Risk-Neutral Pricing of Financial Instruments in Emission Markets

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\textsuperscript{1}Joint work with Sam Howison.
Introduction
The Need for Emission Reduction
Which Policy Options Should we Use?

Aim is to reduce emission of atmospheric gases such as carbon dioxide, methane, ozone and water vapour.

Different policy options:

- Emission norm (direct regulation),
- Emission tax (market based),
- Emissions trading (market based).
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How Does it Work?

For each country that participates in an emissions trading scheme

- Impose binding limit (cap) on the cumulative emissions during one year (compliance period) and penalise excess emissions (monetary penalty).
- Divide cap into equal amounts, which define one unit (AAU).
- Print paper certificates (allowances) representing one AAU and distribute to firms (initial allocation).
- Trade allowances.
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\rightarrow Leads to a liquid market and price formation.
Modelling Approaches

Full equilibrium models:
1. R. Carmona et al., Market design for emission trading schemes, 2009
2. R. Carmona et al., Optimal stochastic control and carbon price formation, 2009

Purely risk-neutral models:

Hybrid approach:
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Aim: to explain the price of allowances and of derivatives written on them as a function of demand for a pollution-causing good and cumulative emissions.
From Electricity to CO$_2$ Emissions
Market Setup

Introducing the Key Drivers of the Pricing Model

- $[0, T]$ — finite time interval
- $(\Omega, F, (F_t), \mathbb{P})$ — $(F_t)$ generated by $(W_t) \in \mathbb{R}^2$

- $(\xi_t)$ — aggregate supply of electricity $(0 \leq \xi_t \leq \xi_{\text{max}})$
- $(D_t)$ — aggregate demand for electricity $(0 \leq D_t \leq \xi_{\text{max}})$

Walrasian equilibrium assumption,

$$D_t = \xi_t.$$
Assumption
The market administrator arranges generators’ bids in increasing order of price (*merit order*).
The Bid- and Emissions Stack
Price Setting and Emission Measurement in Electricity Markets

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Definition
The business-as-usual bid stack is given by $b^{\text{BAU}} : [0, \xi_{\text{max}}] \rightarrow \mathbb{R}^+$,

$$\frac{db^{\text{BAU}}}{d\xi}(\xi) > 0.$$
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**Definition**

The **marginal emissions stack** is given by $e : [0, \xi_{\text{max}}] \rightarrow \mathbb{R}^+$. Assume that $e$ has at most a finite number of minima and maxima.
The Bid- and Emissions Stack
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Definition
The **marginal emissions stack** is given by \( e : [0, \xi_{\text{max}}] \to \mathbb{R}^+ \).

Assume that \( e \) has at most a finite number of minima and maxima.

To obtain the business-as-usual **instantaneous emissions**, \( \mu_E^{\text{BAU}} : [0, \xi_{\text{max}}] \to \mathbb{R}^+ \), integrate the marginal emissions stack up to current level of demand:

\[
\mu_E^{\text{BAU}}(D) := \int_0^D e(u) \, du.
\]
Load Shifting

The Effects of Emissions Trading on the BAU Economy

With emissions trading, bids increase by

\[(\text{cost of carbon}) \times (\text{marginal emissions rate})\].
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Given an allowance price \(A \geq 0\), the bid stack now becomes

\[b(\xi; A) := b^{\text{BAU}}(\xi) + Ae(\xi).\]

For \(A > 0\) this function may no longer be monotonic!
The Bid Stack under BAU

Supply (as a fraction of the market capacity)

Price (Euro/MWh)

\( b_{\text{BAU}}(\cdot) \)

The Emissions Stack under BAU

Supply (as a fraction of the market capacity)

Marginal Emissions (tCO\(_2\)/MWh)

\( e(\cdot) \)
The Bid Stack under Cap-and-Trade

Supply (as a fraction of the market capacity)

Price (Euro/MWh)

\[ b_{BAU}(\cdot) \quad \text{and} \quad b(\cdot;A), A=100 \]

The Emissions Stack under Cap-and-Trade

Supply (as a fraction of the market capacity)

Marginal Emissions (tCO₂/MWh)
Allowance Pricing
Market Assumptions

Applying the Risk-Neutral Pricing Methodology

Traded assets in the market are

- Allowance certificates
- Derivatives written on the certificate
- Riskless money market account

Assumption

There exists an equivalent (risk-neutral) martingale measure \( \hat{\mathbb{P}} \), under which, for \( 0 \leq t \leq T \), the discounted price of any tradable asset is a martingale.
Market Assumptions
Concretising the Demand and Emissions Process

Demand evolves according to a time-homogeneous Itô diffusion; i.e. for $0 \leq t \leq T$,

$$dD_t = \mu_D(D_t)dt + \sigma_D(D_t)d\tilde{W}_t^1, \quad D_0 = d \in (0, \xi_{\text{max}}),$$

with $\sigma_D(0) = \sigma_D(\xi_{\text{max}}) = 0$. 
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Cumulative emissions have drift $\mu_E$ and we allow for uncertainty by adding a volatility term $\sigma_E$. Then, for $0 \leq t \leq T$,

$$dE_t = \mu_E(A_t, D_t)dt + \sigma_E(E_t)d\tilde{W}_t^2, \quad E_0 = 0,$$

with $\sigma_E(0) = 0$. 
Market with One Compliance Period

Characterising the Allowance Price at $t = T$

- $\Gamma_{\text{cap}} \geq 0$ — initial allocation of certificates
- $\pi \geq 0$ — monetary penalty
- $\{E_T \geq \Gamma_{\text{cap}}\}$ — non-compliance event

Terminal value of the allowance certificate is

$$A_T = \pi \mathbb{I}_{\{E_T \geq \Gamma_{\text{cap}}\}}.$$
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As a traded asset, $A_t$ is given by

$$A_t = e^{-r(T-t)} \pi \tilde{E} \left[ \mathbb{I}\{E_T \geq \Gamma_{\text{cap}}\} \mid \mathcal{F}_t \right], \quad \text{for } 0 \leq t \leq T.$$

Martingale Representation Theorem:

$$d \left( e^{-rt} A_t \right) = Z_t^1 d\tilde{W}_t^1 + Z_t^2 d\tilde{W}_t^2, \quad \text{for } 0 \leq t \leq T$$

and some $\mathcal{F}_t$-adapted process $(Z_t) := (Z_t^1, Z_t^2)$. 
Combining the processes for demand, cumulative emissions and the allowance certificate leads to the FBSDE

\[
\begin{aligned}
    dD_t &= \mu_D(D_t)dt + \sigma_D(D_t)d\tilde{W}_t^1, & D_0 &= d \in (0, \xi_{\text{max}}), \\
    dE_t &= \mu_E(A_t, D_t)dt + \sigma_E(E_t)d\tilde{W}_t^2, & E_0 &= 0, \\
    dA_t &= rA_t dt + Z_t^1 d\tilde{W}_t^1 + Z_t^2 d\tilde{W}_t^2, & A_T &= \pi \mathbb{1}_{\{E_T \geq \Gamma_{\text{cap}}\}}.
\end{aligned}
\]

- \((D_t, E_t)\) — forward part
- \((A_t)\) — backward part
- \((Z_t)\) — generator
Market with One Compliance Period

PDE Representation of the FBSDE Solution

Let $A_t = \alpha(t, D_t, E_t)$, then

\[
\frac{\partial \alpha}{\partial t} + \frac{1}{2} \sigma_D^2(D) \frac{\partial^2 \alpha}{\partial D^2} + \frac{1}{2} \sigma_E^2(E) \frac{\partial^2 \alpha}{\partial E^2} + \mu_D(D) \frac{\partial \alpha}{\partial D} + \mu_E(\alpha, D) \frac{\partial \alpha}{\partial E} - r\alpha = 0,
\]

with terminal condition

\[
\alpha(T, D, E) = \pi \mathbb{I}_{\{E \geq \Gamma_{\text{cap}}\}}, \quad 0 \leq D \leq \xi_{\text{max}}, \quad E \geq 0.
\]

The solution $\alpha$ also satisfies

\[
\lim_{E \to \infty} \alpha(t, D, E) = e^{-r(T-t)}\pi, \quad 0 \leq t \leq T, \quad 0 \leq D \leq \xi_{\text{max}}.
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Allowance Certificate Price at t=0.5 T

Cumulative Emissions (as a fraction of the initial allocation) 0 Demand (as a fraction of the market capacity)

Price (Euro)

Allowance Certificate Price at t=T

Cumulative Emissions (as a fraction of the initial allocation) 0 Demand (as a fraction of the market capacity)

Price (Euro)
Market with Multiple Compliance Periods

Generalising the Pricing Problem

\[ [0, T_1], \ldots, [T_{N_{cp}-1}, T_{N_{cp}}(= T)] : N_{cp} \text{ compliance periods.} \]

Subsequent compliance periods joint by connecting mechanisms:

1. banking
2. withdrawal
3. borrowing

\[ \rightarrow \text{ Terminal condition } A'_{T_i} : \]

- path-dependant
- more expensive
Market with Multiple Compliance Periods

Banking

Banking: an additional incentive to reduce emissions.

In the event of compliance, a number \( \hat{\Gamma}_i^{\text{cap}} - E_i \) of certificates with price \( A_{Ti}^{i} \) are exchanged for certificates valid during the next compliance period, with price \( A_{Ti}^{i+1} \).
Market with Multiple Compliance Periods

Withdrawal

Withdrawal: additional punishment for excess emissions.

In the event of non-compliance, a number

\[ \min \left( E_i - \hat{\Gamma}_{cap}^i, \Gamma_{cap}^{i+1} \right) \]

certificates with price \( A^{i+1}_{T_i} \) are subtracted from \( \Gamma_{cap}^{i+1} \).
Allowance Certificate Price at $t=0.5 \ T_1$
(market with two compliance periods; banking and withdrawal)

Allowance Certificate Price at $t=T_1$
(market with two compliance periods; banking and withdrawal)
Option Pricing
Our example of choice is a European call \((C_t(\tau))_{t \in [0, \tau]}\) with maturity \(\tau\), where \(0 \leq \tau \leq T\), and strike \(K \geq 0\). Its payoff is

\[ C_\tau(\tau) := (A_\tau - K)^+ \]
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- Require knowledge of \(A_t\) \(\Rightarrow\) need to solve problem for \(A_t\) and for \(C_t\) in parallel.
- The option price does not affect the rate at which firms emit \(\Rightarrow\) expect the option pricing problem to be linear.
The European Call on the Allowance Certificate

The Pricing PDE

Letting \( C_t = \nu(t, D_t, E_t) \),

\[
\frac{\partial \nu}{\partial t} + \frac{1}{2} \sigma_D(D) \frac{\partial^2 \nu}{\partial D^2} + \frac{1}{2} \sigma_E(E) \frac{\partial^2 \nu}{\partial E^2} + \mu_D(D) \frac{\partial \nu}{\partial D} + \mu_E(\alpha(t, D, E), D) \frac{\partial \nu}{\partial E} - r \nu = 0,
\]

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\nu(T, D, E) = (A_T - K)^+, \quad 0 \leq D \leq \xi_{\text{max}}, \quad E \geq 0.
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Further, \( \nu \) satisfies

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\lim_{E \to \infty} \nu(t, D, E) = e^{-r(T-t)} \left( \pi - e^{r(T-\tau)} K \right)^+, \quad 0 \leq t \leq \tau, \quad 0 \leq D \leq \xi_{\text{max}}.
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<th>Cumulative Emissions (as a fraction of the initial allocation)</th>
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<td></td>
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**Call Option Price at t=τ**

- The graph illustrates the call option price at time $t=\tau$.
- The $x$-axis represents the demand (as a fraction of the market capacity).
- The $y$-axis represents the cumulative emissions (as a fraction of the initial allocation).
- The $z$-axis represents the price in Euros.
Conclusion
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Current work shows:

- **Bid stack →** which generators are active in the market → rate at which market emits CO$_2$.

- **Allowances →** derivatives on demand for polluting goods and cumulative emissions.

- **Borrowing, banking and withdrawal →** impact the allowance price in multi-period setting.

- **Options on allowances →** can be priced in this hybrid framework.
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Future work:

- Impact of large abatement project.
- Impact of cost of carbon on demand.
- FBSDEs with degeneracy in forward part.
Thank you for your attention.
Questions?
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Supply (as a fraction of the market capacity)
Price (Euro/MWh)

\[ b_{\text{BAU}}(\cdot) \]
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Supply (as a fraction of the market capacity)

Marginal Emissions (tCO₂/MWh)