On Clearing Coupled Day-Ahead Electricity Markets

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joint work with
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Day-Ahead Electricity Auctions

Day-Ahead Auction

1. collect demand and supply orders for the following day
2. start auction
   - maximize economic surplus
   - accept/reject orders
   - find clearing price
3. return execution schedule for the following day

Infrastructure

- Market Areas
- Interconnectors

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Basic elements of energy markets

Bids / Orders are characterized by

- quantity (demand/supply, area, hour)
- price limit
- allow partial execution (yes/no)

Order Types

- Hourly Bids
- Block Bids
- Flexible Bids

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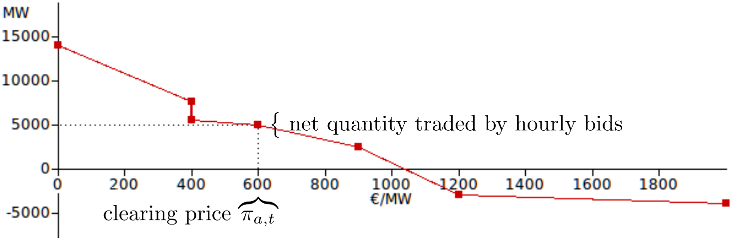
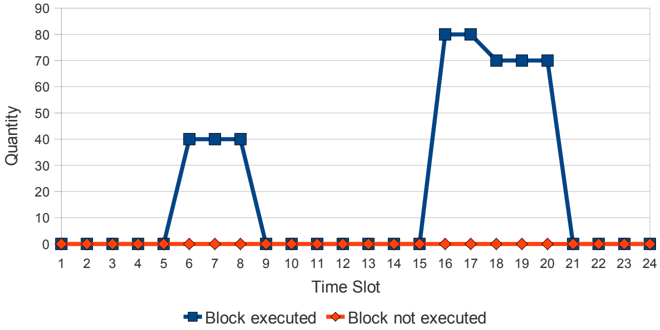
Hourly Bids



Aggregation of hourly bid curves provides

- one hourly net curve per area and hour
- bid curve trading quantities depending on the price
- downward sloping curves

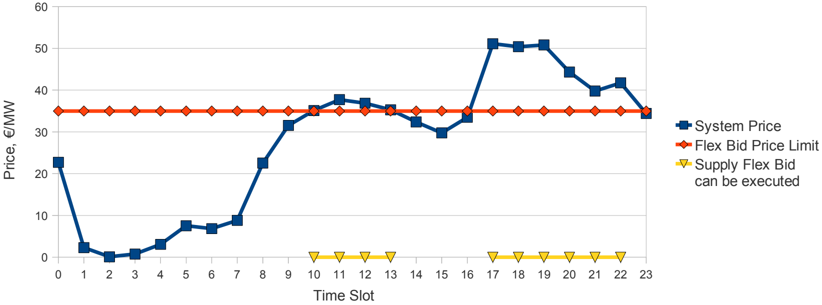
Block Bids



A block bid allows for

- trading electricity in several hours
- using only one price limit for all hours:
  (net gain) $< 0 \quad \Rightarrow \quad$ don't execute

Flexible Bids



Flexible bids allow for

- trading electricity without defining the delivery hour
- using a limit price to avoid a loss
  (gain in hour t) $< 0 \quad \Rightarrow \quad$ don't execute in hour t

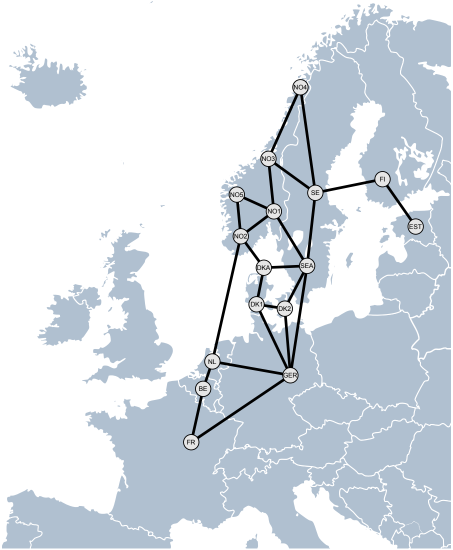
Interconnectors

Function of Interconnectors

- balance net demand of adjacent areas
- harmonize prices

Transmission Constraints

- Available Transmission Capacity (ATC): *limits the flow*
- Ramp Rate: *limits the change of flow*



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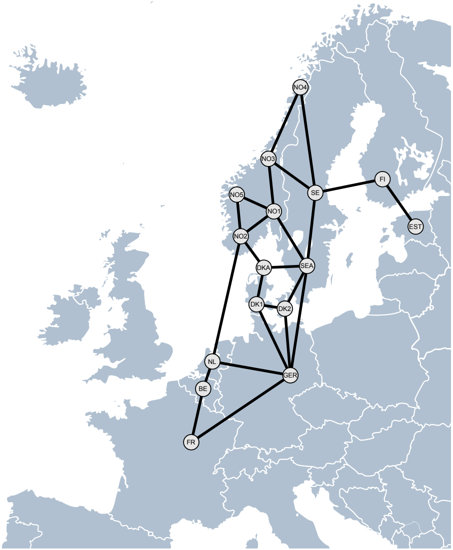

Maximization of the Economic Surplus,
subject to Transmission Constraints: Overview

$$\max \sum_{h\in H} \Delta q_h \int_0^{\delta_h} \mathbf{p}_h(u)\,\mathrm{d}u + \sum_{b\in B, t\in T} p_b q_{b,t}\beta_b + K \qquad \text{(MIQP)}$$

$$\text{s.t.}\ \forall a \in A, t \in T: \quad \sum_{h\in H_{a,t}} \Delta q_h \delta_h + \sum_{b\in B_a} q_{b,t}\beta_b = \sum_{c\in C_a^-} \tau_{c,t} - \sum_{c\in C_a^+} \tau_{c,t}$$

$$\forall c \in C, t \in T: \quad \underline{\tau}_{c,t} \le \tau_{c,t} \le \overline{\tau}_{c,t}$$

$$\forall c \in C, t \in T: \quad -\widetilde{\tau}_c \le \tau_{c,t} - \tau_{c,t-1} \le \widetilde{\tau}_c$$

$$\forall c \in C, t \in T: \quad \tau_{c,t} \in \mathbb{R}$$

$$\forall h \in H: \quad \delta_h \in [0,1]$$

$$\forall b \in B: \quad \beta_b \in \{0,1\}$$

Mixed Integer Quadratic Programm

Objective: Economic Surplus

$$\max \sum_{h \in H} \Delta q_h \int_0^{\delta_h} \mathbf{p}_h(u)\,\mathrm{d}u + \sum_{b \in B, t \in T} p_b q_{b,t} \beta_b + K$$

Notation:

$\beta_b \in \{0,1\}$	execution state of block bid $b \in B$
$p_b,\ q_{b,t}$	price limit; net demand of block $b \in B$ in hour $t \in T$
$\delta_h \in [0,1]$	execution state of hourly net curve segment $h \in H$
$\mathbf{p}_h(u)$	parametrization of the price of curve segment $h \in H$

Mixed Integer Quadratic Programm

Constraints:

1. net demand = net import:

$$\forall a \in A, t \in T \quad \sum_{h \in H_{a,t}} \Delta q_h \delta_h + \sum_{b \in B_a} q_{b,t} \beta_b = \sum_{c \in C_a^-} \tau_{c,t} - \sum_{c \in C_a^+} \tau_{c,t}$$

2. ATC: $\forall c \in C, t \in T \quad \underline{\tau}_{c,t} \leq \tau_{c,t} \leq \overline{\tau}_{c,t}$

3. ramp rate: $\forall c \in C, t \in T \quad -\tilde{\tau}_c \leq \tau_{c,t} - \tau_{c,t-1} \leq \tilde{\tau}_c$

Notation:

$\tau_{c,t} \in \mathbb{R}$	transmission on connector $c \in C$, hour $t \in T$
$h \in H_{a,t}$	net curve segments in area $a \in A$, hour $t \in T$
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$c \in C_a^+$	connectors leaving area a

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Optimality Conditions

Optimal solution to (MIQP) can be computed with MIQP solvers (e.g. IBM CPLEX 12.1)

Analyze optimal solution:

- fix the combinatorial bid selection
- apply Karush-Kuhn-Tucker-Condition for QPs (cf. [1])
- obtain dual variables, also called shadow prices

Shadow Prices π to (MIQP) satisfy (cf. [3])

Filling Condition

Flow Price Condition

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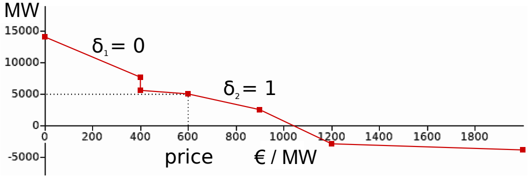
Filling Condition



Filling Condition

Price change is linear in the partial execution of the active segment:

Filling condition is satisfied, if for all segments $h \in H_{a,t}$ in area a, hour t

$$\pi_{a,t} \begin{cases} \leq \mathbf{p}_h(\delta_h), & \text{if } \delta_h = 1 \\ = \mathbf{p}_h(\delta_h), & \text{if } \delta_h \in (0,1) \\ \geq \mathbf{p}_h(\delta_h), & \text{if } \delta_h = 0 \end{cases} . \tag{1}$$

Flow Price Condition

Simplified Flow Price Condition

For adjacent areas it holds:

Prices deviate $\Rightarrow$ a transmission constraint is active.

No active transmission constraint $\Rightarrow$ prices coincide.

(π, τ) satisfies simplified flow price condition, if for all connectors $c = (r, s)$, hours t

$$\pi_{r,t} \neq \pi_{s,t} \quad \Rightarrow \quad \begin{cases} \tau_{c,t} \in \{\underline{\tau}_{c,t}, \overline{\tau}_{c,t}\} & \text{active ATC} \\ \vee \quad |\tau_{c,t} - \tau_{c,t-1}| = \widetilde{\tau}_c & \text{active ramping} \\ [\vee \quad |\tau_{c,t} - \tau_{c,t+1}| = \widetilde{\tau}_c\,] & \text{active ramping} \end{cases} .$$

Block Price Condition

Block can be executed, if it does not incur a loss.

$$\forall a \in A, b \in B_a : \quad \beta_b = 1 \qquad \Rightarrow \qquad \sum_{t \in T} (p_b - \pi_{a,t})\, q_{b,t} \geq 0 \; .$$

Notation:

$\pi_{a,t}$	price in area a, time t
β_b	execution state of block $b \in B_a$ in area a
$p_b,\ q_{b,t}$	price limit; net demand in hour $t \in T$

Optimal Solution to (MIQP)

For an optimal bid selection β^* of (MIQP) the existence of shadow prices that satisfy all block price conditions is *not* given.

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Infeasibility Cut: Bid Cut

Given: bid selection β^*
Assume: no shadow prices exist, satisfying all block price conditions. Define the following cut:

Bid Cut

L: executed blocks incurring a loss at prices π^*.

$$L = \{b \in B \mid \beta_b^* = 1 \text{ and } \sum_{t \in T} (p_b - \pi_{a,t}^*) q_{b,t} < 0\},$$

$\mathrm{Cut}(L)$: prohibits execution of at least one bid of L.

$$\mathrm{Cut}(L) : \quad \sum_{b \in L} \beta_b \leq |L| - 1.$$

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Bid Cut Heuristic

Algorithm:

1. $(\beta^*, \delta^*, \tau^*) \leftarrow$ Solve (MIQP).
2. $(\pi^*, L) \leftarrow$ Find shadow prices for $(\beta^*, \delta^*, \tau^*)$.
3. **if** $|L| > 0$, **then**
4. Add Cut(L) to the model (MIQP).
5. Go to step 1.
6. **end if**

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Bid Cut Heuristic, Discussion

(+) Fast Algorithm (4 seconds for 10 market areas)

(+) Good Relative Gap (1.926×10^{-6})

(o) Bid Cut slightly to strong

(o) Not Optimal in pathological cases (4% of the cases)

Branch-and-Cut Decomposition

Replace Bid Cut by a less restrictive Cut:

Exact Bid Cut (cf. [2])

Exclude only one infeasible bid selection β^*:

$$\mathrm{Cut}(\beta^*): \quad \sum_{b \in B: \beta_b^* = 0} \beta_b + \sum_{b \in B: \beta_b^* = 1} (1 - \beta_b) \geq 1$$

Improved version:

Branching over the Bid Cut

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Improved version:

Branching over the Bid Cut

Branch-and-Cut Decomposition, Discussion

(+) Optimal Algorithm

(+) Good Relative Gap (1.924×10^{-6})

(o) Slight improvement of Relative Gap (0.002×10^{-6})

(–) Exact Bid Cut not strong enough

(–) Slow ($\geq$ 10 minutes in 62% of the cases)

Results

79 realistic test cases:

10 market areas, ca. 600 blocks, ca. 31.700 curve segments

Algorithm	*Optimal Bid Selection*	*Relative Gap*	*Computing Time*
Bid Cut Heuristic	$\geq$ 38%	1.926E-6	$\approx$ 4.1 sec
B&C Decomposition	$\geq$ 38%	1.924E-6	$\geq$ 10 min

Optimal bid selection: relative gap $\leq 1 \times 10^{-12}$ within 10 minutes

Thank you
for your attention!

References

References

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