over risk.
Conclusions

- Overall, there seems to be some gains from including heteroscedasticity and time-varying variances in hedge ratios calculations, although it is not completely guaranteed that improving statistical price modelling provides better performance.

- Correlation results are important for EU ETS allowances price risk management, as they show that December Futures will provide a good risk reduction for hedgers participating in EU ETS markets.

- As the market evolves and more data becomes available, it is expected more useful results obtained through dynamic models or even others given that empirical research is evolving constantly.
And finally...

Thanks for your attention!
Questions?