LIQUIDITY RISKS ON POWER EXCHANGE

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Liquidity is a key variable for assets manager

- It enlarges the capacity of the market to accommodate order flows
- It guarantees the ability to quickly buy/sell sufficient quantities of an asset without significantly affecting its price
- Portfolios can easily be converted into cash

Two important issues about liquidity

- How to measure liquidity: unobserved variable that embeds several dimension (volume, depth, resiliency,...)
- Effect on the pricing of financial contract
Energy trading on power exchanges:

- Exceptional volatility of electricity makes derivatives particularly relevant
- Even though volumes are increasing, the market is still less liquid than for other commodities

Review and analysis of EU wholesale energy markets (source: DG-TREND)

- Regardless of liquidity problems, the pricing of financial power derivatives is challenging
We analyze illiquidity through equilibrium based models

- Agents (risk-averse) want to hedge their stochastic profit
  \[ \Rightarrow \text{Prices are determined by an equilibrium among players} \]
  \[ \Rightarrow \text{Optimal position in financial contracts} \]

- Methodology widely used
  \[ \Rightarrow \text{Impact of power derivatives on investment (Willems and Morbee, 2008)} \]
  \[ \Rightarrow \text{Extension to dynamic equilibrium (Büller and Müller-Merbach, 2008)} \]
  \[ \Rightarrow \text{Pricing of weather derivatives in a multi-commodity setting (Lee and Oren, 2008)} \]
  \[ \Rightarrow \text{...} \]

- All research concludes to very high hedge ratios

Perspective of the analysis

- Insufficient liquidity restricts the construction of portfolios
  \[ \Rightarrow \text{liquidity constraints in agents optimization} \]
Two stage forward equilibrium

- \((\Omega, \mathcal{P})\) finite probability space
- \(N\) : set of players (producer, retailer,..)
- \(C\) : set of financial contracts (futures, options, FTRs,...)

- The seller of a contract \(c\) gets the following pay-off \((P^f_c - P^s_{c,\omega})\).
- \(\pi^{\text{spot}}_{\nu,\omega}\) spot profit (at \(t = 1\)) of market participant \(\nu\)
- Agent hedges profit by concluding financial contracts

\[
\Pi_{\nu,\omega} = \sum_c x^\nu_c (P^f_c - P^s_{c,\omega}) + \pi^{\text{spot}}_{\nu,\omega}
\]
Agents hedging optimization:
- We assume perfect competition ⇒ price taking agents.
  - The spot market equilibrium is not influenced by the forward equilibrium
  - Financial contracts are pure hedge tools
  - Strategic behavior: stochastic equilibrium program with equilibrium constraints (Zhang, Xu and Wu, 2008)

- We model risk aversion by risk function $\rho(X): \mathcal{Z} \to \mathbb{R}$
  - ex: mean-variance, exponential utility function, VaR, CVaR, ...

**Optimal Hedging in a Liquid Market**

$$\mathcal{P}_\nu(P^f_c) \equiv \max_{x^\nu_c} \rho_\nu(\Pi_\nu)$$

$$\Pi_{\nu, \omega} = \sum_c x^\nu_c (P^f_c - P^s_{c, \omega}) + \pi^{\text{spot}}_{\nu, \omega}$$
**Forward Equilibrium**

**Equilibrium in a liquid market**

A forward equilibrium, is a tuple \([(x^\nu_c)^{\nu=1}_{\nu}, (P^f_c)^{C=1}_{c=1}]\) such that:

- \(\forall \nu \in N, x^\nu_c\) is an optimal solution of \(\mathcal{P}^\nu(P^f_c)\)

- it satisfies the market clearing conditions: \(\forall c \in C, \sum_{\nu \in N} x^\nu_c = 0\)

Existence: compactness of strategies
- yes, if monetary concave risk function

Uniqueness?: strong monotonicity in the gradient map of the risk function

- All research concludes to high hedge ratio (ex: Bessembinder-Lemon: from 0.7 to 1.2)
- Such level of trades have never been observed
- Some contracts are known to be illiquid (ex: explicit auctions)
Liquidity modeling

- We model the liquidity on the basis of the total volume traded
- We impose some liquidity bounds

\[ \sum_{\nu \in N} |x^\nu_c| \leq LIQ_c \]

- Insufficient liquidity restricts the construction of the agent’s portfolio

Optimal hedging in a market with liquidity bounds

\[ P^\nu(P^f_c, x^-\nu_c) \equiv \max_{x^\nu_c} \rho_\nu(\Pi_\nu) \]
\[ \Pi_{\nu,\omega} = \sum_c x^\nu_c (P^f_c - P^s_{c,\omega}) + \pi^{\text{spot}}_{\nu,\omega} \]
\[ |x^\nu_c| + \sum_{-\nu} |x^-\nu_c| \leq LIQ_c \]

◊ The agent’s hedging problem depends on the strategies of other players \( x^-\nu_c \)

◊ The problem becomes a Generalized Nash Equilibrium Problem (GNEP) with shared constraints
FORWARD EQUILIBRIUM WITH LIQUIDITY CONSTRAINTS

GENERALIZED NASH EQUILIBRIUM IN AN ILLIQUID MARKET

A forward equilibrium, is a tuple \([(x^\nu_c)_{\nu=1}^N, (P_c^f)_{c=1}^C]\) such that:

- \(\forall \nu \in N, x^\nu_c\) is an optimal solution of \(\mathcal{P}^\nu(P^f_c, x^{\neg \nu}_c)\)
- it satisfies the market clearing conditions: \(\forall c \in C, \sum_{\nu \in N} x^\nu_c = 0\)

- GNEP may have multiple, possibly infinite solution (continuum)
- Concept criticized by economist for a meaningful game
- Insufficient liquidity is a market failure (as externalities)
- Practically, find a large set of equilibria to illustrate the type of inefficiency arising from illiquidity
- Numerically, heuristic have been developed recently (Fukushima, 2008)
Liquidity, Concavity and Arbitrage

In a **liquid** market, no arbitrage is guaranteed in the equilibrium solution if:

1. Risk aversion is modeled through concave risk function (i.e. concavity, monotonicity, cash invariance)

   Any concave risk function can be represented as (Föllmer et al., 2002):
   \[
   \rho_\nu(\Pi_\nu) = \inf_{Q \in \mathcal{P}} \left( E_Q[\Pi_\nu] + \alpha(Q) \right)
   \]

2. The investor’s problem becomes

   \[
   \mathcal{P}_\nu \equiv \max_{x^\nu_c \in \mathbb{R}^c} \left\{ \inf_{Q \in \mathcal{P}} E_Q[\pi^\text{spot}_\nu] + \sum_c x^\nu_c (P^f_c - E_Q[P^s_c]) + \alpha(Q) \right\}
   \]

3. By duality theory, it can be restated as

   \[
   \mathcal{P}_\nu \equiv \min_{Q \in \mathcal{P}} \left( E_Q[\pi^\text{spot}_\nu] + \alpha(Q) \right)
   \]
   \[
   \text{s.t.} \quad P^f_c = E_Q[P^s_c] \quad (x^\nu_c)
   \]

4. \( Q \) is equivalent to \( \mathbb{P} \) (share the same set of measure zero)
- In an **illiquid** market, the equilibrium solution may contain arbitrage, even if agents have concave risk function:

\[
P^{\nu}(P_c^f, x_c^{-\nu}) \equiv \min_{\lambda_c^{\nu} \geq 0} \max_{\mu_c^{\nu} \geq 0} \left\{ \inf_{Q \in \mathcal{P}} \mathbb{E}_Q[\pi^{\text{spot}}] + \alpha(Q) \right. \\
+ \sum_{c \in C} (x_c^{\nu} \mathbb{E}_Q[P_c^f - P_s^c] + \lambda_c^{\nu} (LIQ_c - x_c^{\nu} - |x_c^{-\nu}|)) \\
\left. + \mu_c^{\nu} (LIQ_c + x_c^{\nu} - |x_c^{-\nu}|) \right\}
\]

- The optimality conditions give \( P_c^f = \mathbb{E}_Q[P_c^s] + \lambda_c^{\nu} - \mu_c^{\nu} \)
- Perfectly competitive environment; Organized as a stylized US-like market

- Market participants:
  - Producers: unlimited capacity, bid their marginal cost of supply $C_{\nu}$
    \[
    C_{\nu}^T(q_{\nu}) = a_{\nu}q_{\nu} + b_{\nu}\frac{q_{\nu}^2}{2} ; \quad C_{\nu}(q_{\nu}) = a_{\nu} + b_{\nu}q_{\nu}
    \]
  - Retailers: serve consumers at a fixed price, bid their inverse demand function $P_{\nu}(q_{\nu})$
    \[
    P_{\nu}(q_{\nu}) = a_{\nu} - b_{\nu}q_{\nu}
    \]
  - System Operator collects the bids and maximizes total Welfare leading to
    \[
    \max_{q_{\nu} \in \mathbb{R}^N_+} \left[ \sum_{\nu \in N_r} \int_0^{q_{\nu}} P_{\nu}(\xi_{\nu})d\xi_{\nu} - \sum_{\nu \in N_p} \int_0^{q_{\nu}} C_{\nu}(\xi_{\nu})d\xi_{\nu} \right]
    \]
    s.t. \[
    \sum_{\nu \in N} q_{\nu} = 0
    \]
    \[
    -K_{\ell} \leq \sum_{\nu \in N} \text{PTDF}_{\nu,\ell} q_{\nu} \leq K_{\ell}
    \]
- Uncertainty and spot scenarios (#250)
  - Demand sensitive to weather variation ($\alpha_\nu$)
  - Transmission line outage (line 1-6)
  - Gas, coal, and carbon emission prices (not treated here)

- Risk exposure:

<table>
<thead>
<tr>
<th></th>
<th>$\mathbb{E}[\pi_\nu^{\text{spot}}]$</th>
<th>vol($\pi_\nu^{\text{spot}}$)</th>
<th>CVaR$_{75%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2197</td>
<td>11%</td>
<td>1517</td>
</tr>
<tr>
<td>4</td>
<td>652</td>
<td>75%</td>
<td>-83</td>
</tr>
<tr>
<td>6</td>
<td>1979</td>
<td>87%</td>
<td>-309</td>
</tr>
</tbody>
</table>

- Price (nodal & transmission)

<table>
<thead>
<tr>
<th></th>
<th>$\mathbb{E}[P^s]$</th>
<th>$\text{Var}[P^s]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.72</td>
<td>3.08</td>
</tr>
<tr>
<td>6</td>
<td>53.05</td>
<td>76.7</td>
</tr>
<tr>
<td>1→6</td>
<td>28.3</td>
<td>85.2</td>
</tr>
</tbody>
</table>
- Type of derivatives: Energy futures and FTRs

- Risk function used:
  \[ \rho_\nu(\Pi_\nu) = (1 - \beta) \mathbb{E}[\Pi_\nu] + \beta \text{CVaR}_\alpha(\Pi_\nu) \]

- Important reduction of risk

<table>
<thead>
<tr>
<th></th>
<th>( \mathbb{E}[\pi^{\text{spot}}] )</th>
<th>vol(( \pi^{\text{spot}} ))</th>
<th>( \mathbb{E}[\Pi] )</th>
<th>vol(( \Pi ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2197</td>
<td>11%</td>
<td>2198</td>
<td>1.8%</td>
</tr>
<tr>
<td>6</td>
<td>1979</td>
<td>87%</td>
<td>1890</td>
<td>48%</td>
</tr>
</tbody>
</table>

Agents hedge ratio:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedge ratio</td>
<td>0.92</td>
<td>0.82</td>
<td>0.26</td>
<td>0.63</td>
<td>0.6</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Illustration: illiquid market

- We impose liquidity constraints on energy futures and on total volume of FTRs
  - 66% of expected spot quantities for the FUTURE
  - Total of available network capacities for FTRs
- For illustrating the impact of liquidity constraints, we aim at finding the largest set of equilibria (# 5000)

◊ Range of risk premia:

<table>
<thead>
<tr>
<th></th>
<th>$P^f_c - \mathbb{E}[P^s_c]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUTURE 6</td>
<td>$[-1.85, 0.65]$</td>
</tr>
<tr>
<td>FTR 1→6</td>
<td>$[-1.52, 0.78]$</td>
</tr>
</tbody>
</table>

◊ Range of profit distribution

<table>
<thead>
<tr>
<th></th>
<th>$\mathbb{E}[\Pi_\nu]$</th>
<th>$\text{vol}(\Pi_\nu)$</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[2116, 2226]</td>
<td>[1.8%, 22%]</td>
<td>[1, 712]</td>
</tr>
<tr>
<td>6</td>
<td>[1860, 2502]</td>
<td>[41%, 48%]</td>
<td>[113, 677]</td>
</tr>
</tbody>
</table>
Illustration: liquidity in FTR and future market

- Insufficient liquidity in the FTR market impacts the futures market

<table>
<thead>
<tr>
<th>Volume FTRs</th>
<th>Volume Futures</th>
<th>$P_c^f$ (FUTURE 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>960</td>
<td>570 (92%)</td>
<td>[52.9, 53.6]</td>
</tr>
<tr>
<td>720</td>
<td>478 (69%)</td>
<td>[52.9, 53.7]</td>
</tr>
<tr>
<td>480</td>
<td>320 (56%)</td>
<td>[52.9, 53.8]</td>
</tr>
<tr>
<td>240</td>
<td>269 (39%)</td>
<td>[53.5, 54.0]</td>
</tr>
<tr>
<td>0</td>
<td>152 (22%)</td>
<td>54.8</td>
</tr>
</tbody>
</table>

**Table:** Induced energy futures volume for a given liquidity bounds on FTRs