Energy Trading

# Power spot price models with negative prices

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2nd International Ruhr Energy Conference, October 2010

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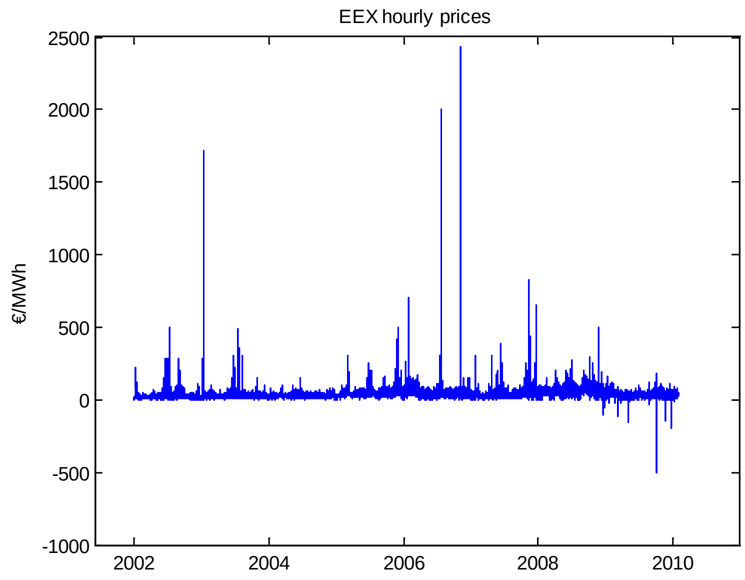
# Negative power prices

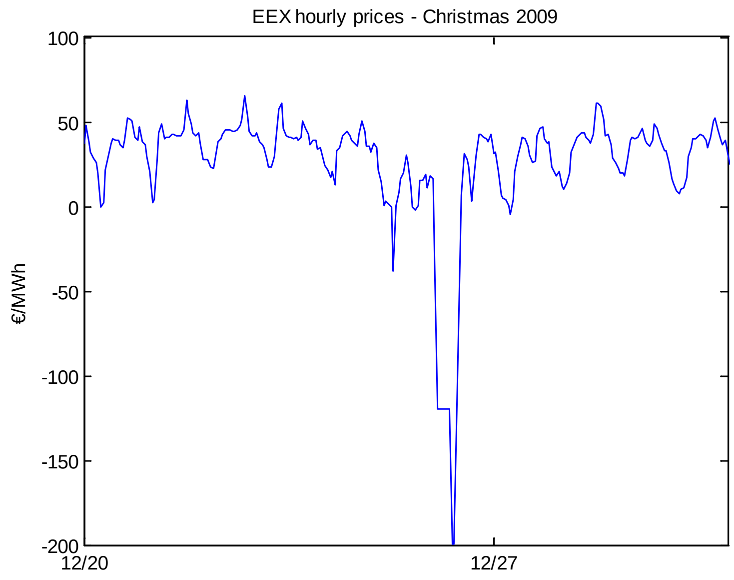
# Negative power prices

Negative spot prices are a (relatively) new phenomenon in European power markets:

- Negative hourly spot auction results are permitted at German EEX  since autumn 2008 (floor price: -3000 €/MWh)
- Permitted at Scandinavian Nordpool since end of 2009

→ Others European markets to follow?



# Negative power prices

In other regions of the world negative power prices are an established characteristic of markets for a many years already (being features of the market design from the beginning on). Examples:

- USA: ERCOT (West Texas), PJM
- Australia: AEMO, region South Australia
- Canada: Ontario (AMPCO) , Alberta (AESO)

Also for another energy ccommodity negative prices occured: Spot price of natural gas at the British NBP

→ Exceptional or more to come?

# Negative power prices

In general, negative prices occur when very low demand coincides with very high supply. Especially if the incident is on short notice since there is limited flexibility on the supply side making reaction and adaptation difficult

- Factors contributing to exceptional slumps in demand: Public holidays, mild winter weather, slumps in industrial activities (eg, production cuts due to the economic crisis)
- **Limited flexibility of power plant operation**: Strong output variation or interruption on short time scales either technically difficult (eg, low-flexibility lignite and nuclear base load plants) or expensive (eg, start-up costs) → **Optimal overall profit when power generation continued and negative revenues for a brief period accepted**
- High wind power infeed: Obligation of grid operator to take wind production with priority, guaranteed tariffs for produced MWh → Grid operator may bid wind power for negative price to achieve market clearing
- Transmission bottleneck: High production in production area cannot be transported to demand area: Eg, wind production Denmark cannot be transported to northern Scandinavian hydro pump storages due to cable transmission bottlenecks

-> Economically reasonable to allow for negative prices to optimize market clearing

*Literature:*
Dettmer & Jacob (2009)
Viehmann & Sämisch (2009):

# Modelling motivation & concept

# Motivation

Negative prices pose an obvious, basic problem to stochastic price modelling: the typical **initial step from prices to log-prices is not possible**

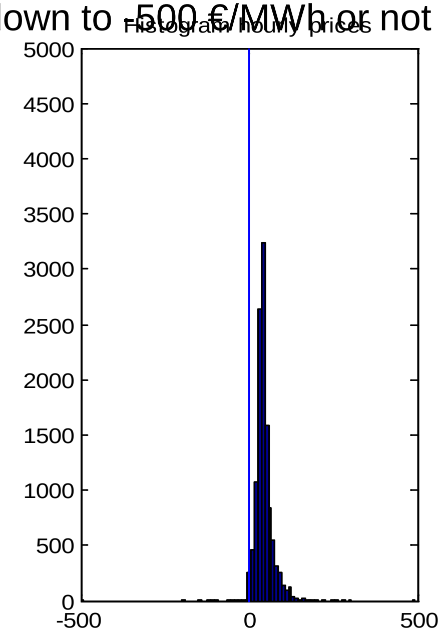
- So far, approaches (to the author's knowledge) are more "fixing" by "workarounds" than systematic: E.g., shift zero price down to a negative lowest level. Or, simply leave out the negative occurrences
  ← Motivated by the relative rarity of negative occurrences
- But: **Practitioners do not feel comfortable with workarounds**, they need to integrate price reality into mark-to-market valuation or risk measurement. E.g., when holding a structured Power Offpeak position, including negative price spikes down to -500 €/MWh or not makes a significant valuation difference

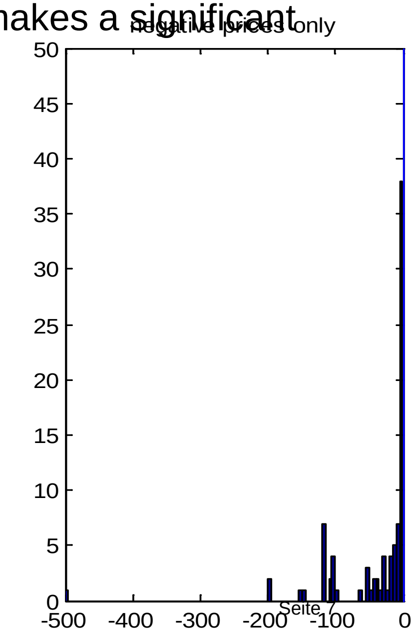
Example EEX:

- 32 days / 88 hours with negative prices. Minimum reached, so far, -500 €/MWh
- Critical conditions: deep night, holidays, high wind infeed

→ Negative prices/spikes have „replaced" positive spikes (price risk!) from the pre-economical crisis „normal demand world"

*Literature:*
Sewalt & De Jong (2003)
Sprenger & Laege (2009)



# Area hyperbolic sine transformation

Proposal: Apply area hyperbolic sine transformation instead of log!

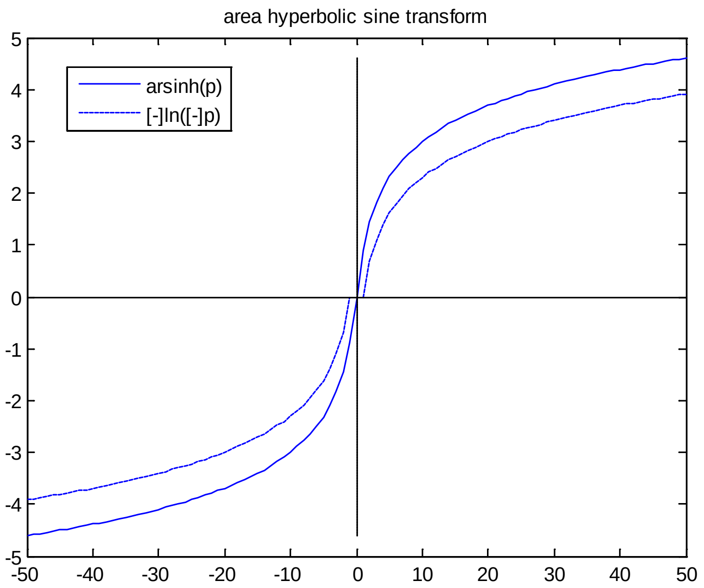
$$x = \sinh^{-1}\left(\frac{p-\xi}{\lambda}\right)$$

$\zeta$ : offset , $\lambda$ : scale

Properties:

- Same asymptotics as (-)ln[(-)*p*] for large |*p*|
- Approx linear around turning point

→ Natural extension of log transformation



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# Model motivation

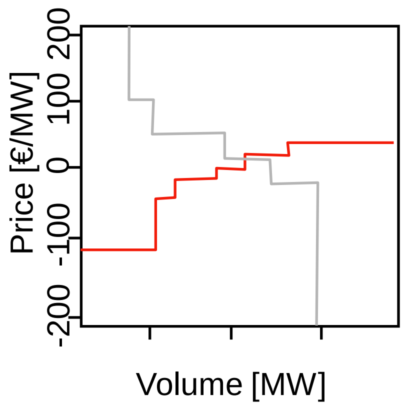
Recall a basic rationale behind the log transformation:

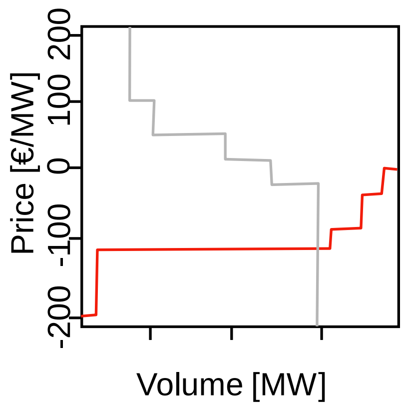
- Variablity of prices d*p* increases with price level: d*p* ~ *p* → study returns d*p*(*t*)/*p*(*t*) or log returns ln(*p*(*t*))-ln(*p*(*t*-1))
- ( *p*>0 )

This is established for power prices >0 → Negative prices (obviously) exhibit the analogous characteristic when decreasing *p*:

- E.g., see bid / ask curves of spot auctions: The curves are not only becoming steeper – and thus the auction outcome more variable / more spiky – when going to high positive prices, but also when going to very negative prices

→ Rules out the „shift zero price level down to a negative lowest level" workaround




EEX bid/demand (grey) and ask/supply (red) curves of hours 1 (bottom) and 14 (top) for the auctions of Dec 26th, 2009.

*Schematically re-drawn*

# A hyperbolic sine stochastic spot price process

Start spot price analysis by area hyperbolic sine transformation:

$$x = \sinh^{-1}(p)$$

Assume a simple OU process for the spot price:

$$dx = k(m - x)dt + \sigma dW$$

Application of Ito's formula yields:

$$\frac{dp}{\sqrt{1+p^2}} = \left[ k\left(m - \sinh^{-1} p\right) + 0.5\sigma^2 \frac{p}{\sqrt{1+p^2}} \right] dt + \sigma dW$$

- Analogous to the usual log-normal process
- log-normal process asymptotically & linear in p for small |*p*|

Asymptotic distribution:

$$x \sim N\left(m, \sqrt{\frac{\sigma^2}{2k}}\right) = N(m, \sigma_{OU})$$

- *f*(*p*) is the density of the **Johnson (SU) distribution**

$$f(p) = \frac{1}{\sigma_{OU}\sqrt{2\pi}\sqrt{1+p^2}} \exp\left( \frac{\left(\sinh^{-1} p - m\right)^2}{2\sigma_{OU}^2} \right)$$

$\rightarrow$ **The hyperbolic sine case is similiarly easily solvable as the the log case**

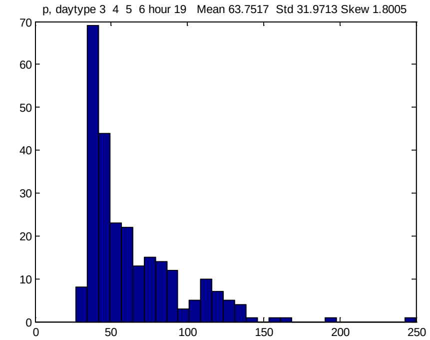
Hypebolic sine processes have been employed before in modelling volatility smiles (positive prices)
Brigo et al , Carr et al (1999)

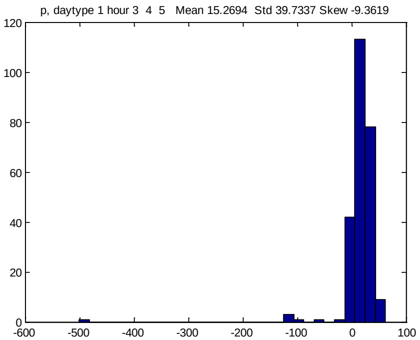
# Spot price distributions

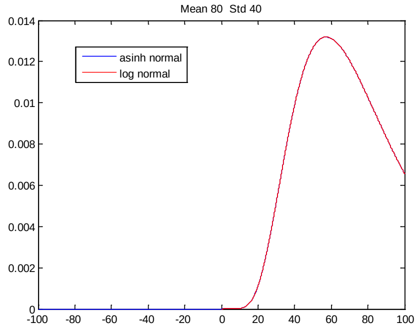
Empirical comparison of some spot price distributions (EEX) and theoretical distrebutions (Johnson with different means and variances)

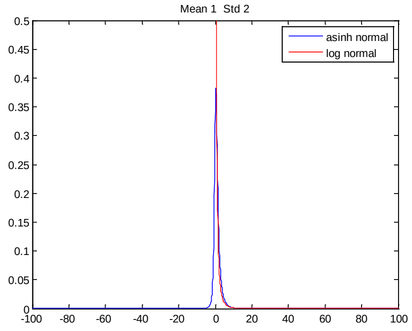
- High price hour type (working day early evening): up spiking, right skewed
- Lowest price hour type (Sunday & holidays deep nights): down spiking, left skewed

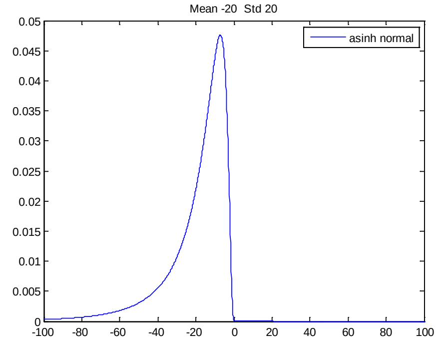
red dotted line: log-normal distribution with same mean and variance



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# Spot price models

# Spot price models

Basis: Regime-switchting model for daily spot prices (De Jong 2006, De Jong & Schneider 2009)

- Regime for up- and down spiking each. Spike height = Poisson compound Gaussian

For transformed and de-seasonalized prices:

*Mean-reverting regime M:* $dx_t^M = \alpha(\mu - x_{t-1}^M) + \sigma \cdot \varepsilon_t$

*Spike regimes:* $x_t^S = \mu + \sum_{i=1}^{n_t+1} Z_{t,i}$

*High spike regime H:* $Z_{t,i} \sim N(\mu^H, \sigma^H), \quad n_t \sim POI(\lambda^H), \quad \mu^H > 0$

*Low spike regime L:* $Z_{t,i} \sim N(\mu^L, \sigma^L), \quad n_t \sim POI(\lambda^L), \quad \mu^L < 0$

*Markov transition matrix:*

$$\Pi = \begin{bmatrix} 1-\pi^{MH}-\pi^{ML} & \pi^{MH} & \pi^{ML} \\ \pi^{HM} & 1-\pi^{HM} & 0 \\ \pi^{LM} & 0 & 1-\pi^{LM} \end{bmatrix}$$

# Spot price models

From transformed prices *p*(*t*) the deterministic component *s*(*t*) is removed $\sinh^{-1}(p_t) = s_t + x_t$

- Remove trend and season by moving average *b*MA(t)*
  - Spikes are removed by a simple 3*sigma procedure when calculating *MA(t)*
- Day type dummies *d**1(*D*), *D*={1,..,7} (holiday=Sunday, bidge day=Saturday).
- Determine coefficients *b*, *d* by linear regression

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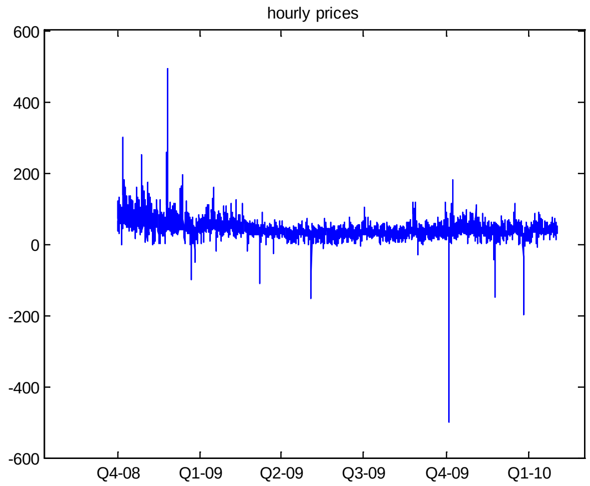
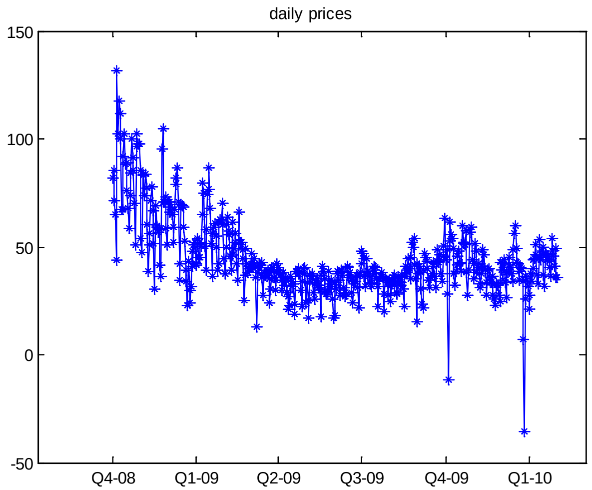
# Spot price models - EEX

# Spot price models - EEX

**Model daily prices *P*(*t*) by regime-switching model**.

- Model hourly prices *S*(*t,h*) by sampling hourly profiles from history of similiar days and adjust to daily price.
- Data: Oct 2008 (first negative price at EEX) – Jan 2010
  - The two days with daily (average) price < 0 are taken removed. They consitute the only two examples of an extreme daily dwonward spikes ← Not (yet) enough samples for estimating a specific spike dynamics




De Jong (2006)
De Jong & Schneider (2009)

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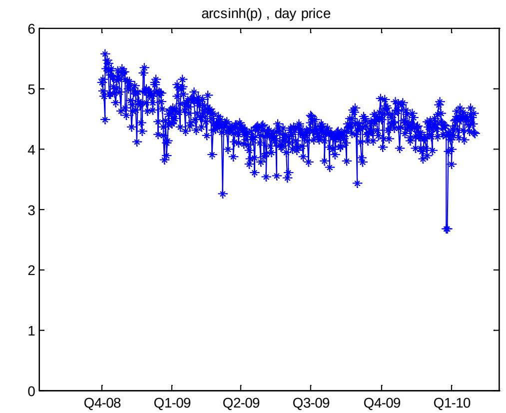
# EEX: Parameter estimation

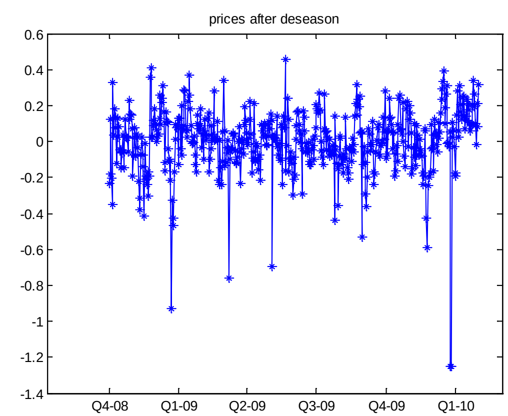
Parameters of the daily process are estimated by maximum likelihood / Bayes scheme (see De Jong (2006)) →

- Spikes are rare and short lived (Markov probabilities)
- Down spikes are only a bit more frequent, but, clearly stronger than up spikes (intuitively correct)

Parameter estimates regime switch model (5b)

| | | | | | |
|---|---|---|---|---|---|
| Time-series parameters: | | | | Switch probabilities | |
| Normal regime | α | 0.42 | | from N to H | 0.95% |
| | μ | 0.02 | | from H to N | 71.15% |
| | σ | 0.11 | | from N to L | 1.93% |
| High-spike regi | μH | 0.35 | | from L to N | 78.50% |
| | σH | 0.06 | | | |
| | λH | 0.10 | | | |
| Low-spike regir | μL | 0.24 | | | |
| | σL | 0.04 | | | |
| | λL | 2.00 | | | |




Transformed daily price asinh(*p*(*t*)) (top) , and deterministic part removed (bottom)

# EEX: Hourly profiles sampling and scaling

Simulation of hourly prices: for a simulated daily price *P*sim(*t*) sample randomly a day's hourly profile *S*hist(*h*) from history

- Match day type *d*(*t*), month, and spike regime
- Adjust *S*sim(*t*,*h*)=S-Transformation(*S*hist(*h*)) such that mean(*S*sim(*t*,*h*))=*P*sim(t)

In earlier versions of the model the transformation was simply the factor *P*sim/*P*hist : No longer reasonable because, eg, for *P*sim>*P*hist („stronger market") it would make negative hours even more negative (factor>1) which would only be reasonable for a „weaker market"

→ **Find a plausible transformation, accounting for negative prices and the hyperbolic sine property**

- **Transfer the principle: find a parameter that shifts the mean of all hours' price distribution & preserves the distribution type**

# EEX: Hourly profiles sampling and scaling

The appropriate rescaling transformation depends on the distribution type of the hourly prices *S*(*h*).

Factor scaling *l* * *S*(*h*) is appropriate for log-normal distributions as the factor shifts the mean and preserves the distribution type

$$\ln S_h \sim N \qquad \ln S_h' = \ln l \cdot S_h = \ln l + \ln S_h \sim N$$

with $$l = \frac{L'}{L} = \frac{\text{mean}(S_h')}{\text{mean}(S_h)}$$

$\rightarrow$ Apply same principle to the „Johnson distribution world" – shift the mean:

$$\sinh^{-1} S_h \sim N \qquad \sinh^{-1} S_h' = \delta + \sinh^{-1} S_h \sim N$$

with $\delta \equiv \sinh^{-1} \Delta$

We get: $$S_h' = \sinh\left(\sinh^{-1}\Delta + \sinh^{-1} S_h\right) = \Delta \cdot \cosh\left(\sinh^{-1} S_h\right) + S_h \cosh\left(\sinh^{-1}\Delta\right)$$

Some substitutions and a small approximation yields: $L' \approx \Delta \cdot \text{mean}(|S_h|) + \cosh \sinh^{-1} \Delta \cdot L$

Contrary to the log-normal case this cannot explicitely solved for $\Delta$, but (easily) numerically

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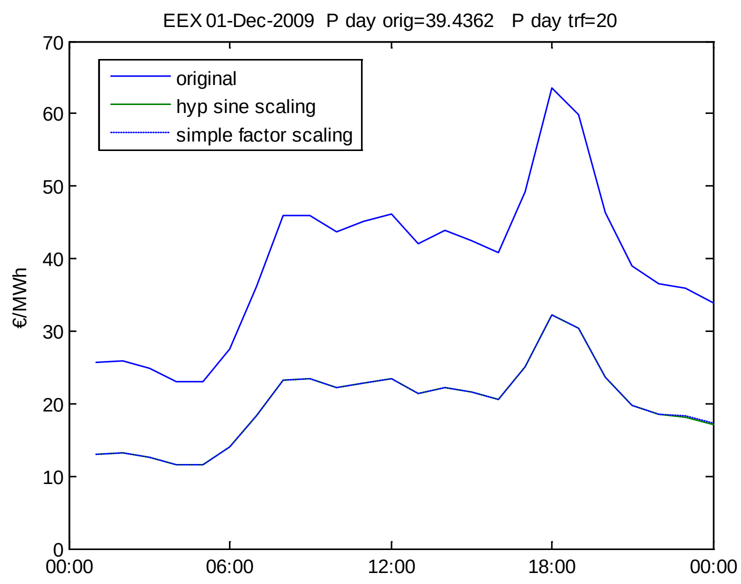
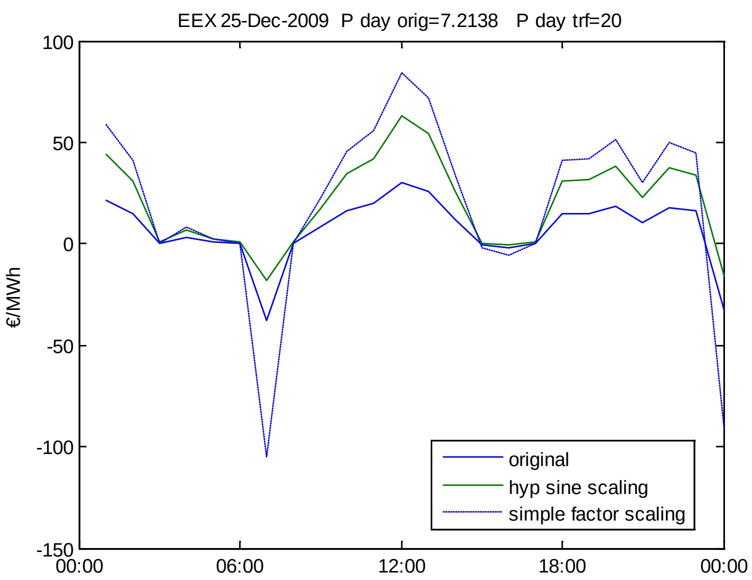
# EEX: Hourly profiles sampling and scaling

Exemplary re-scaling of two hourly spot profiles

- For „normal“ profiles – well above *S*=0 - both transformation act almost identical
- For profiles with (partly) *S*(*h*)<0, only the hypebolic scaling produces reasonable results (shifting the whole profile upwards)



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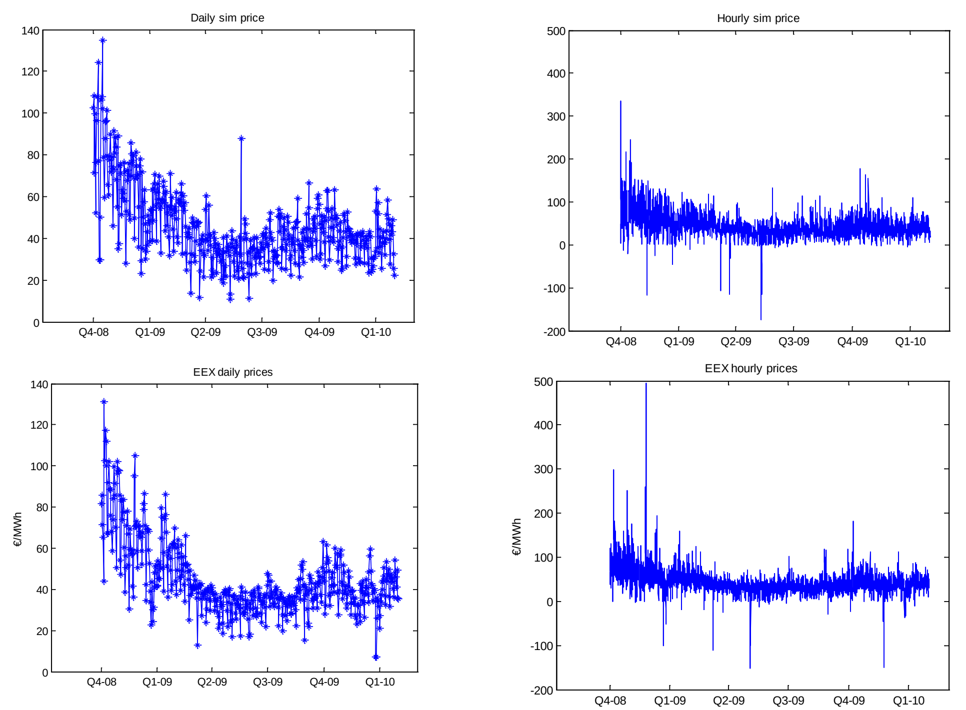
# EEX: Simulation results

- Daily deseasonalized price process *x*(*t*) simulated by regime-switching model
- Simulation period and deterministic part from historical prices $P_t^{sim} = \sinh(s_t^{hist} + x_t^{sim})$
- Hourly prices by sampling and scaling $S_h^{sim}$

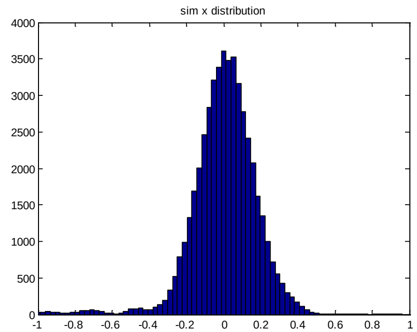
**→ Simulation – trajectories and distributions - matches history well**

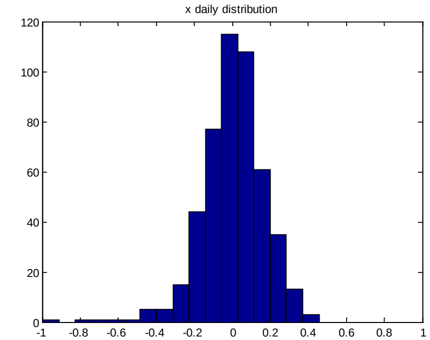


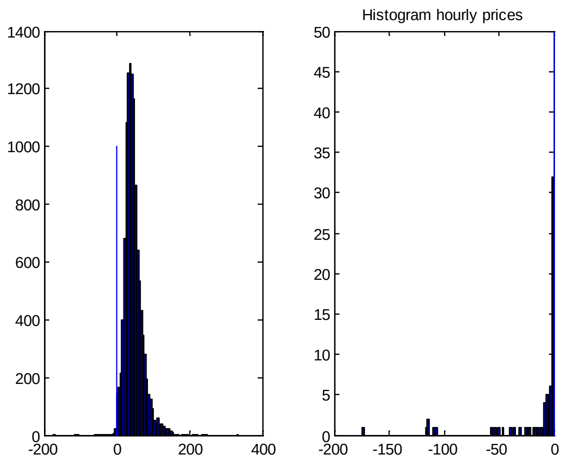

Simulated trajectories (top) , history (bottom)

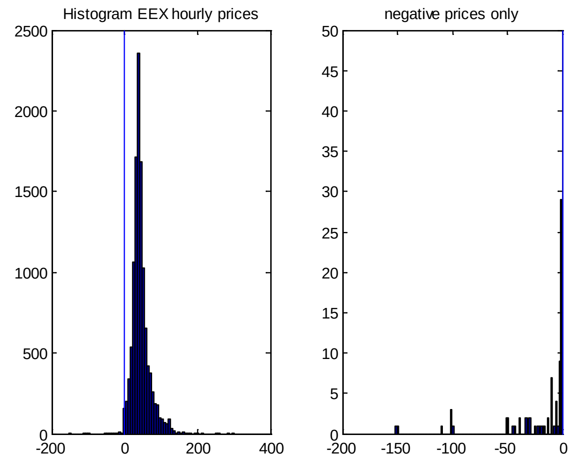
# EEX: Simulation results

Simulation (left) , historical prices (right)



# EEX: Simulation results

The moments of simulated and historical price distributions match well over all

- Kurtosis of simulation does not fully capture history (a typical issue in power spot price modelling)

| | historical prices | sim prices |
|---|---|---|
| x | | |
| mean | 0.00 | |
| standard deviatio | 0.18 | 0.18 |
| skewness | -1.88 | -1.56 |
| kurtosis | 13.03 | 11.19 |
| P, daily prices | | |
| mean | 44.74 | 44.86 |
| standard deviatio | 17.69 | 17.68 |
| skewness | 1.51 | 1.44 |
| kurtosis | 5.92 | 5.93 |
| S, hourly prices | | |
| mean | 44.74 | 44.83 |
| standard deviatio | 24.46 | 23.94 |
| skewness | 1.89 | 1.57 |
| kurtosis | 20.02 | 10.36 |

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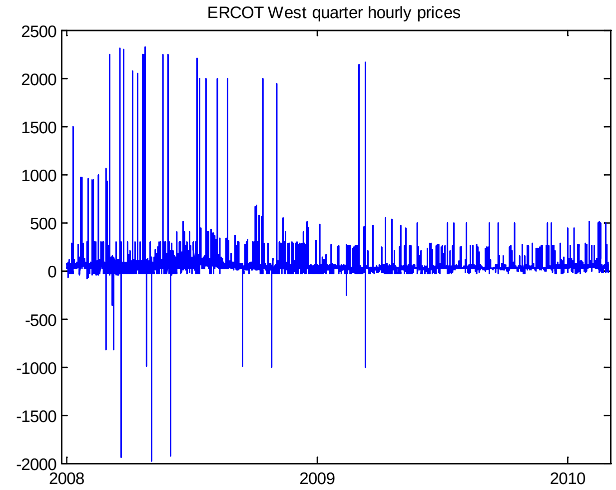
# ERCOT West

# ERCOT West

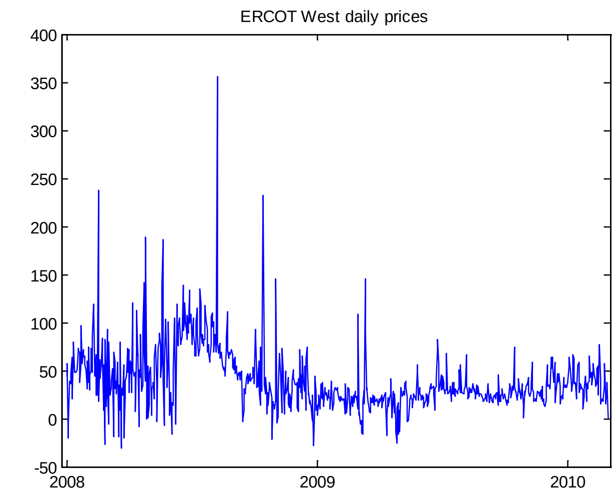
ERCOT settlement price (= „market clearing price for energy in a zonal type market")

- Quarter hourly prices [$/MW]
- Data period: Jan 2008 – Feb 2010 (aligning with EEX data)




Characteristics of data:

- Very spiky
- **Negative (spike) events very frequent**: 370 days with 8520 negative prices. 34 days with negative total daily average
- Negative prices considered to be associated to strong wind power infeed
- Structural changes in price time series apparent (compare structural change in negative price occurrence at EEX in 2010)




→ **Modelling goals**:

- Model only daily prices (intra-day profiles very irregular)

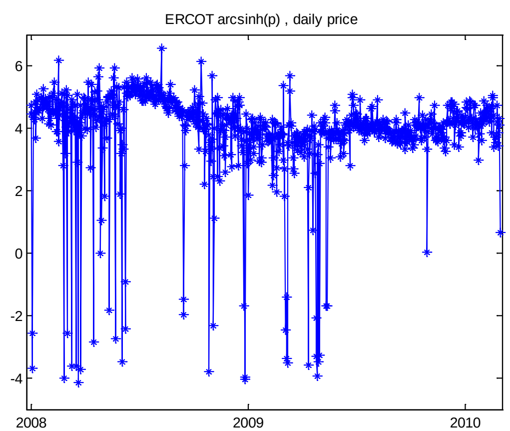
# ERCOT West: Enhanced hyperbolic sine transformation

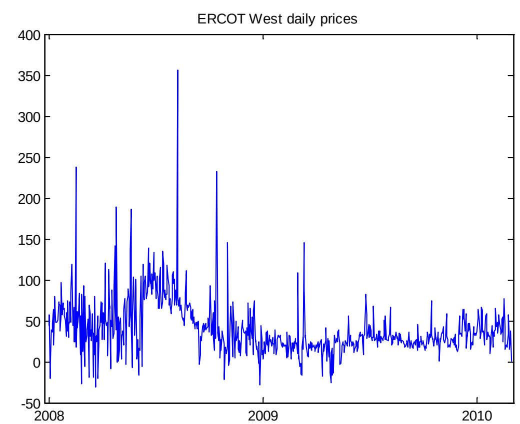
$$x = \sinh^{-1}\left(\frac{p-\xi}{\lambda}\right)$$

Area hyperbolic sine transform of daily prices: simplest transform (with offset 0 and scale 1) does not yield satisfying result

- Downward spikes over-emphasized, upwards spikes almost completely suppressed
- Reason: Rapid, jump-like transition - compared to the range of original price levels – of the asinh transformation from effective ln(*p*) to effective –ln(-*p*)   (see also next slide)

→ Introduce a more „natural transition“ by considering offset and scale



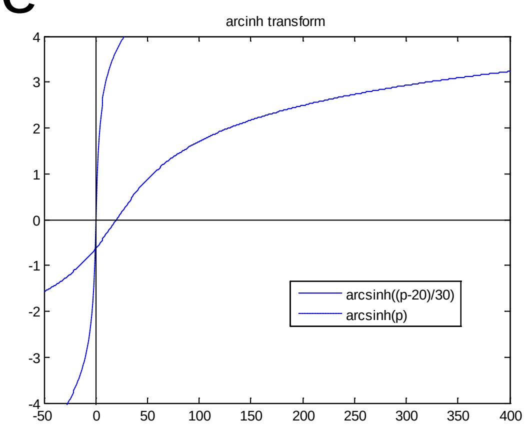




(Left) Transform of daily prices with offset 0 and scale 1, original daily prices (right)

# ERCOT West: Enhanced hyperbolic sine transformation

$$x = \sinh^{-1}\left(\frac{p-\xi}{\lambda}\right)$$

Ad hoc: ξ=20, λ=30

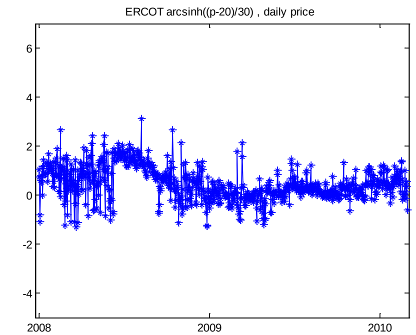



- Transformation looks much more natural now - spikes and intermediate distribution of prices preserved (in rescaled form)

Reason: **linear range of transformation extended** to match intermediate range of prices

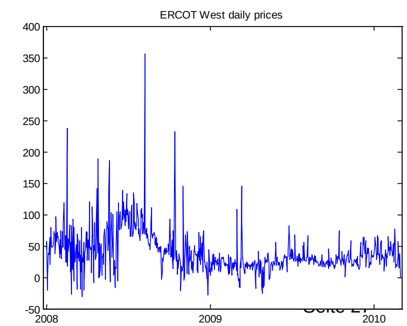
- Weron (...) has already pointed out for (positive only) power spot prices that the usual log transform produces **artificial downward spikes**. He therefore employs no transformation at all (= linear transformation). This, however, leaves the spike distribution extreme and hard to model.

→ Here, the adavantages of both approaches are combined, the disadvantages avoided




- A **fundamental motivation**: Looking at the merit order curve (generation stack) suggests that the minimal price variability should occur in the intermediate price part of the curve, eg, generation from hard coal in Germany. Both in the high and low ends of the curve, price formation is very sensitive to changes in demand and supply (leading to positive and negative spikes)

→ Future work: devise an estimation model for the parameters




Weron (2008)

# ERCOT West: Parameter estimation

As mentioned above, the ERCOT price process appears to be not homogeneous in time with respect to spike characteristics (frequency, height). As the focus of this work is not on spike modelling itself (but negative prices), a **simplified, „homogenized history“** only is modelled

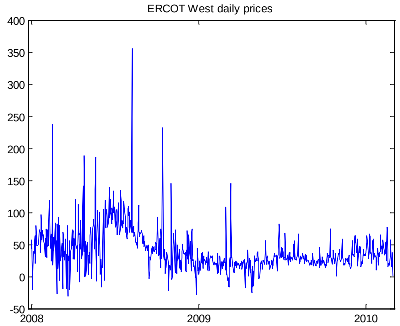
→ Employ simplified parameter estimation

- **Fit only the total price distribution** (not a day-to-day process): parameters of normal and compound normal (spikes) distributions are fitted (maximum likelihood)
- Define regime transition probabilities by dividing data into spikes/non-spike days (maximum likelihood)

Results:

- Only one spike regime (captures both up and down spikes)
- Spikes are short-lived but frequent (as expected)

| | |
|---|---|
| mean-reversion rate | 0.59 |
| level normal price regime | 0.03 |
| vol normal price regime | 0.32 |
| level spike regime | 0.00 |
| vol spike regime | 0.74 |
| Poisson parameter spike regime | 0.50 |
| Switch prob N->S | 21% |
| Switch prob S->N | 61% |

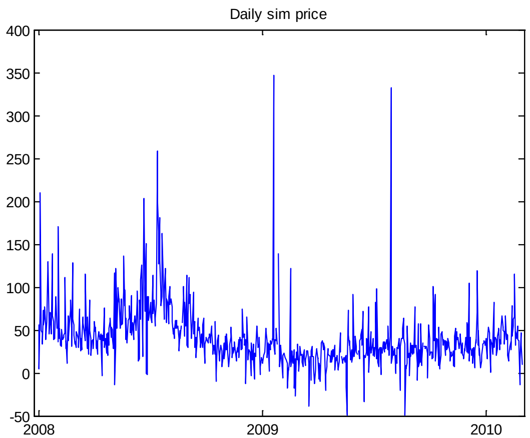


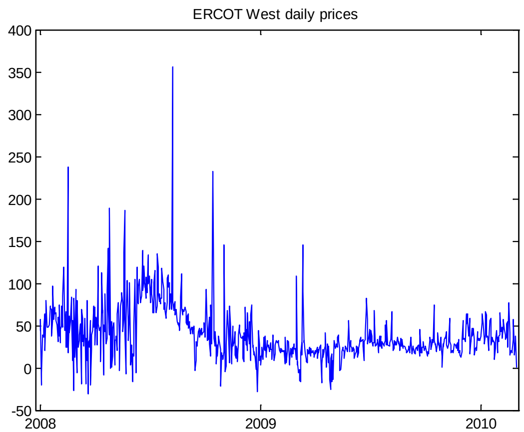
# ERCOT West: Simulation results

As for EEX:

- Simulation period and deterministic part from historical prices

Simulation issue: Due to the combination of the homogenized spike process with the quite inhomogeneous historical deterministic component, some spikes with unrealistically large amplitude are produced (≈0.5% of the data). These are capped/floored to the historically observed max/min prices




(Left) Simulated price scenario: General dynamics is matched. However, observe effect of artificial homogeneity for this example trajectory: some spikes occurring at (historically) wrong times (other trajectories match better).

# ERCOT West: Simulation results

The moments of simulated and historical price distributions match reasonably

| | historical prices | sim prices |
|---|---|---|
| x | | |
| mean | | 0.04 |
| standard deviatio | 0.53 | 0.52 |
| skewness | -0.43 | 0.19 |
| kurtosis | 6.49 | 5.51 |
| P, daily prices | | |
| mean | 39.40 | 40.63 |
| standard deviatio | 32.40 | 32.83 |
| skewness | 2.50 | 2.73 |
| kurtosis | 18.39 | 19.78 |

# Application example: value of a simple spot option

# A European put option on the spot price

**Consider a power generator, selling its production in the spot market, wanting to secure his revenues agains very low or even negative prices** → Purchases a strip of put options on the daily spot price with low strike price

Price model:

- For the sake of demonstrating the core effect, consider a simple mean-reverting process (no spike regimes, no seasonal component)
- Compare results based on the hyperbolic sine and the log transformation

→ Because of the simplicity of the process we know the distribution (of the spot price *p* of a day) explicitely: Johnson distribution & log-normal distribution

$$f_{SU}(p) = \frac{1}{\lambda \cdot \sigma_{SU}\sqrt{2\pi}\sqrt{1+\left(\frac{p-\xi}{\lambda}\right)^2}} \exp\left(\frac{\left(\sinh^{-1}\left(\frac{p-\xi}{\lambda}\right) - m_{SU}\right)^2}{2\sigma_{SU}^2}\right)$$

$$f_{LN}(p) = \frac{1}{\sigma_{LN}\sqrt{2\pi}\,p} \exp\left(\frac{(\ln p - m_{LN})^2}{2\sigma_{LN}^2}\right)$$

# A European put option on the spot price

Start with a given expected value E(*p*) and variance Var(*p*) of the spot price & calculate the parameters of each distribution

→ Calculate option value for each distribution case

$$V = \mathrm{E}\left[\max\left[(K - p);0\right]\right]$$

Example

Strike: $K = 10$

Expected price level: $\mathrm{E}\left[p\right] = 40$

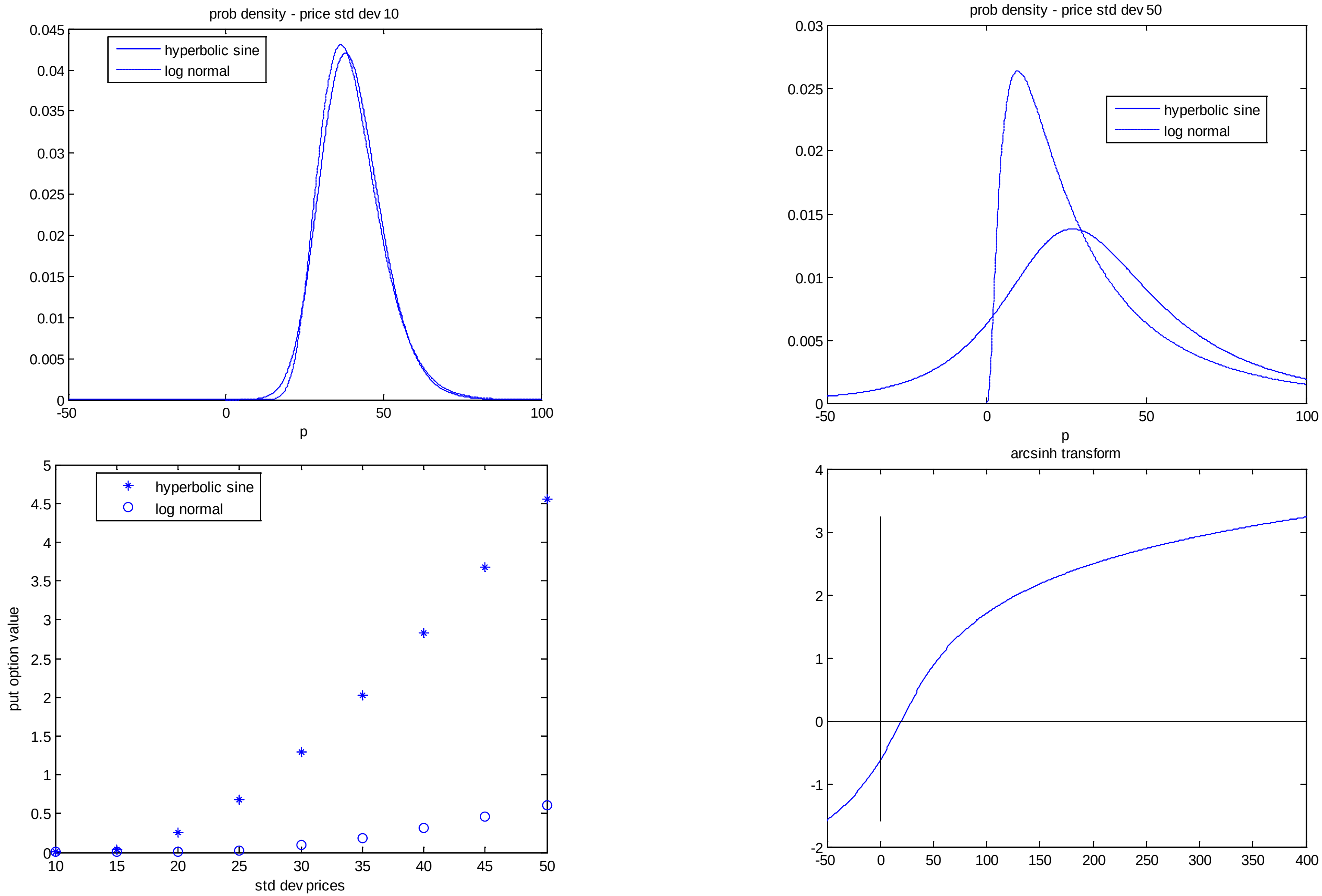
Hyperbolic sine transformation (same parameters as for ERCOT modelling): $\xi = 20$ $\lambda = 30$

Results:

- Hyperbolic sine case option value > log-normal case option value always (however, almost the same value if p bounded well away from 0 ⇔ Jonhson distribution almost log-normal)
- Difference obvious, when there is a probability for negative spot prices

→ However, holds also when probability p<0 neglibile. This is because Johnson distribution has more mass near p=0

# A European put option on the spot price



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# Conclusions

# Conclusions

Negative prices are an inherent feature of the commodity power (they are not going to vanish)

- Aim for sound theoretical treatment instead of workarounds

Area hyperbolic sine transformation well and naturally suited starting point for modelling negative power prices

- Natural extension of the log transformation to the negative axis
- Mathematically almost as easily tractable as the log case

Moreover

- Matches fundamental properties of power prices (linear intermediate price transformation range) better than the log transformation (causes artificial downward spikes)
- Can also be applied to markets with positive prices only

Issues, to dos

- Structural market changes / market rules. Compare EEX negative prices 2009, 2010, future?

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Thank you for your attention!

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