Risk-Neutral Pricing of Financial Instruments in Emission Markets

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[1] Joint work with Sam Howison.

Introduction

The Need for Emission Reduction

Which Policy Options Should we Use?

Aim is to reduce emission of atmospheric gases such as carbon dioxide, methane, ozone and water vapour.

Different policy options:

- Emission norm (direct regulation),
- Emission tax (market based),
- Emissions trading (market based).

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Emissions Trading

How Does it Work?

For each country that participates in an emissions trading scheme

- Impose binding limit (*cap*) on the cumulative emissions during one year (*compliance period*) and penalise excess emissions (monetary *penalty*).
- Divide cap into equal amounts, which define one unit (*AAU*).
- Print paper certificates (*allowances*) representing one AAU and distribute to firms (*initial allocation*).
- Trade allowances.

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$\hookrightarrow$ Leads to a liquid market and price formation.

Modelling Approaches

Full equilibrium models:

1. R. Carmona et al., Market design for emission trading schemes, 2009
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Aim: to explain the price of allowances and of derivatives written on them as a function of demand for a pollution-causing good and cumulative emissions .

From Electricity to CO_2 Emissions

Market Setup

Introducing the Key Drivers of the Pricing Model

- $[0, T]$ — finite time interval
- $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$ — $(\mathcal{F}_t)$ generated by $(W_t) \in \mathbb{R}^2$

- (ξ_t) — aggregate supply of electricity $(0 \leq \xi_t \leq \xi_{\max})$
- (D_t) — aggregate demand for electricity $(0 \leq D_t \leq \xi_{\max})$

Walrasian equilibrium assumption,

$$D_t = \xi_t.$$

- (E_t) — cumulative emissions up to time t
- (A_t) — allowance certificate price

The Bid- and Emissions Stack

Price Setting and Emission Measurement in Electricity Markets

Assumption

The market administrator arranges generators' bids in increasing order of price (**merit order**).

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Definition

The **business-as-usual bid stack** is given by $b^{\mathsf{BAU}} : [0, \xi_{\mathsf{max}}] \to \mathbb{R}_+$,

$$\frac{\mathrm{d}b^{\mathsf{BAU}}}{\mathrm{d}\xi}(\xi) > 0.$$

▸ Bid Stack

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▸ Bid Stack

$\hookrightarrow$ Allows us to deduce the *generation order*.

The Bid- and Emissions Stack

Price Setting and Emission Measurement in Electricity Markets

Definition

The **marginal emissions stack** is given by $e : [0, \xi_{\max}] \to \mathbb{R}_+$.

Assume that e has at most a finite number of minima and maxima.

▸ Emissions Stack

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▸ Emissions Stack

$\hookrightarrow$ To obtain the business-as-usual *instantaneous emissions*, $\mu_E^{\text{BAU}} : [0, \xi_{\max}] \rightarrow \mathbb{R}_+$, integrate the marginal emissions stack up to current level of demand:

$$\mu_E^{\text{BAU}}(D) := \int_0^D e(u)\, \mathrm{d}u.$$

Load Shifting

The Effects of Emissions Trading on the BAU Economy

With emissions trading, bids increase by

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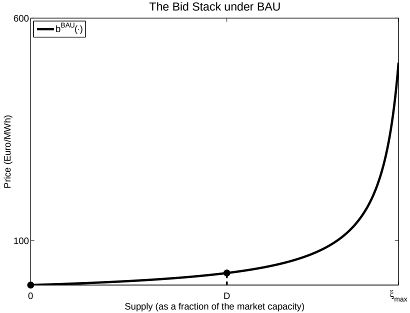
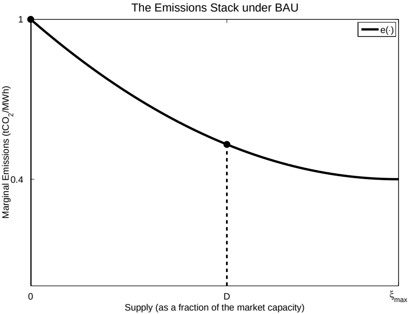
Given an allowance price $A \geq 0$, the bid stack now becomes

$$b(\xi; A) := b^{\text{BAU}}(\xi) + Ae(\xi).$$

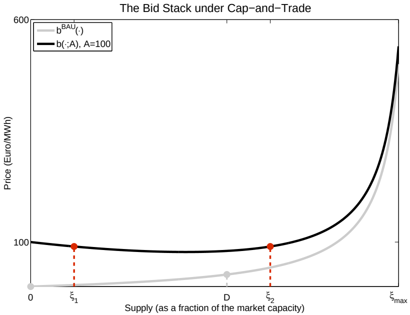
For $A > 0$ this function may no longer be monotonic!

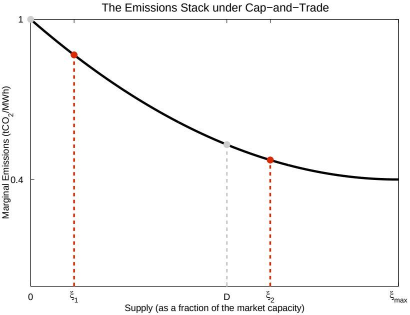


The Bid Stack under BAU
$b^{BAU}(\cdot)$
600
100
Price (Euro/MWh)
0
D
ξ_{max}
Supply (as a fraction of the market capacity)



The Emissions Stack under BAU
$e(\cdot)$
1
0.4
Marginal Emissions (tCO_2/MWh)
0
D
ξ_{max}
Supply (as a fraction of the market capacity)


The Bid Stack under Cap−and−Trade
600
100
Price (Euro/MWh)
b^BAU(·)
b(·;A), A=100
0
ξ1
D
ξ2
ξmax
Supply (as a fraction of the market capacity)


The Emissions Stack under Cap−and−Trade
1
0.4
Marginal Emissions (tCO2/MWh)
0
ξ1
D
ξ2
ξmax
Supply (as a fraction of the market capacity)

Allowance Pricing

Market Assumptions

Applying the Risk-Neutral Pricing Methodology

Traded assets in the market are

- Allowance certificates
- Derivatives written on the certificate
- Riskless money market account

Assumption

There exists an equivalent (risk-neutral) martingale measure $\tilde{\mathbb{P}}$, under which, for $0 \leq t \leq T$, the discounted price of any tradable asset is a martingale.

Market Assumptions

Concretising the Demand and Emissions Process

Demand evolves according to a time-homogeneous Itô diffusion; i.e. for $0 \leq t \leq T$,

$$\mathrm{d}D_t = \mu_D(D_t)\mathrm{d}t + \sigma_D(D_t)\mathrm{d}\tilde{W}_t^1, \qquad D_0 = d \in (0, \xi_{\max}),$$

with $\sigma_D(0) = \sigma_D(\xi_{\max}) = 0$.

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Cumulative emissions have drift μ_E and we allow for uncertainty by adding a volatility term σ_E. Then, for $0 \leq t \leq T$,

$$\mathrm{d}E_t = \mu_E(A_t, D_t)\mathrm{d}t + \sigma_E(E_t)\mathrm{d}\tilde{W}_t^2, \quad E_0 = 0,$$

with $\sigma_E(0) = 0$.

Market with One Compliance Period

Characterising the Allowance Price at $t = T$

- $\Gamma_{\text{cap}} \geq 0$ — inital allocation of certificates
- $\pi \geq 0$ — monetary penalty
- $\{E_T \geq \Gamma_{\text{cap}}\}$ — non-compliance event

Terminal value of the allowance certificate is

$$A_T = \pi \mathbb{I}_{\{E_T \geq \Gamma_{\text{cap}}\}}.$$

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As a traded asset, A_t is given by

$$A_t = e^{-r(T-t)} \pi \tilde{\mathbb{E}} \left[\mathbb{I}_{\{E_T \geq \Gamma_{\text{cap}}\}} \middle| \mathcal{F}_t \right], \quad \text{for } 0 \leq t \leq T.$$

Martingale Representation Theorem:

$$\mathrm{d}\left(e^{-rt} A_t\right) = Z_t^1 \mathrm{d}\tilde{W}_t^1 + Z_t^2 \mathrm{d}\tilde{W}_t^2, \quad \text{for } 0 \leq t \leq T$$

and some $\mathcal{F}_t$-adapted process $(Z_t) := \left(Z_t^1, Z_t^2\right)$.

Market with One Compliance Period

FBSDE Formulation of the Pricing Problem

Combining the processes for demand, cumulative emissions and the allowance certificate leads to the FBSDE

$$\begin{cases} \mathrm{d}D_t = \mu_D(D_t)\mathrm{d}t + \sigma_D(D_t)\mathrm{d}\tilde{W}_t^1, & D_0 = d \in (0, \xi_{\max}), \\ \mathrm{d}E_t = \mu_E(A_t, D_t)\mathrm{d}t + \sigma_E(E_t)\mathrm{d}\tilde{W}_t^2, & E_0 = 0, \\ \mathrm{d}A_t = rA_t\mathrm{d}t + Z_t^1\mathrm{d}\tilde{W}_t^1 + Z_t^2\mathrm{d}\tilde{W}_t^2, & A_T = \pi\mathbb{I}_{\{E_T \geq \Gamma_{\text{cap}}\}}. \end{cases}$$

- (D_t, E_t) — forward part
- (A_t) — backward part
- (Z_t) — generator

Market with One Compliance Period

PDE Representation of the FBSDE Solution

Let $A_t = \alpha(t, D_t, E_t)$, then

$$\frac{\partial \alpha}{\partial t} + \frac{1}{2}\sigma_D^2(D)\frac{\partial^2 \alpha}{\partial D^2} + \frac{1}{2}\sigma_E^2(E)\frac{\partial^2 \alpha}{\partial E^2} + \mu_D(D)\frac{\partial \alpha}{\partial D} + \mu_E(\alpha, D)\frac{\partial \alpha}{\partial E} - r\alpha = 0,$$

with terminal condition

$$\alpha(T, D, E) = \pi \mathbb{I}_{\{E \geq \Gamma_{\text{cap}}\}}, \quad 0 \leq D \leq \xi_{\max}, \ E \geq 0.$$

The solution α also satisfies

$$\lim_{E \to \infty} \alpha(t, D, E) = e^{-r(T-t)}\pi, \quad 0 \leq t \leq T, \ 0 \leq D \leq \xi_{\max}.$$

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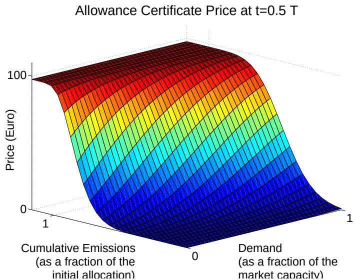

Allowance Certificate Price at t=0.5 T
Price (Euro)
100
0
1
0
1
Cumulative Emissions
(as a fraction of the
initial allocation)
Demand
(as a fraction of the
market capacity)
Allowance Certificate Price at t=T
Price (Euro)
100
0
1
0
1
Cumulative Emissions
(as a fraction of the
initial allocation)
Demand
(as a fraction of the
market capacity)

Market with Multiple Compliance Periods

Generalising the Pricing Problem

$[0, T_1], \ldots, \left[T_{N_{cp}-1}, T_{N_{cp}}(= T)\right]$: N_{cp} compliance periods.

Subsequent compliance periods joint by connecting mechanisms:

1. banking
2. withdrawal
3. borrowing

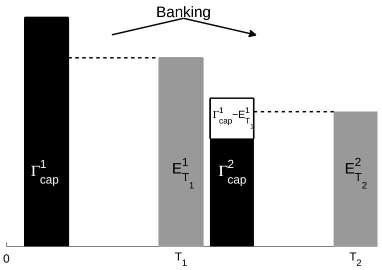
$\hookrightarrow$ Terminal condition $A^i_{T_i}$:

- path-dependant
- more expensive

Market with Multiple Compliance Periods

Banking

Banking: an additional incentive to reduce emissions.

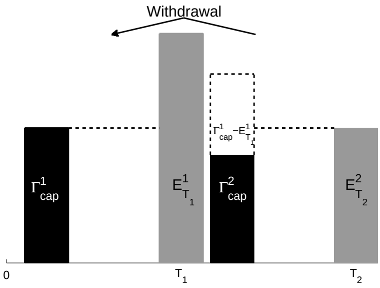


In the event of compliance, a number $\left(\hat{\Gamma}^i_{cap} - E^i\right)$ of certificates with price $A^i_{T_i}$ are exchanged for certificates valid during the next compliance period, with price $A^{i+1}_{T_i}$.

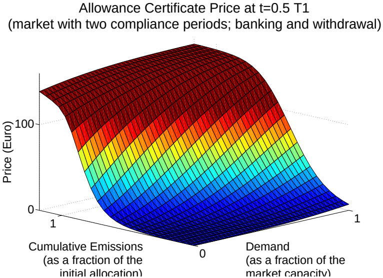
Market with Multiple Compliance Periods

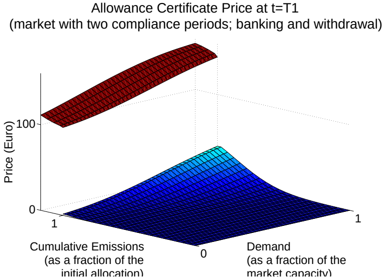
Withdrawal

Withdrawal: additional punishment for excess emissions.



In the event of non-compliance, a number $\min\left(E^i - \hat{\Gamma}^i_{cap}, \Gamma^{i+1}_{cap}\right)$ of certificates with price $A^{i+1}_{T_i}$ are subtracted from Γ^{i+1}_{cap}.


Allowance Certificate Price at t=0.5 T1
(market with two compliance periods; banking and withdrawal)
Price (Euro)
100
0
1
0
1
Cumulative Emissions
(as a fraction of the
initial allocation)
Demand
(as a fraction of the
market capacity)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Price (Euro)
100
0
1
0
1
Cumulative Emissions
(as a fraction of the
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Demand
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Option Pricing

The European Call on the Allowance Certificate

Formulating the Pricing Problem

Our example of choice is a European call $(C_t(\tau))_{t\in[0,\tau]}$ with maturity τ, where $0 \leq \tau \leq T$, and strike $K \geq 0$. Its payoff is

$$C_\tau(\tau) := (A_\tau - K)^+ \, .$$

The European Call on the Allowance Certificate

Formulating the Pricing Problem

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- Require knowledge of A_t $\Rightarrow$ need to solve problem for A_t and for C_t in parallel.
- The option price does not affect the rate at which firms emit $\Rightarrow$ expect the option pricing problem to be linear.

The European Call on the Allowance Certificate

The Pricing PDE

Letting $C_t = v(t, D_t, E_t)$,

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma_D^2(D)\frac{\partial^2 v}{\partial D^2} + \frac{1}{2}\sigma_E^2(E)\frac{\partial^2 v}{\partial E^2} + \mu_D(D)\frac{\partial v}{\partial D} + \mu_E(\alpha(t, D, E), D)\frac{\partial v}{\partial E} - rv = 0,$$

with terminal condition

$$v(T, D, E) = (A_\tau - K)^+, \quad 0 \leq D \leq \xi_{\max}, \; E \geq 0.$$

Further, v satisfies

$$\lim_{E\to\infty} v(t, D, E) = e^{-r(T-t)}\left(\pi - e^{r(T-\tau)}K\right)^+, \quad 0 \leq t \leq \tau, \; 0 \leq D \leq \xi_{\max}.$$

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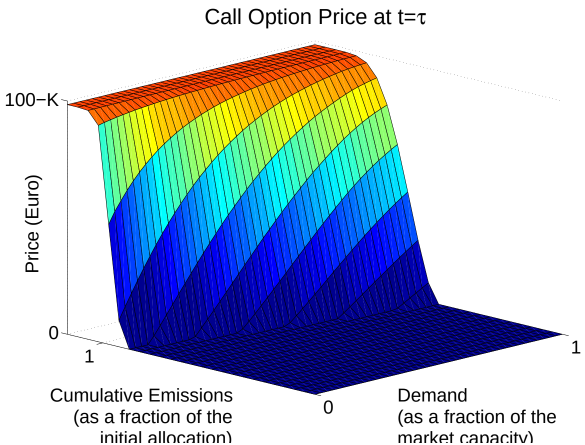
Call Option Price at t=τ
100−K
0
Price (Euro)
1
0
1
Cumulative Emissions
(as a fraction of the
initial allocation)
Demand
(as a fraction of the
market capacity)

Conclusion

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Current work shows:

- Bid stack → which generators are active in the market → rate at which market emits CO_2.
- Allowances → derivatives on demand for polluting goods and cumulative emissions.
- Borrowing, banking and withdrawal → impact the allowance price in multi-period setting.
- Options on allowances → can be priced in this hybrid framework.

Conclusion

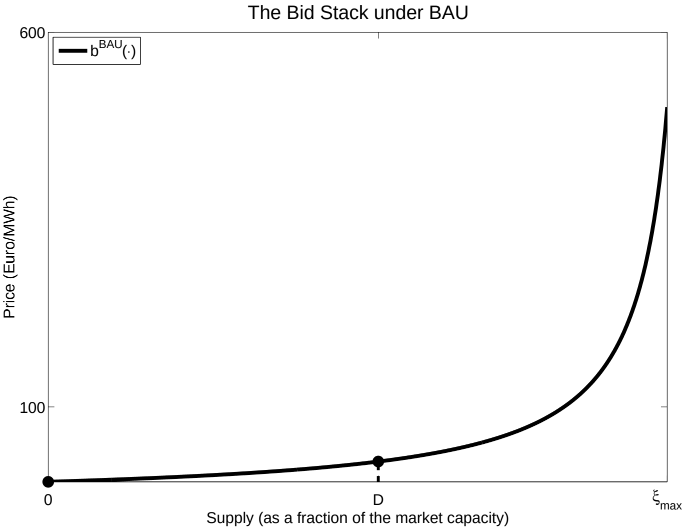
Current work shows:

- Bid stack $\rightarrow$ which generators are active in the market $\rightarrow$ rate at which market emits CO_2.
- Allowances $\rightarrow$ derivatives on demand for polluting goods and cumulative emissions.
- Borrowing, banking and withdrawal $\rightarrow$ impact the allowance price in multi-period setting.
- Options on allowances $\rightarrow$ can be priced in this hybrid framework.

Future work:

- Impact of large abatement project.
- Impact of cost of carbon on demand.
- FBSDEs with degeneracy in forward part.

Thank you for your attention.
Questions?



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600
100
$b^{BAU}(\cdot)$
Price (Euro/MWh)
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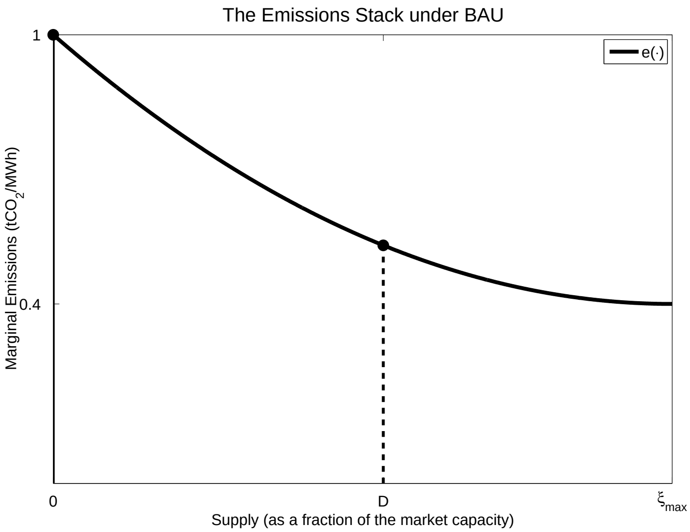
ξ_{max}
Supply (as a fraction of the market capacity)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D
ξmax
Supply (as a fraction of the market capacity)