# LIQUIDITY RISKS ON POWER EXCHANGE

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# Introduction

Liquidity is a key variable for assets manager

- It enlarges the capacity of the market to accommodate order flows
- It guarantees the ability to quickly buy/sell sufficient quantities of an asset without significantly affecting its price
- Portfolios can easily be converted into cash

Two important issues about liquidity

- How to measure liquidity : unobserved variable that embeds several dimension ( volume, depth, resiliency,...)
- Effect on the pricing of financial contract

Energy trading on power exchanges:

- Exceptional volatility of electricity makes derivatives particularly relevant
- Even though volumes are increasing, the market is still less liquid than for other commodities


**Review and analysis of EU wholesale energy markets (source: DG-TREND)**

- Regardless of liquidity problems, the pricing of financial power derivatives is challenging

We analyze illiquidity through equilibrium based models

- Pioneer paper of Bessembinder and Lemon(2002)
- Agents (risk-averse) want to hedge their stochastic profit
  - => Prices are determined by an equilibrium among players
  - => Optimal position in financial contracts
- Methodology widely used
  - => Impact of power derivatives on investment (Willems and Morbee, 2008)
  - => Extension to dynamic equilibrium (Büller and Müller-Merbach, 2008)
  - => Pricing of weather derivatives in a multi-commodity setting (Lee and Oren, 2008)
  - => ...
- All research concludes to very high hedge ratios

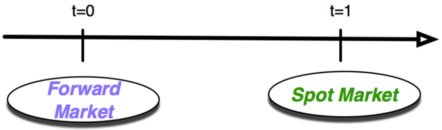
Perspective of the analyzis

- Insufficient liquidity restricts the construction of portfolios

$\Rightarrow$ liquidity constraints in agents optimization

Two stage forward equilibrium

- $(\Omega, \mathcal{P})$ finite probability space
- $N$ : set of players (producer, retailer,..)
- $C$: set of financial contracts (futures, options, FTRs,...)




- The seller of a contract $c$ gets the following pay-off $(P_c^f - P_{c,\omega}^s)$.
- $\pi_{\nu,\omega}^{\mathtt{spot}}$ spot profit (at $t = 1$) of market participant $\nu$
- Agent hedges profit by concluding financial contracts

$$\Pi_{\nu,\omega} = \sum_c x_c^\nu (P_c^f - P_{c,\omega}^s) + \pi_{\nu,\omega}^{\mathtt{spot}}$$

Agents hedging optimization :

- We assume perfect competition $\Rightarrow$ price taking agents.
  - The spot market equilibrium is not influenced by the forward equilibrium
  - Financial contracts are pure hedge tools
  - Strategic behavior: stochastic equilibrium program with equilibrium constraints (Zhang, Xu and Wu, 2008)
- We model risk aversion by risk function $\rho(X) : \mathcal{Z} \rightarrow \mathbb{R}$
  - ex: mean-variance, exponential utility function, VaR, CVaR, ...

### Optimal hedging in a liquid market

$$\mathcal{P}^{\nu}(P_c^f) \equiv \max_{x_c^{\nu}} \; \rho_{\nu}(\Pi_{\nu})$$
$$\Pi_{\nu,\omega} = \sum_c x_c^{\nu}(P_c^f - P_{c,\omega}^s) + \pi_{\nu,\omega}^{\texttt{spot}}$$

# Forward Equilibrium

## Equilibrium in a liquid market

A forward equilibrium, is a tuple $[(x_c^\nu)_{\nu=1}^N, (P_c^f)_{c=1}^C]$ such that:

- $\forall \nu \in N, x_c^\nu$ is an optimal solution of $\mathcal{P}^\nu(P_c^f)$
- it satisfies the market clearing conditions: $\forall c \in C, \sum_{\nu \in N} x_c^\nu = 0$

Existence : compactness of strategies

- yes, if monetary concave risk function

Uniqueness? : strong monotonicity in the gradient map of the risk function

- All research concludes to high hedge ratio (ex: Bessembinder-Lemon: from 0.7 to 1.2)
- Such level of trades have never been observed
- Some contracts are known to be illiquid (ex: explicit auctions)

# Liquidity modeling

- We model the liquidity on the basis of the total volume traded
- We impose some liquidity bounds

$$\sum_{\nu \in N} |x_c^\nu| \leq LIQ_c$$

- Insufficient liquidity restricts the construction of the agent's portfolio

## Optimal hedging in a market with liquidity bounds

$$\mathcal{P}^\nu(P_c^f, x_c^{-\nu}) \equiv \begin{array}{l} \max_{x_c^\nu} \;\; \rho_\nu(\Pi_\nu) \\ \Pi_{\nu,\omega} = \sum_c x_c^\nu (P_c^f - P_{c,\omega}^s) + \pi_{\nu,\omega}^{\texttt{spot}} \\ |x_c^\nu| + \sum_{-\nu} |x_c^{-\nu}| \leq LIQ_c \end{array}$$

- ◇ The agent's hedging problem depends on the strategies of other players $x_c^{-\nu}$
- ◇ The problem becomes a Generalized Nash Equilibrium Problem (GNEP) whit shared constraints

# Forward Equilibrium with Liquidity constraints

## Generalized Nash Equilibrium in an illiquid market

A forward equilibrium, is a tuple $[(x_c^\nu)_{\nu=1}^N, (P_c^f)_{c=1}^C]$ such that:

- $\forall \nu \in N, x_c^\nu$ is an optimal solution of $\mathcal{P}^\nu(P_c^f, x_c^{-\nu})$
- it satisfies the market clearing conditions: $\forall c \in C, \sum_{\nu \in N} x_c^\nu = 0$

- GNEP may have multiple, possibly infinite solution (continuum)
- Concept criticized by economist for a meaningful game
- Insufficient liquidity is a market failure (as externalities)
- Practically, find a large set of equilibria to illustrate the type of inefficiency arising from illiquidity
- Numerically, heuristic have been developed recently (Fukushima, 2008)

In a **liquid** market, no arbitrage is guaranteed in the equilibrium solution if:

1. Risk aversion is modeled through concave risk function (i.e. concavity, monotonicity, cash invariance)
   - Any concave risk function can be represented as (Föllmer et al., 2002):

$$\rho_\nu(\Pi_\nu) = \inf_{\mathbb{Q}\in\mathcal{P}} \left(\mathbb{E}_\mathbb{Q}[\Pi_\nu] + \alpha(\mathbb{Q})\right)$$

   - The investor's problem becomes

$$\mathcal{P}^\nu \equiv \max_{x_c^\nu\in\mathbb{R}^c} \left\{ \inf_{\mathbb{Q}\in\mathcal{P}} \mathbb{E}_\mathbb{Q}[\pi_\nu^{\texttt{spot}}] + \sum_c x_c^\nu (P_c^f - \mathbb{E}_\mathbb{Q}[P_c^s]) + \alpha(\mathbb{Q}) \right\}$$

   - By duality theory, it can be restated as

$$\begin{aligned} \mathcal{P}^\nu \equiv \quad & \min_{\mathbb{Q}\in\mathcal{P}} \left(\mathbb{E}_\mathbb{Q}[\pi_\nu^{\texttt{spot}}] + \alpha(\mathbb{Q})\right) \\ \text{s.t.} \quad & P_c^f = \mathbb{E}_\mathbb{Q}[P_c^s] \qquad (x_c^\nu) \end{aligned}$$

2. $\mathbb{Q}$ is equivalent to $\mathbb{P}$ (share the same set of measure zero)

- In an **illiquid** market, the equilibrium solution may contain arbitrage, even if agents have concave risk function :

$$\mathcal{P}^{\nu}(P_c^f, x_c^{-\nu}) \equiv \min_{\substack{\lambda_c^{\nu}\geq 0 \\ \mu_c^{\nu}\geq 0}} \max_{x_c^{\nu}\in\mathbb{R}^c} \Big\{ \inf_{\mathbb{Q}\in\mathcal{P}} \mathbb{E}_{\mathbb{Q}}[\pi_{\nu}^{\texttt{spot}}] + \alpha(\mathbb{Q}) + \sum_{c\in C} \big( x_c^{\nu}\mathbb{E}_{\mathbb{Q}}[P_c^f - P_c^s] + \lambda_c^{\nu}(LIQ_c - x_c^{\nu} - |x_c^{-\nu}|) + \mu_c^{\nu}(LIQ_c + x_c^{\nu} - |x_c^{-\nu}|)\big) \Big\}$$

- The optimality conditions give $P_c^f = \mathbb{E}_{\mathbb{Q}}[P_c^s] + \lambda_c^{\nu} - \mu_c^{\nu}$

- Perfectly competitive environment ; Organized as a stylized US-like market
- Market participants:
  - Producers : unlimited capacity, bid their marginal cost of supply $C_\nu$

$$C_\nu^T(q_\nu) = a_\nu q_\nu + b_\nu \frac{q_\nu^2}{2} \quad ; \quad C_\nu(q_\nu) = a_\nu + b_\nu q_\nu$$

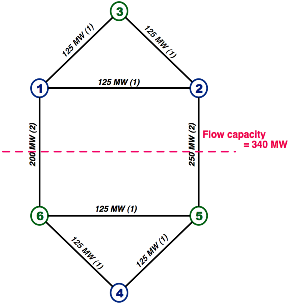
  - Retailers : serve consumers at a fixed price, bid their inverse demand function $P_\nu(q_\nu)$

$$P_\nu(q_\nu) = a_\nu - b_\nu q_\nu$$

  - System Operator collects the bids and maximizes total Welfare leading to

$$\begin{aligned} \max_{q_\nu \in \mathbb{R}_+^N} \quad & \left[ \sum_{\nu \in N_r} \int_0^{q_\nu} P_\nu(\xi_\nu) d\xi_\nu - \sum_{\nu \in N_p} \int_0^{q_\nu} C_\nu(\xi_\nu) d\xi_\nu \right] \\ \text{s.t.} \quad & \sum_{\nu \in N} q_\nu = 0 \\ & -K_\ell \leq \sum_{\nu \in N} \mathtt{PTDF}_{\nu,\ell} \; q_\nu \leq K_\ell \end{aligned} \tag{1}$$

- Uncertainty and spot scenarios (#250)
  - Demand sensitive to weather variation ($\alpha_\nu$)
  - Transmission line outage (line 1-6)
  - Gas, coal, and carbon emission prices (not treated here)



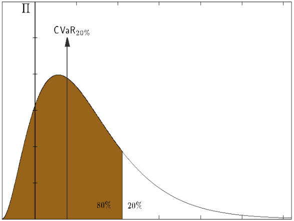
6 nodes example

- Risk exposure:

| | $\mathbb{E}\left[\pi_\nu^{spot}\right]$ | $\text{vol}\left(\pi_\nu^{spot}\right)$ | $\text{CVaR}_{75\%}$ |
|---|---|---|---|
| 1 | 2197 | 11% | 1517 |
| 4 | 652 | 75% | -83 |
| 6 | 1979 | 87% | -309 |

- Price (nodal & transmission)

| | $\mathbb{E}\left[P^s\right]$ | $\mathbb{V}\text{ar}\left[P^s\right]$ |
|---|---|---|
| 1 | 24.72 | 3.08 |
| 6 | 53.05 | 76.7 |
| 1→6 | 28.3 | 85.2 |



6 nodes example

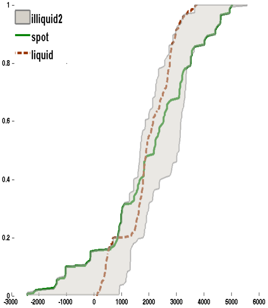
- Type of derivatives : Energy futures and FTRs
- Risk function used :
  $\rho_\nu(\Pi_\nu) = (1-\beta)\,\mathbb{E}\left[\Pi_\nu\right] + \beta\ \mathrm{CVaR}_\alpha\left(\Pi_\nu\right)$
- Important reduction of risk

| | $\mathbb{E}\left[\pi^{spot}\right]$ | $\mathrm{vol}\left(\pi^{spot}\right)$ | $\mathbb{E}\left[\Pi\right]$ | $\mathrm{vol}\left(\Pi\right)$ |
|---|---|---|---|---|
| **1** | 2197 | 11% | 2198 | 1.8% |
| **6** | 1979 | 87% | 1890 | 48% |

Agents hedge ratio :

| | **1** | **2** | **3** | **4** | **5** | **6** |
|---|---|---|---|---|---|---|
| Hedge ratio | 0.92 | 0.82 | 0.26 | 0.63 | 0.6 | 1.4 |

- We impose liquidity constraints on energy futures and on total volume of FTRs
  - 66% of expected spot quantities for the FUTURE
  - Total of available network capacities for FTRs
- For illustrating the impact of liquidity constraints, we aim at finding the largest set of equilibria (# 5000)




◇ Range of risk premia:

| | $P_c^f - \mathbb{E}[P_c^s]$ |
|---|---|
| FUTURE 6 | $[-1.85\ ,\ 0.65]$ |
| FTR 1→6 | $[-1.52\ ,\ 0.78]$ |

◇ Range of profit distribution

| | $\mathbb{E}[\Pi_\nu]$ | vol$(\Pi_\nu)$ | Volume |
|---|---|---|---|
| **1** | [2116 , 2226] | [1.8% , 22%] | [1 , 712] |
| **6** | [1860 , 2502] | [41% , 48%] | [113 , 677] |

# Illustration: liquidity in FTR and future market

- Insufficient liquidity in the FTR market impacts the futures market

| Volume FTRs | Volume Futures | $P_c^f$ (FUTURE 6) |
|---|---|---|
| 960 | 570 (92%) | $[52.9, 53.6]$ |
| 720 | 478 (69%) | $[52.9, 53.7]$ |
| 480 | 320 (56%) | $[52.9, 53.8]$ |
| 240 | 269 (39%) | $[53.5, 54.0]$ |
| 0 | 152 (22%) | 54.8 |

TABLE: Induced energy futures volume for a given liquidity bounds on FTRs