On Clearing Coupled Day-Ahead Electricity Markets

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Electricity Markets

Mixed Integer Quadratic Programm
- Maximization of the Economic Surplus
- Optimality Conditions
- Block Price Condition

Algorithms
- Fast Heuristic
- Exact Algorithm

Results
Day-Ahead Electricity Auctions

Day-Ahead Auction

1. collect demand and supply orders for the following day
2. start auction
   • maximize economic surplus
   • accept/reject orders
   • find clearing price
3. return execution schedule for the following day

Infrastructure

• Market Areas
• Interconnectors
Day-Ahead Electricity Auctions

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Infrastructure

- Market Areas
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Basic elements of energy markets

Bids / Orders are characterized by

- quantity (demand/supply, area, hour)
- price limit
- allow partial execution (yes/no)

Order Types

- Hourly Bids
- Block Bids
- Flexible Bids
Basic elements of energy markets

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Order Types

- Hourly Bids
- Block Bids
- Flexible Bids
Hourly Bids

Aggregation of hourly bid curves provides
- one hourly net curve per area and hour
- bid curve trading quantities depending on the price
- downward sloping curves
Block Bids

A block bid allows for

- trading electricity in several hours
- using only one price limit for all hours:
  \[(\text{net gain}) < 0 \implies \text{don’t execute}\]
Flexible Bids

Flexible bids allow for

- trading electricity without defining the delivery hour
- using a limit price to avoid a loss

\[(\text{gain in hour } t) < 0 \implies \text{don’t execute in hour } t\]
Interconnectors

**Function of Interconnectors**

- balance net demand of adjacent areas
- harmonize prices

**Transmission Constraints**

- Available Transmission Capacity (ATC): *limits the flow*
- Ramp Rate: *limits the change of flow*
Interconnectors

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Transmission Constraints

- Available Transmission Capacity (ATC): *limits the flow*
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Maximization of the Economic Surplus, subject to Transmission Constraints: Overview

\[
\begin{align*}
\max & \quad \sum_{h \in H} \Delta q_h \int_{0}^{\delta_h} p_h(u) \, du + \sum_{b \in B, t \in T} p_b q_b, t \beta_b + K \\
\text{s.t.} & \quad \forall a \in A, t \in T : \sum_{h \in H_a, t} \Delta q_h \delta_h + \sum_{b \in B_a} q_b, t \beta_b = \sum_{c \in C_a^-} \tau_{c, t} - \sum_{c \in C_a^+} \tau_{c, t} \\
& \quad \forall c \in C, t \in T : \tau_{c, t} \leq \tau_{c, t} \leq \bar{\tau}_{c, t} \\
& \quad \forall c \in C, t \in T : -\bar{\tau}_c \leq \tau_{c, t} - \tau_{c, t-1} \leq \bar{\tau}_c \\
& \quad \forall c \in C, t \in T : \tau_{c, t} \in \mathbb{R} \\
& \quad \forall h \in H : \delta_h \in [0, 1] \\
& \quad \forall b \in B : \beta_b \in \{0, 1\}
\end{align*}
\]
Mixed Integer Quadratic Programm

Objective: Economic Surplus

\[
\max \sum_{h \in H} \Delta q_h \int_{0}^{\delta_h} p_h(u) \, du + \sum_{b \in B, t \in T} p_b q_{b,t} \beta_b + K
\]

Notation:

\( \beta_b \in \{0, 1\} \) execution state of block bid \( b \in B \)

\( p_b, q_{b,t} \) price limit; net demand of block \( b \in B \) in hour \( t \in T \)

\( \delta_h \in [0, 1] \) execution state of hourly net curve segment \( h \in H \)

\( p_h(u) \) parametrization of the price of curve segment \( h \in H \)
Mixed Integer Quadratic Programm

Constraints:

1. net demand = net import:

\[ \forall a \in A, t \in T \sum_{h \in H_{a,t}} \Delta q_{h} \delta_{h} + \sum_{b \in B_{a}} q_{b,t} \beta_{b} = \sum_{c \in C_{a}^{-}} \tau_{c,t} - \sum_{c \in C_{a}^{+}} \tau_{c,t} \]

2. ATC:

\[ \forall c \in C, t \in T \tau_{c,t} \leq \tau_{c,t} \leq \bar{\tau}_{c,t} \]

3. ramp rate:

\[ \forall c \in C, t \in T - \bar{\tau}_{c} \leq \tau_{c,t} - \tau_{c,t-1} \leq \bar{\tau}_{c} \]

Notation:

- \( \tau_{c,t} \in \mathbb{R} \): transmission on connector \( c \in C \), hour \( t \in T \)
- \( h \in H_{a,t} \): net curve segments in area \( a \in A \), hour \( t \in T \)
- \( b \in B_{a} \): block bids in area \( a \)
- \( c \in C_{a}^{+} \): connectors leaving area \( a \)
Mixed Integer Quadratic Program

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Optimality Conditions

Optimal solution to (MIQP) can be computed with MIQP solvers (e.g. IBM CPLEX 12.1)

**Analyze optimal solution:**

- fix the combinatorial bid selection
- apply Karush-Kuhn-Tucker-Condition for QPs (cf. [1])
- obtain dual variables, also called shadow prices

Shadow Prices $\pi$ to (MIQP) satisfy (cf. [3])

- Filling Condition
- Flow Price Condition
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Filling Condition

Price change is linear in the partial execution of the active segment:

Filling condition is satisfied, if for all segments $h \in H_{a,t}$ in area $a$, hour $t$

$$\pi_{a,t} \begin{cases} \leq p_h(\delta_h), & \text{if } \delta_h = 1 \\ = p_h(\delta_h), & \text{if } \delta_h \in (0, 1) \\ \geq p_h(\delta_h), & \text{if } \delta_h = 0 \end{cases}$$  \hspace{1cm} (1)
Flow Price Condition

For adjacent areas it holds:
- Prices deviate $\Rightarrow$ a transmission constraint is active.
- No active transmission constraint $\Rightarrow$ prices coincide.

$(\pi, \tau)$ satisfies simplified flow price condition, if for all connectors $c = (r, s)$, hours $t$

$$\pi_{r,t} \neq \pi_{s,t} \Rightarrow \begin{cases} \tau_{c,t} \in \{\tau_{c,t} - \tau_{c,t-1}, \tau_{c,t+1} - \tau_{c,t}\} & \text{active ATC} \\ \vee |\tau_{c,t} - \tau_{c,t-1}| = \tilde{\tau}_c & \text{active ramping} \\ \vee |\tau_{c,t} - \tau_{c,t+1}| = \tilde{\tau}_c & \text{active ramping} \end{cases}$$
Block Price Condition

Block can be executed, if it does not incur a loss.

\[ \forall a \in A, b \in B_a : \quad \beta_b = 1 \quad \Rightarrow \quad \sum_{t \in T} (p_b - \pi_{a,t}) q_{b,t} \geq 0. \]

Notation:

- \( \pi_{a,t} \) price in area \( a \), time \( t \)
- \( \beta_b \) execution state of block \( b \in B_a \) in area \( a \)
- \( p_b, q_{b,t} \) price limit; net demand in hour \( t \in T \)

Optimal Solution to (MIQP)

For an optimal bid selection \( \beta^* \) of (MIQP) the existence of shadow prices that satisfy all block price conditions is not given.
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Infeasibility Cut: Bid Cut

**Given**: bid selection $\beta^*$

**Assume**: no shadow prices exist, satisfying all block price conditions. Define the following cut:

$\text{Bid Cut}$

$L$: executed blocks incurring a loss at prices $\pi^*$.

$$L = \{ b \in B \mid \beta^*_b = 1 \text{ and } \sum_{t \in T} (p_b - \pi^*_{a,t})q_{b,t} < 0 \}$$

Cut ($L$): prohibits execution of at least one bid of $L$.

$$\text{Cut}(L) : \sum_{b \in L} \beta_b \leq |L| - 1.$$
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Bid Cut Heuristic

Algorithm:

1. \((\beta^*, \delta^*, \tau^*) \leftarrow \text{Solve (MIQP)}.\)
2. \((\pi^*, L) \leftarrow \text{Find shadow prices for } (\beta^*, \delta^*, \tau^*).\)
3. \text{if } |L| > 0, \text{ then}
4. \quad \text{Add Cut}(L) \text{ to the model (MIQP).}
5. \quad \text{Go to step 1.}
6. \text{end if}
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Bid Cut Heuristic, Discussion

(+): Fast Algorithm (4 seconds for 10 market areas)

(+): Good Relative Gap ($1.926 \times 10^{-6}$)

(o): Bid Cut slightly to strong

(o): Not Optimal in pathological cases (4% of the cases)
Branch-and-Cut Decomposition

Replace Bid Cut by a less restrictive Cut:

**Exact Bid Cut (cf. [2])**

Exclude only one infeasible bid selection $\beta^*$:

$$\text{Cut}(\beta^*) : \sum_{b \in B: \beta_b^* = 0} \beta_b + \sum_{b \in B: \beta_b^* = 1} (1 - \beta_b) \geq 1$$

**Improved version:**
Branching over the Bid Cut
Branch-and-Cut Decomposition

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**Improved version:**
Branching over the Bid Cut
Branch-and-Cut Decomposition, Discussion

(+) Optimal Algorithm
(+) Good Relative Gap ($1.924 \times 10^{-6}$)
(o) Slight improvement of Relative Gap ($0.002 \times 10^{-6}$)
(−) Exact Bid Cut not strong enough
(−) Slow ($\geq 10$ minutes in 62% of the cases)
Results

79 realistic test cases:
10 market areas, ca. 600 blocks, ca. 31,700 curve segments

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Optimal Bid Selection</th>
<th>Relative Gap</th>
<th>Computing Time</th>
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</thead>
<tbody>
<tr>
<td>Bid Cut Heuristic</td>
<td>$\geq 38%$</td>
<td>1.926E-6</td>
<td>$\approx 4.1$ sec</td>
</tr>
<tr>
<td>B&amp;C Decomposition</td>
<td>$\geq 38%$</td>
<td>1.924E-6</td>
<td>$\geq 10$ min</td>
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Optimal bid selection: relative gap $\leq 1 \times 10^{-12}$ within 10 minutes
Thank you for your attention!
References

