Hedging with CO_2 allowances: the ECX market

Mara Madaleno and Carlos Pinho

October 6-8, 2010

Motivation

1. The trading scheme for carbon dioxide (CO_2) emission allowances is one of the major steps towards reducing the environmental burden
2. Allowances: new asset with derivatives
3. CO_2 spot and futures contracts for ECX: lack of hedging analysis
4. Hedging is analyzed for several commodities markets
5. What about hedging using CO_2 contracts? How it behaves?
6. Daily hedging with futures: MV hedge ratios estimated conditionally using multivariate GARCH models and unconditionally by OLS and naïve strategies
7. Utility gains are considered in order to take into account risk-return considerations

Contribution

1. Calculate for the first time hedge ratios for the CO_2 allowances market.
2. Extend the data span considered by previous authors that mostly covered the Phase I period (2005-2007).
3. Use both static and dynamic hedging strategies which allows us to compare different specifications.
4. Help to identify the internal dynamics of widely traded CO_2 emission allowances, essential in pricing of the contracts, while the implications of the study are expected to be functional for risk managers, individual investors and hedgers dealing with the carbon allowances trading markets.

Overview

- Methodology
- Data and Summary Statistics
- Empirical Results
- Conclusions

Methodology: Optimal hedge ratio

- $h_t = \frac{Cov_t(s_{t+1}, f_{t+1})}{Var_t(f_{t+1})}$
- Effectiveness of the hedging strategy measured by: the degree of hedging effectiveness (EH) - variance reduction - and utility maximization - from a utility gains standpoint
- Variance Reduction:
  $EH = \frac{Var(\Delta S_t) - Var(\Delta h_t)}{Var(\Delta S_t)} = 1 - \left[\frac{Variance_{hedgedPortfolio}}{Variance_{unhedgedPortfolio}}\right]$
- Expected Utility gains:
  $\underset{h}{Max} U\left[E(r_{p,t}), \sigma_{p,t}; \eta\left(r_{p,t}\right)\right] = \underset{h}{Max} E\left(r_{p,t}\right) - 0,5\eta\left(r_{p,t}\right)\sigma^2_{p,t}$;
  $E[U(r_{p,t})|\psi_{t-1}] = E[r_{p,t}|\psi_{t-1}] - \lambda Var[r_{p,t}|\psi_{t-1}]$

Methodology: Static hedge

- When a hedge where the futures position have the same size but the opposite sign than the position held in the spot market is considered, we have what is called a naïve hedge ratio ($h_t = 1, \forall t$).
- Empirically, the one period hedge ratio is estimated by the slope from the following ordinary least squared (OLS) regression equation:

$$s_{t+1} = \alpha + h^* f_{t+1} + \varepsilon_t$$

where ε_t is the error term from OLS estimation, s_{t+1} and f_{t+1} are the changes in the spot and futures prices, respectively, between time t and $t+1$, and h^* is the minimum hedge ratio.

Methodology: Time varying hedge ratio

- Multivariate GARCH models
- 1) BEKK

$$\begin{bmatrix} \sigma^2_{ss,t} & \sigma^2_{sf,t} \\ \sigma^2_{fs,t} & \sigma^2_{ff,t} \end{bmatrix} = \begin{bmatrix} c_{ss} & c_{sf} \\ 0 & c_{ff} \end{bmatrix}' \begin{bmatrix} c_{ss} & c_{sf} \\ 0 & c_{ff} \end{bmatrix} + \begin{bmatrix} a_{ss} & a_{sf} \\ a_{fs} & a_{ff} \end{bmatrix}' \begin{bmatrix} \varepsilon^2_{s,t-1} & \varepsilon_{s,t-1}\varepsilon_{f,t-1} \\ \varepsilon_{f,t-1}\varepsilon_{s,t-1} & \varepsilon^2_{f,t-1} \end{bmatrix} \begin{bmatrix} a_{ss} & a_{sf} \\ a_{fs} & a_{ff} \end{bmatrix} + + \begin{bmatrix} b_{ss} & b_{sf} \\ b_{fs} & b_{ff} \end{bmatrix}' \begin{bmatrix} \sigma^2_{ss,t-1} & \sigma^2_{sf,t-1} \\ \sigma^2_{fs,t-1} & \sigma^2_{ff,t-1} \end{bmatrix} \begin{bmatrix} b_{ss} & b_{sf} \\ b_{fs} & b_{ff} \end{bmatrix}$$

- $\varepsilon_t | \phi_{t-1} \sim BN(0, H_t)$ with $\varepsilon_t = \begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix}$ and $H_t = \begin{bmatrix} \sigma^2_{ss,t} & \sigma^2_{sf,t} \\ \sigma^2_{fs,t} & \sigma^2_{ff,t} \end{bmatrix}$
- $\varepsilon_t | \phi_{t-1} \sim t(0, H_t, v)$; where v is the degrees of freedom parameter of a conditional bivariate t-student distribution.

Methodology: Time varying hedge ratio

- 2) Diagonal BEKK

$$
\begin{aligned}
\sigma^2_{ss,t} &= c_{ss} + a_{ss}\varepsilon^2_{s,t-1} + b_{ss}\sigma^2_{ss,t-1} \\
\sigma^2_{sf,t} &= c_{sf} + a_{sf}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + b_{sf}\sigma^2_{sf,t-1} \\
\sigma^2_{ff,t} &= c_{ff} + a_{ff}\varepsilon^2_{f,t-1} + b_{ff}\sigma^2_{ff,t-1}
\end{aligned}
$$

- $\varepsilon_t|\phi_{t-1} \sim BN(0, H_t)$
- $\varepsilon_t|\phi_{t-1} \sim t(0, H_t, v)$; where v is the degrees of freedom parameter of a conditional bivariate t-student distribution.

- 3) Bollerslev's CCC
- In the Bollerslev's (1990) model, covariances between i and j are allowed to vary only through the product of standard deviations with a correlation coefficient which is constant through time (constant correlation model or CCC). The dynamic of standard deviations is governed by the GARCH(1,1) variances' dynamic or any univariate GARCH model. Keeping the covariance matrix $\Sigma_t = [\sigma_{ij,t}]$, we have

$$\sigma_{ii,t} = \omega_{ii} + \beta_{ii}\sigma_{ii,t-1} + \alpha_{ii}\eta^2_{i,t}$$

and

$$\sigma_{ij,t} = \rho_{ij}\sqrt{\sigma_{ii,t}\sigma_{jj,t}}$$

- 4) DCC model of Engle
- Correlations between returns may not be constant trough time
- The general form of the dynamic conditional correlation (DCC) model introduced by Engle (2002) is defined by

$$
\begin{aligned}
\Sigma_t &= D_t R_t D_t \\
\text{where} \quad R_t &= Q_t^{*-1} Q_t Q_t^{*-1} \\
\text{and} \quad Q_t &= \left(1 - \sum_{p=1}^{P} \alpha_p - \sum_{q=1}^{Q} \beta_q\right) \overline{Q} + \sum_{p=1}^{P} \alpha_p \left(\eta_{t-p} \eta'_{t-p'}\right) + \sum_{q=1}^{Q} \beta_q Q_{t-q}
\end{aligned}
$$

where D_t is a $n \times n$ diagonal matrix of time varying standard deviations, D_t is a $n \times n$ time varying correlation matrix, $\overline{Q}$ is the unconditional covariance matrix using standardized residuals from the univariate estimates, and Q_t^* is a diagonal matrix of the square root of the diagonal elements of Q_t. Time varying correlation matrix defined as $R_t = \left[\rho_{ij,t}\right]$ with $\left[\rho_{ij,t}\right] = \frac{q_{ij,t}}{\sqrt{q_{ii} q_{jj}}}$. DCC differs from CCC mainly in that it allows the correlation matrix to be changed over time.

Data and Descriptive Statistics

1. June 24, 2005 to October 9, 2009 for futures ECX and Bluenext spot
2. Daily returns
3. One future contract to hedge the spot price variation: Future Contract Maturing in December 2009 (FutDec09) to cover the all period

Table 1: Descriptive Statistics

ECX Series	mean	variance	skewness	kurtosis
Spot CO2	0.044	4.045	0.671	45.072
FutDec05	0.132	2.831	-1.811	12.494
FutDec06	-0.223	4.864	-0.292	44.226
FutDec07	-0.918	7.423	-0.821	18.152
FutDec08	0.110	2.944	-1.558	10.310
FutDec09	-0.009	3.353	-1.718	20.844
FutDec10	-0.002	3.322	-1.660	20.104
FutDec11	0.005	3.335	-1.600	18.576
FutDec12	0.011	3.404	-1.564	16.965

Estimated spot and futures volatility for each multivariate model

Figure 1: Conditional volatility for the spot CO_2 allowances in the ECX market

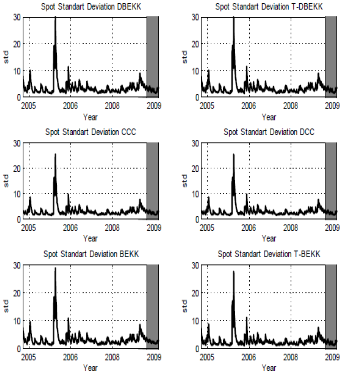
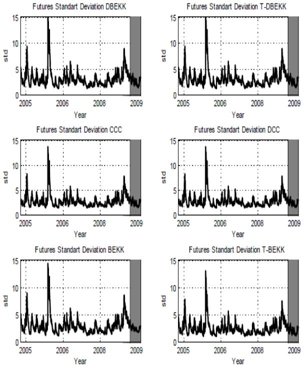
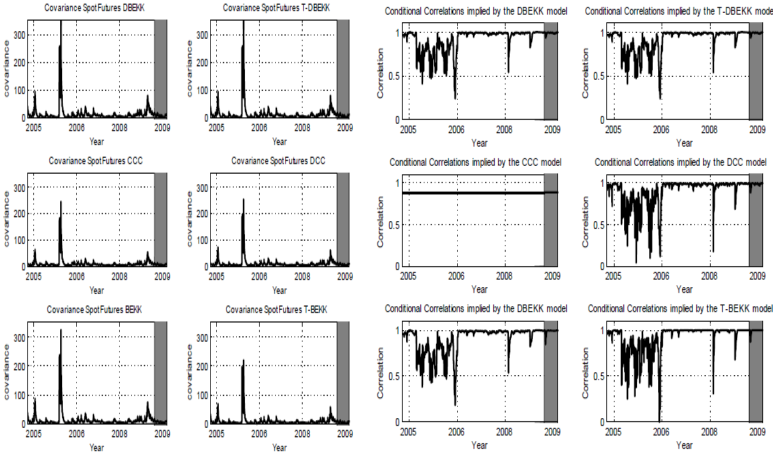


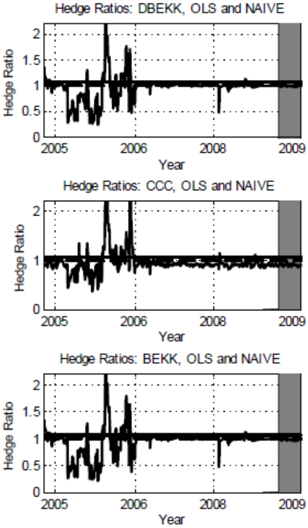
Figure 2: Conditional volatility for the Futures December 2009 CO_2 allowances in the ECX market



Estimated Covariance and Conditional Correlation



Conditional Hedge Ratio



Variance Reduction measure: Hedging effectiveness

	In the Sample	Out of Sample
Spot variance (no hedging) $(h=0)$	13.61	4.53
Hedging	Risk reduction	Risk reduction
Naive $(h=1)$	74.32	98.62
OLS $\left(h=\frac{\sigma_{FS}}{\sigma_F^2}\right)$	74.04	99.04
Diag-BEKK $\left(h_t=\frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$	85.53	99.04
T-Diag-BEKK $\left(h_t=\frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$	86.63^{+}	99.06
CCC $\left(h_t=\frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$	83.72	97.72
DCC $\left(h_t=\frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$	85.64	99.01
BEKK $\left(h_t=\frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$	85.34	99.00
T-BEKK $\left(h_t=\frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$	85.03	99.07^{+}

Utility Gains for alternative risk aversion levels and different models

$\eta = 1$; Risk Averse

	In Sample				Out Sample			
	Variance	Return	Exp Util[a]	Gain[b]	Variance	Return	Exp Util[a]	Gain[b]
$\eta = 1$								
Unhedge	13.61	−0.01	−13.63	–	4.53	−0.13	−4.67	–
Naive	3.50	0.00	−3.50	10.13	0.06	0.01	0.00	4.67
OLS	3.53	0.00	−3.53	10.09	0.04	0.00	−0.04	4.62
Diag-BEKK	1.97	0.01	−1.96	11.66	0.04	−0.00	−0.04	4.62
T-Diag-BEKK	1.82	0.00	−1.82	11.81	0.04	−0.00	−0.04	4.63
CCC	2.22	−0.01	−2.23	11.40	0.10	−0.02	−0.12	4.55
DCC	1.96	−0.00	−1.96	11.67	0.04	−0.00	−0.05	4.62
BEKK	2.00	0.01	−1.99	11.64	0.05	−0.00	−0.05	4.62
T-BEKK	2.04	0.00	−2.03	11.59	0.04	−0.00	−0.04	4.63

Utility Gains for alternative risk aversion levels and different models

$\eta = 2$; Risk Neutral

	In Sample				Out Sample			
	Variance	Return	Exp Util[a]	Gain[b]	Variance	Return	Exp Util[a]	Gain[b]
$\eta = 2$								
Unhedge	13.61	−0.01	−27.24	–	4.53	−0.13	−9.20	–
Naive	3.50	0.00	−6.99	20.25	0.06	0.01	−0.01	9.19
OLS	3.53	0.00	−7.07	20.17	0.04	0.00	−0.09	9.12
Diag-BEKK	1.97	0.01	−3.93	23.31	0.04	−0.00	−0.09	9.12
T-Diag-BEKK	1.82	0.00	−3.64	23.60	0.04	−0.00	−0.09	9.12
CCC	2.22	−0.01	−4.44	22.80	0.10	−0.02	−0.22	8.98
DCC	1.96	−0.00	−3.91	23.33	0.04	−0.00	−0.09	9.11
BEKK	2.00	0.01	−3.98	23.26	0.05	−0.00	−0.09	9.11
T-BEKK	2.04	0.00	−4.07	23.17	0.04	−0.00	−0.08	9.12

Utility Gains for alternative risk aversion levels and different models

$\eta = 4$; Risk Lover

	In Sample				Out Sample			
	Variance	Return	Exp Util[a]	Gain[b]	Variance	Return	Exp Util[a]	Gain[b]
$\eta = 4$								
Unhedge	13.61	−0.01	−54.47	–	4.53	−0.13	−18.27	–
Naive	3.50	0.00	−13.98	40.49	0.06	0.01	−0.03	18.25
OLS	3.53	0.00	−14.14	40.34	0.04	0.00	−0.17	18.10
Diag-BEKK	1.97	0.01	−7.87	46.60	0.04	−0.00	−0.17	18.10
T-Diag-BEKK	1.82	0.00	−7.28	47.19	0.04	−0.00	−0.17	18.10
CCC	2.22	−0.01	−8.88	45.59	0.10	−0.02	−0.43	17.84
DCC	1.96	−0.00	−7.83	46.64	0.04	−0.00	−0.18	18.09
BEKK	2.00	0.01	−7.98	46.49	0.05	−0.00	−0.18	18.09
T-BEKK	2.04	0.00	−8.15	46.32	0.04	−0.00	−0.17	18.10

[a] Expected Utility: $E\left[U\left(r_{p,t}\right)|\psi_{t-1}\right] = E\left[r_{p,t}|\psi_{t-1}\right] - \lambda\, Var\left(r_{p,t}|\psi_{t-1}\right)$

[b] Utility Gain of Hedging Models over Unhedged Position

Conclusions

- Empirically estimate optimal hedge ratios in the EU ETS CO_2 allowances markets
- Results indicate that taking into account transaction costs of rebalancing daily the hedged portfolio in dynamic MGARCH models will imply that their better statistical performance in the EU ETS market becomes seriously questioned.
- Taking into account the data leptokurtosis through the error distribution assumption indicates superior gains, measured by variance reduction, obtained from the multivariate model BEKK (Diagonal), for both in sample and out of sample results (BEKK). Moreover, utility gains increase with the investor's preference over risk.

Conclusions

- Overall, there seems to be some gains from including heteroscedasticity and time-varying variances in hedge ratios calculations, although it is not completely guaranteed that improving statistical price modelling provides better performance.
- Correlation results are important for EU ETS allowances price risk management, as they show that December Futures will provide a good risk reduction for hedgers participating in EU ETS markets.
- As the market evolves and more data becomes available, it is expected more useful results obtained through dynamic models or even others given that empirical research is evolving constantly.

And finally...

Thanks for your attention!
Questions?