Estimation of realised volatility and correlation using High-Frequency Data: An analysis of Nord Pool Electricity futures.

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Research agenda for our study

- Using high-frequency (intra-day) electricity futures data and the concept of realised volatility and realised correlation to:

  1. Describe the distributional properties and temporal dependencies of realized volatility and correlation for Nord Pool electricity forward data from 2005 to 2009.
  2. Compare whether the financial electricity market behavior differ significantly from financial- and other commodity markets.

- Analyzing intra-daily electricity futures data has only to a limited extend been studied before.

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Why study volatility and correlation for electricity futures?

- Pricing of future options (one or more underlying contracts).
- Calculation of hedge ratios.
- Value at Risk and Expected Shortfall calculations.
- Portfolio analysis / optimisation.

Why use methods based on intra-daily data for volatility and correlation estimates?

- More information provided.
- Easy to understand and implement (though there is more work organising the data….).
- Standard time series methods can be applied for modelling and forecasting volatility and correlation.
- Better forecasts of volatility, correlation and risk measures?

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Literature realized volatility and correlation:

- **Base references:**
  - Andersen and Bollerslev (1998)
  - Andersen et al. (2003)
  - Barndorff-Nielsen and Shephard (2004)
  - Andersen et al. (2006)
- **Applied studies stock markets:**
  - Andersen et al. (2001b)
  - Areal and Taylor (2002)
  - Thomakos and Wang (2003)
  - Chen et al. (2006)
- **Applied studies bond markets:**
  - Thomakos and Wang (2003)
- **Applied studies currency markets:**
  - Andersen et al. (2001a)
  - Dacorogna et al. (2001)
  - Thomakos and Wang (2003)
  - Chen et al. (2006)
- **Applied studies agricultural markets:**
  - Chen et al. (2006)
- **Applied studies energy markets:**
  - Wang et al. (2008) (Oil/Gas Futures)
  - Chan et al. (2008) (El spot)
  - Ulrich (2009) (El spot)
  - Haugom et al. (2009) (El futures)
  - Schulz (2010) (El futures)

Our study extend the univariate analysis of Nord Pool Electricity Futures by Haugom et al. (2009) and Schulz (2010) to a multivariate analysis.

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The concept of realized volatility and correlation

Assume the multivariate set of log-electricity futures prices are governed by a jump-diffusion process:

$$dp(t) = \mu(t)dt + \Omega(t)dw(t) + K(t)dq(t)$$

where the drift, **μ(t)**, is N dimensional vector process, the instantaneous volatility, **Ω(t)**, is a N X N matrix such that **Σ(t)= Ω(t)Ω'(t)** is the covariance matrix process of the continuous sample path component and **dw(t)** is a vector of N independent Brownian incremental motions. Further, **K(t)** is the N x N process controlling for the magnitude and transmission of jumps, and **dq(t)** is the N dimensional incremental jump-counting process.

The concept of realized volatility and correlation

The total variation of the vector log price process can be approximated by realised covariance (RCov). (given equally spaced return intervals and many of these during each day).

$$dp(t) = \mu(t)ds + \Omega(t)dw(t) + k(t)dq(t)$$

The continous covariance part.

The jump/cojump or discontinuos part.

While it is trivial to decompose these components for the case of stocks/fx (using bi-power covariance or variants of outlyingness weighted measures), more complexity arises for electricity futures data that have longer periods between trades and many intra-daily zero returns.

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The concept of realized volatility and correlation

The approximation measures **realised covariance (RCov)**, **realised variance (RVar)**, and **realised correlation (RCorr)** of the log-price process for day t are (1: Quarter contract 2: Year contract. M= Return periods each day.)

$$RCov_{1,2t} = \sum_{i=1}^{M} r_{1,t,i} r_{2,t,i}$$

$$RVar_{1,t} = \sum_{i=1}^{M} r^2_{1,t,i}$$

$$RVar_{2,t} = \sum_{i=1}^{M} r^2_{2,t,i}$$

And thus **realised volatility (RV)** and **realised correlation (RCorr)** on day t are:

$$RV_{1,t} = \sqrt{RVar_{1,t}}$$

$$RV_{2,t} = \sqrt{RVar_{2,t}}$$

$$RCorr_{1,2,t} = \frac{RCov_{1,2t}}{RV_{1,t} RV_{2,t}}$$

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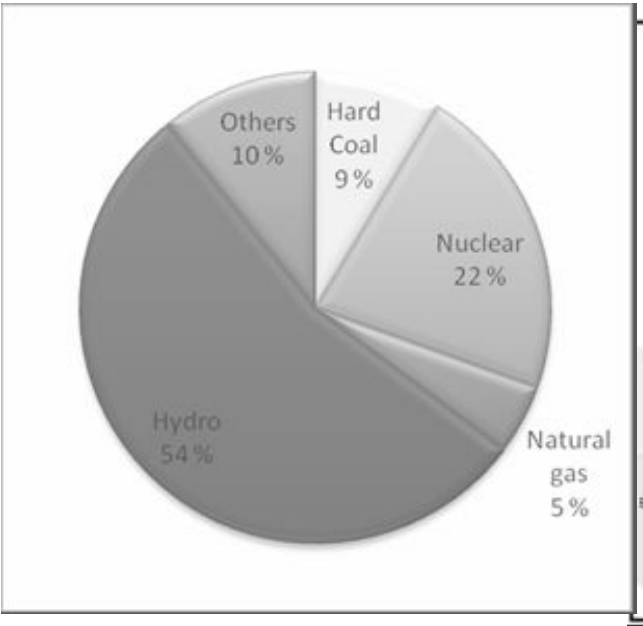
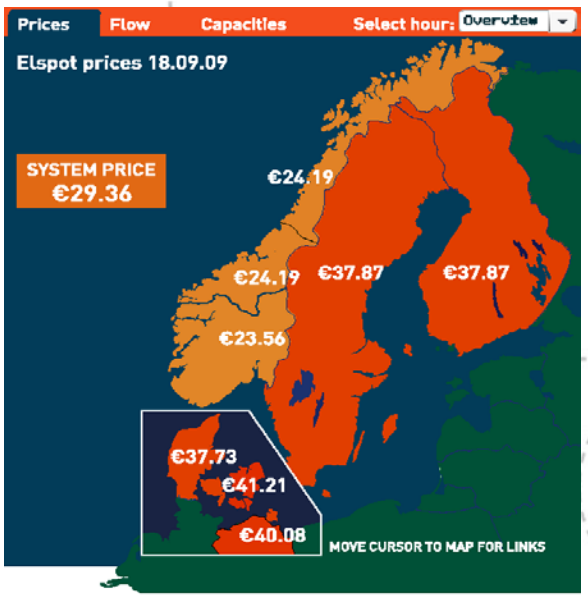
We look at the distributional properties and temporal dependencies for RV and RCorr:

- We calculate realised volatility and correlation for the front quarter and front year electricity futures contract at Nord Pool.

- All estimates in this study were performed using the Oxmetrics™ procedure called RE@LIZED. See Laurent (2009) for details.

Nord Pool electricity spot and futures prices





- Yearly production: approx. 400 TWh
- Covering Norway, Sweden, Denmark, Finland, parts of north east Germany.
- Input: Hydro, Nuclear, Coal, Natural Gas, and Wind.
- Day ahead bidding market – Several areas with 24 hourly prices

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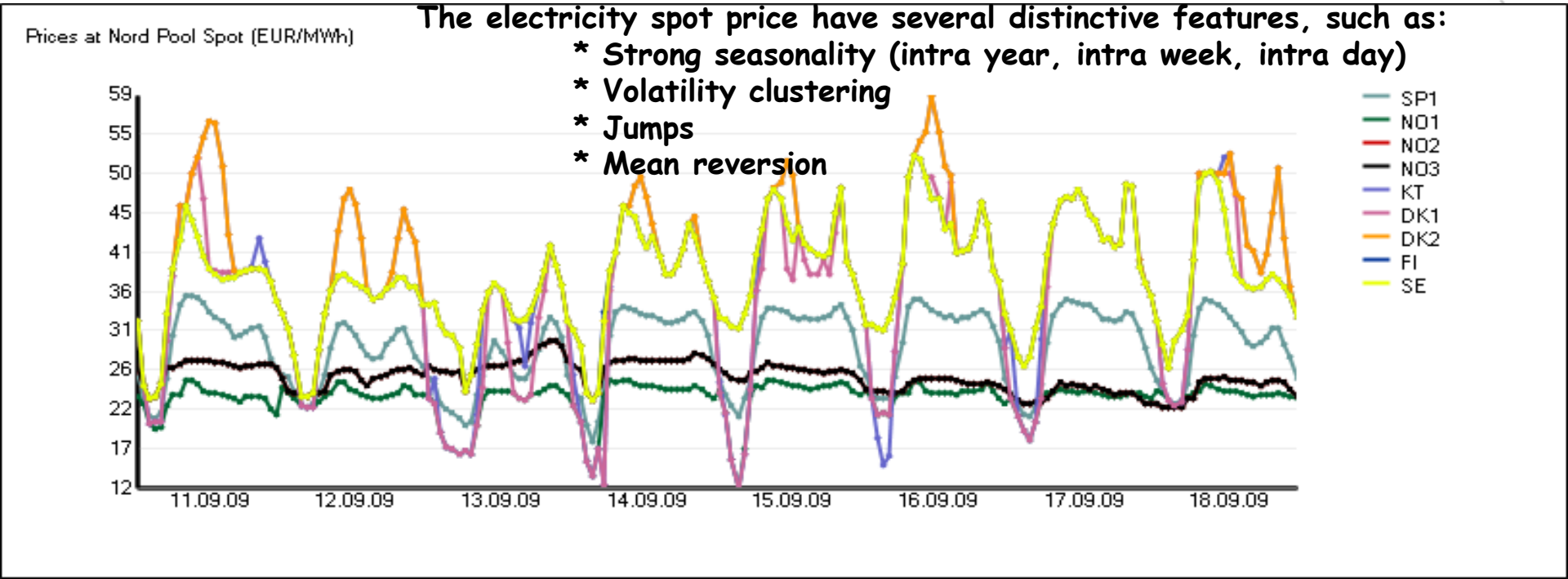
Nord Pool electricity spot prices

- A **daily system price** is calculated across hours and areas in Nord Pool. This price is a "technical" price assuming no capacity constraints in the system.
- All forward and future contracts at Nord Pool has the daily system price as the underlying.
- More info, see www.nordpool.com



Nord Pool electricity forward and future prices

Forwards

Month										
ENOMOCT-09	745	32.40	32.50	32.40	-0.20	36.0	32.75	32.40	-	1426
ENOMNOV-09	720	34.25	35.00	34.70	-0.15	4.0	34.85	34.70	-	777
ENOMDEC-09	744	35.60	35.90	35.65	-0.20	19.0	36.00	35.65	-	1055
ENOMJAN-10	744	37.00	37.50	37.45	-0.13	2.0	37.45	37.45	-	418
ENOMFEB-10	672	37.90	38.00	-		-	-	-	-	110
ENOMMAR-10	743	33.60	35.60	-		-	-	-	-	3
ENOPLMOCT-09	264	36.50	40.25	-		-	-	-	-	0
ENOPLMNOV-09	252	36.25	42.25	-		-	-	-	-	0
Quarter										
ENOQ4-09	2209	34.25	34.30	34.30	-0.15	472.0	34.75	34.15	-	11217
ENOQ1-10	2159	36.65	36.80	36.70	-0.20	9.0	36.80	36.70	-	4150
ENOQ2-10	2184	32.50	32.85	32.75	-0.05	15.0	32.75	32.60	-	1490
ENOQ3-10	2208	32.75	33.10	32.85	0.05	7.0	32.85	32.75	-	1796
ENOQ4-10	2209	38.40	39.00	-		-	-	-	-	1793
ENOQ1-11	2159	41.50	41.95	-		-	-	-	-	1171
ENOQ2-11	2184	34.70	35.25	-		-	-	-	-	222
ENOQ3-11	2208	34.30	35.00	-		-	-	-	-	288
ENOQ4-11	2209	40.50	41.50	41.20	-0.15	1.0	41.20	41.20	-	473
ENOPLQ4-09	792	35.75	41.75	-		-	-	-	-	33
ENOPLQ1-10	768	38.25	44.25	-		-	-	-	-	0
ENOPLQ2-10	780	34.00	40.00	-		-	-	-	-	0
Year										
ENOYR-10	8760	35.30	35.40	35.40	0.05	93.0	35.50	35.15	-	9228
ENOYR-11	8760	38.11	38.35	38.15	-0.10	1.0	38.15	38.15	-	5665
ENOYR-12	8784	40.30	40.80	-		-	-	-	-	2712
ENOYR-13	8760	43.50	44.30	-		-	-	-	-	1167
ENOYR-14	8760	45.50	46.00	-		-	-	-	-	370
ENOPLYR-10	3132	35.75	41.75	-		-	-	-	-	30

- **Weekly (futures), Monthly (futures), Quarterly (futures), Yearly (forwards) contracts. Euro/MWh. *A* given amount of electricity delivered each day in the periods at a fixed price.**

- **Continuously trading forward/futures contracts (open from 08.00 – 15.30 Monday to Friday.**

- **Nearest quarter and year contracts most liquid. We look at these contracts in this study. Contracts are assumed rolled over at maturity.**

- **There are also options on these forwards/futures (not looked at here)**

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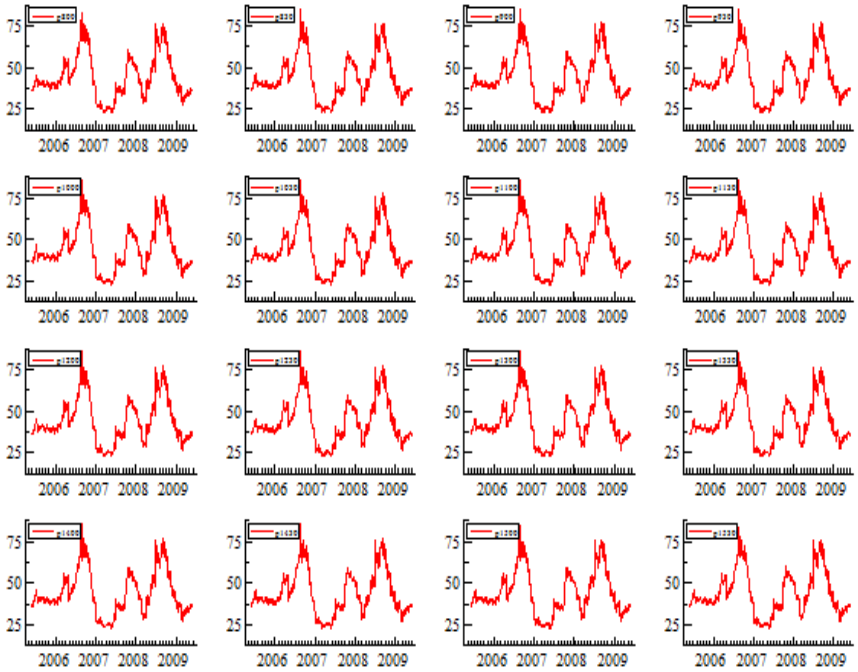
Data

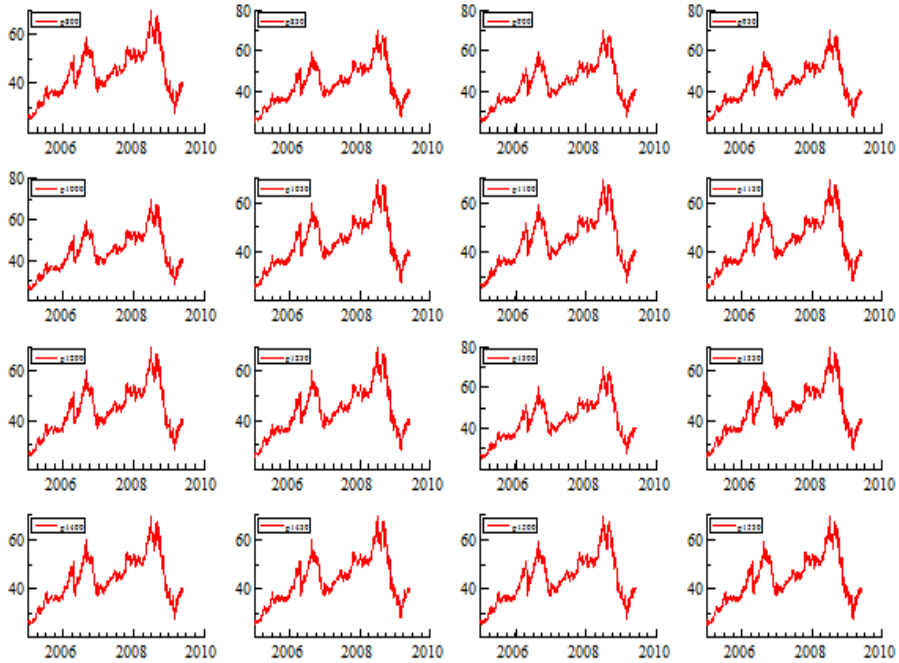
The data consist of intraday prices for two financial contracts (16000 observations from 1June 2005 to 29May2009):

1) One-quarter ahead futures prices.
2) One-year ahead forward prices.
3) Prices are registered as the nearest traded price to each half-hour (800, 830, 900,.....,1530)

Quarterly contracts



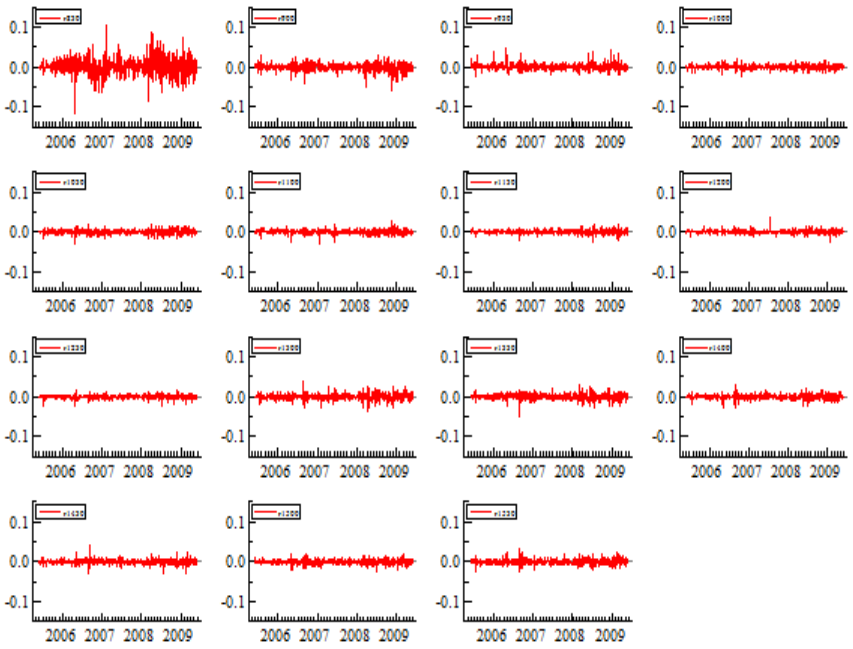
Yearly contracts



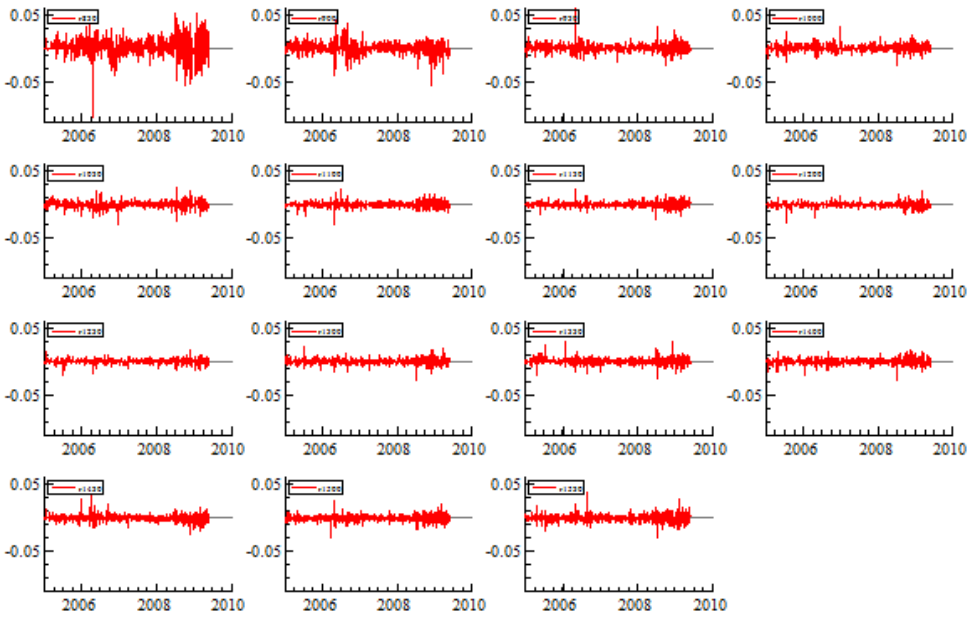
Data

Based on these prices, half-hourly returns for each contract are calculated (830-800, 900-830....,1530-1500).

Quarterly contracts



Yearly contracts



Data

Descriptive statistics half-hourly returns for each contract. Realised volatility and correlation are calculated based on these data.

Quarterly	Hours	N.Obs	Mean	Median	Min.	Max.	Std.Dev.	Skew.	Kurt.
	8:30	995	0.01 %	0.00 %	-11.83 %	10.61 %	1.87 %	0.06	4.86
	9:00	995	-0.10 %	0.00 %	-5.95 %	2.92 %	0.81 %	-1.11	6.42
	9:30	995	0.04 %	0.00 %	-2.21 %	4.81 %	0.66 %	1.49	9.56
	10:00	995	-0.01 %	0.00 %	-2.36 %	2.17 %	0.48 %	-0.17	2.40
	10:30	995	-0.04 %	0.00 %	-3.02 %	1.94 %	0.47 %	-0.19	3.79
	11:00	995	-0.01 %	0.00 %	-2.82 %	2.85 %	0.46 %	-0.38	6.48
	11:30	995	0.01 %	0.00 %	-2.55 %	1.73 %	0.38 %	-0.52	6.02
	12:00	995	0.01 %	0.00 %	-2.67 %	3.69 %	0.38 %	0.52	13.35
	12:30	995	-0.02 %	0.00 %	-2.42 %	1.72 %	0.37 %	-0.86	7.14
	13:00	995	0.01 %	0.00 %	-3.49 %	3.97 %	0.67 %	-0.06	4.39
	13:30	995	-0.01 %	0.00 %	-4.95 %	2.94 %	0.62 %	-0.49	6.25
	14:00	995	0.00 %	0.00 %	-2.71 %	2.86 %	0.51 %	-0.13	5.34
	14:30	995	-0.02 %	0.00 %	-3.06 %	4.14 %	0.51 %	0.22	8.67
	15:00	995	-0.01 %	0.00 %	-2.61 %	2.14 %	0.48 %	-0.25	3.06
	15:30	995	0.05 %	0.00 %	-2.58 %	3.09 %	0.57 %	0.24	3.17
Yearly	Hours	N.Obs	Mean	Median	Min.	Max.	Std.Dev.	Skew.	Kurt.
	8:30	995	0.03 %	0.00 %	-10.24 %	5.23 %	1.28 %	-0.85	7.26
	9:00	995	-0.05 %	0.00 %	-5.61 %	4.11 %	0.72 %	-0.94	9.67
	9:30	995	0.01 %	0.00 %	-2.51 %	5.81 %	0.53 %	1.37	17.25
	10:00	995	0.00 %	0.00 %	-2.64 %	2.31 %	0.43 %	-0.06	5.88
	10:30	995	-0.01 %	0.00 %	-2.41 %	2.45 %	0.42 %	0.13	5.76
	11:00	995	0.02 %	0.00 %	-3.05 %	2.36 %	0.37 %	-0.09	8.92
	11:30	995	0.02 %	0.00 %	-2.18 %	2.35 %	0.34 %	0.33	7.13
	12:00	995	-0.01 %	0.00 %	-2.67 %	1.95 %	0.301 %	-0.73	13.10
	12:30	995	0.00 %	0.00 %	-2.13 %	1.76 %	0.295 %	-1.05	8.49
	13:00	995	0.00 %	0.00 %	-2.84 %	2.29 %	0.37 %	-0.27	8.26
	13:30	995	0.00 %	0.00 %	-2.67 %	2.94 %	0.41 %	0.36	9.45
	14:00	995	0.00 %	0.00 %	-2.87 %	1.77 %	0.38 %	-0.51	6.34
	14:30	995	0.00 %	0.00 %	-2.34 %	3.49 %	0.42 %	0.56	10.15
	15:00	995	-0.01 %	0.00 %	-2.94 %	2.51 %	0.40 %	-0.24	6.91
	15:30	995	0.02 %	0.00 %	-2.86 %	3.94 %	0.49 %	0.46	7.17

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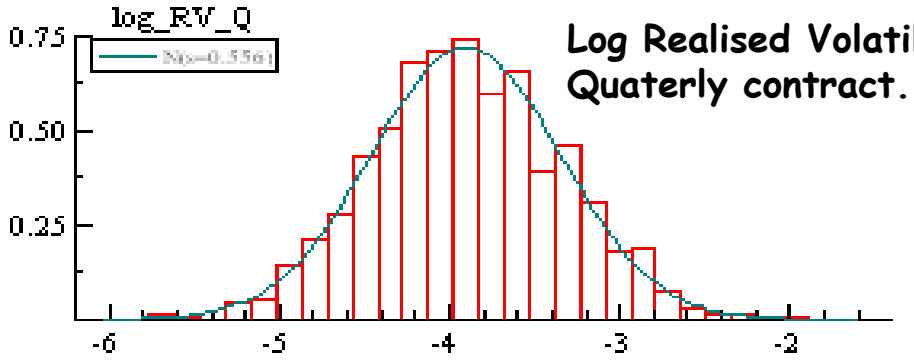
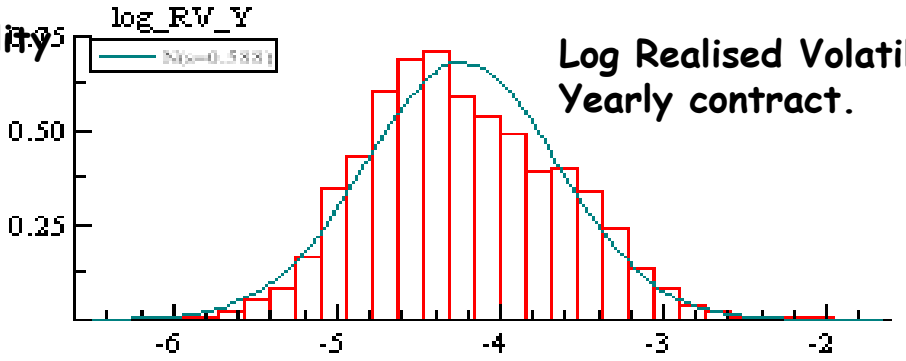
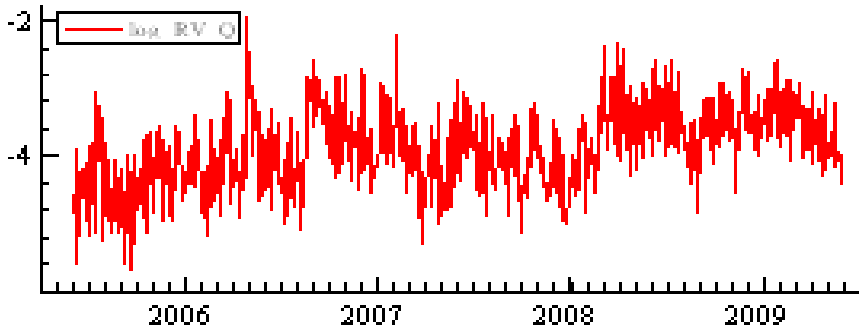
Distributional properties and time series properties

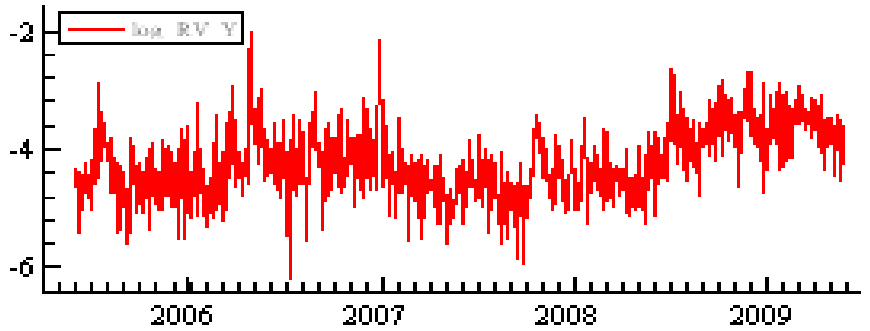


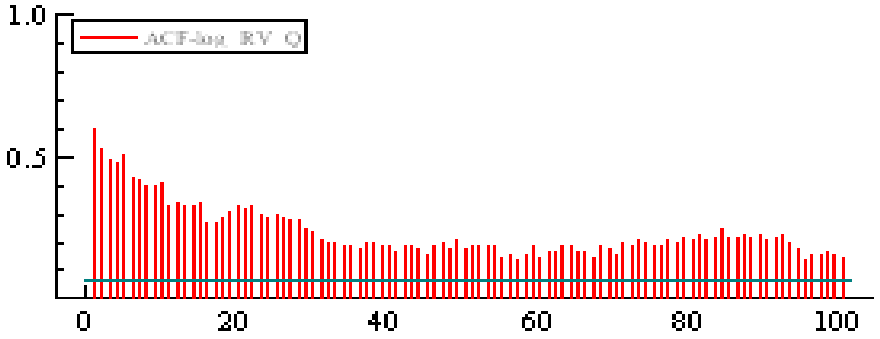
Log Realised Volatility Quaterly contract.

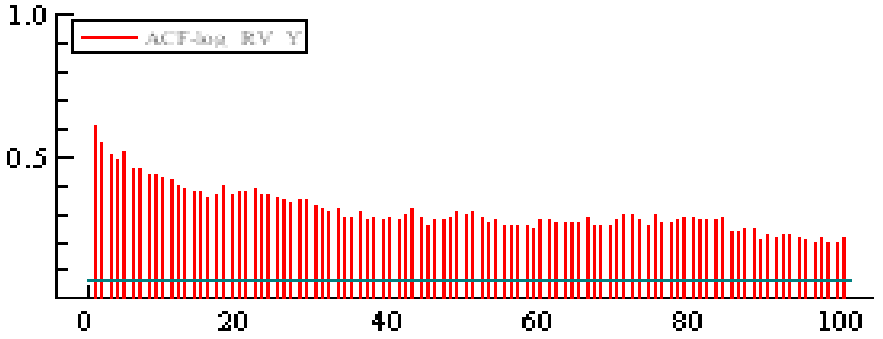


Log Realised Volatility Yearly contract.

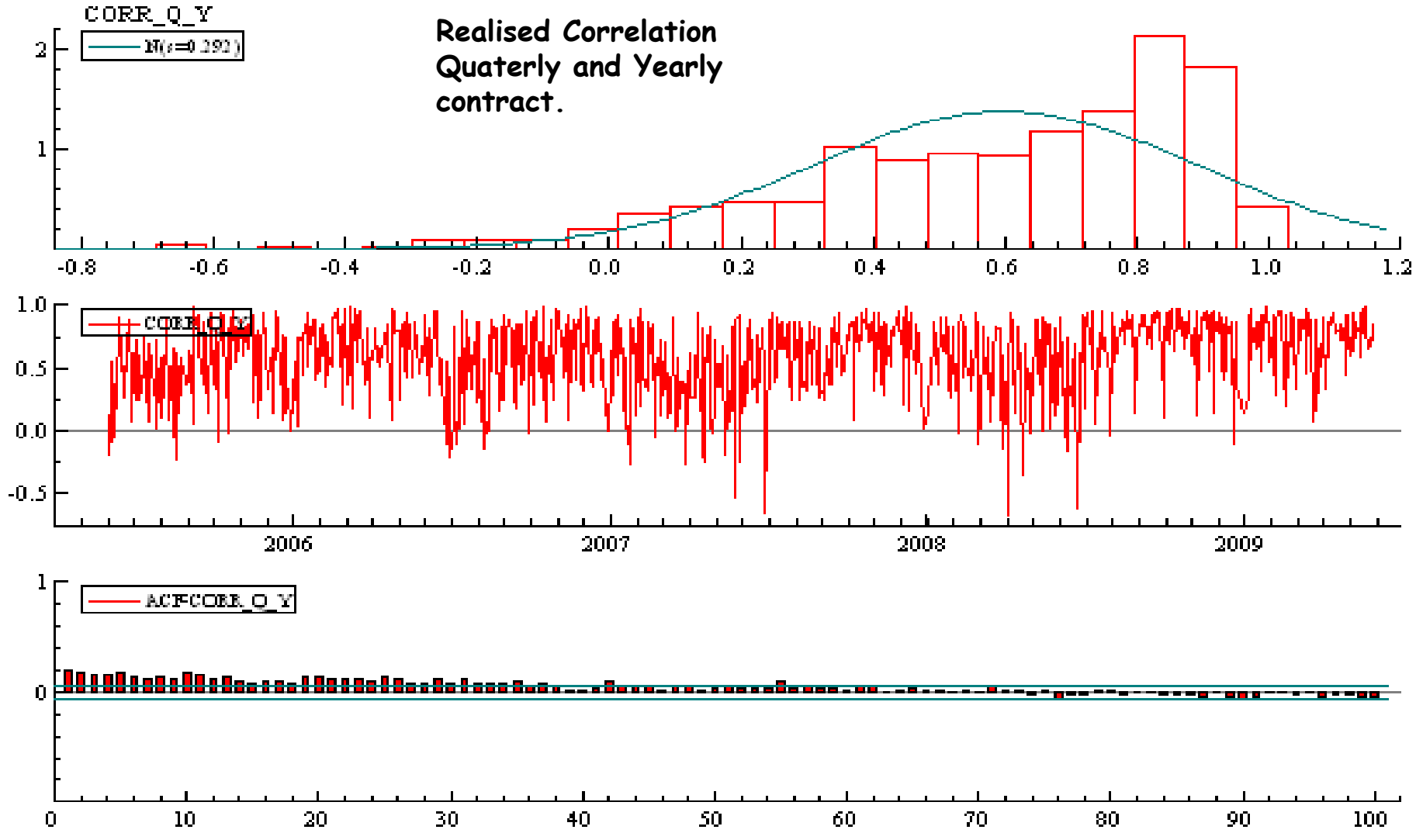








Distributional properties and time series properties



Concluding comments

The overall main findings show that:

1. The logarithmic realized volatility are approximately normal distributed, while realized correlation seems not.
2. Realized volatility has a long memory feature. Can be modelled by an ARFIMA-X model with normally distributed error terms.
3. High correlation between realized correlation and volatilities.

4. We also find that trading volume have a significant positive influence on realised volatility and realised correlation.

These 4 results above are to a large extent consistent with earlier similar stylized facts studies of other financial and commodity markets.

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Concluding comments

The overall main findings show that:

- Opposed to the study of crude oil and natural gas by Wang et al. (2008), we also found some indication of long memory pattern in realised correlation.

- This implies that realised correlation could be modelled by an ARFIMA-X with non-normally distributed error terms.

Further Research

Ongoing research involves:

1. Build time series models (ARFIMA-X) w.r.t. determine and forecast realised volatility and correlation.
2. Building dynamic Quantile regression models with explanatory variables to determine and forecast the distribution of realised volatility and correlation.
3. Compare (and combine) prediction models of realised volatility/correlation with
   - Multivariate GARCH models.
   - Multivariate Stochastic Volatility Models.
   - Implied volatility/correlation from the energy futures option.

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