Understanding the Price Dynamics
of Emission Permits: A Model for Multiple Trading Periods

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Marliese Uhrig-Homburg, Energy & Finance, INREC 2010 | October 6, 2010
1. Introduction

- Cap-and-trade systems for greenhouse gases established in many different countries all over the world
- Emission market is characterized by a set of regulatory rules.
  \[\Rightarrow\] How does the regulatory framework affect price dynamics?
- Understanding the price dynamics crucially important for
  - pricing derivatives
  - sound risk management
  - energy-related operating and investment decisions

\[\Rightarrow\] We propose long-term equilibrium model under uncertainty with and without abatement possibilities
Introduction

- Our equilibrium model for permit prices takes into account
  - sequence of consecutive trading periods
  - inter-period banking, no inter-period borrowing
  - penalty costs and later delivery of lacking permits

  How does additional consideration of consecutive trading periods change finite period view?

- We identify option analogy of emission permits
  - permit \( \equiv \) strip of binary options written on net cumulative emissions
  - underlying not exogenously given but derived endogenously through abatement
Stylized facts of EU ETS have changed

  - within trading periods EUAs are storable (bankable)
  - banking and borrowing not allowed between 2007 and 2008
- meanwhile plans for indefinitely ongoing sequence of trading periods
  - third trading period until 2020
  - no inter-period borrowing but inter-period banking
  - presumable figures for permit allocation in following trading periods
theoretical models

- equilibrium models considering one trading period
  - companies choose optimal trading and abatement strategies
  - companies choose optimal trading strategies only
    - e.g. Chesney/Taschini (2008)

- models considering two trading periods
  - Kijima et al (2009): either banking and borrowing or neither of them
  - Cetin, Verschuere (2009): no banking

empirical studies

- burgeoning literature

- mostly based on data from trial period
Introduction

INREC 2010

Agenda

1. equilibrium model for multiple trading periods
   ▶ takes into account most important features of EU ETS
     ▶ penalty costs and later delivery of lacking permits
     ▶ inter-period banking, no inter-period borrowing
   ▶ both with and without abatement possibilities

2. properties of the EUA price dynamics
   ▶ exploit option analogy of EUAs

3. implications for derivative pricing
   ▶ appropriate price distributions for option pricing
   ▶ insights into valuation of inter-period futures
2. Equilibrium model

CO$_2$–regulated company

- stochastic emission rate (Business As Usual)

$$dy_t = \mu(y_t)dt + \sigma(y_t)dw_t$$

- company may
  - buy or sell EUAs in market ($z_t$)
  - pay penalty for not complying
  - abate $u_t$ of CO$_2$ emissions with abatement costs $C(u_t)$

- total expected emissions in $[0, T_k]$ (abatements/trading taken into account)

$$x_{t,T_k} = -\int_0^t u_s ds - \int_0^t z_s ds + E_t(\int_0^{T_k} y_s ds)$$
CO₂—regulated company

- $n$ consecutive trading periods $[0, T_1], [T_1, T_2] \ldots [T_{n-1}, T_n]$ with inter-period banking but no inter-period borrowing

- initial endowment $e_{k-1}$ of EUAs at beginning of each period $[T_{k-1}, T_k]$

- penalties are incurred if
  - net realized emissions $x_{T_k}$ from 0 until $T_k$ exceed
  - cumulative amount $e_{T_k} = \sum_{T_j < T_k} e_j$ of permits allocated before $T_k$, i.e. remaining permits $R(x_{T_k}) = e_{T_k} - x_{T_k} < 0$

- penalty costs at end of each trading period $T_k$ for lacking EUAs

\[ P(x_{T_k}) = \min[0, pR(x_{T_k})] \]
2. Equilibrium model

CO\textsubscript{2}—regulated company

\begin{itemize}
  \item trial period [0, T]
  \item multiple periods [0, T\textsubscript{1}], [T\textsubscript{1}, T\textsubscript{2}], ...
\end{itemize}

\begin{itemize}
  \item company’s optimization problem:
\end{itemize}

\[
\max_{u_t, z_t, t \in [0, T_n]} E_0 \left( \int_0^{T_n} e^{-rt} C(u_t) \, dt - \int_0^{T_n} e^{-rt} S(t)z_t \, dt + \sum_{j=1}^{n} e^{-rT_j} P(x_{T_j}) + R(x_{T_n})S_{end} \right)
\]
Consider market consisting of $N$ companies

- equilibrium consists of
  - trading strategies $z^*_i, i = 1 \ldots N$
  - abatement rates $u^*_i, i = 1 \ldots N$
  - EUA spot price $S(t)$

- solving
  - individual cost problems and
  - market clearing condition $\sum_{i=1}^{N} z_{it} = 0$ for all $t$

Technically, we

- first consider last trading period $[T_{n-1}, T_n]$ and
- proceed backwards using dynamic programming
Solution without abatement possibilities

- Marginal value of an emission allowance consists of two components:
  1. penalty payment saved weighted by probability that penalties arise
  2. value one additional allowance can be sold for at $T_n$

- In equilibrium $E_t \left[ 1\{R_n(x^i_{T_n})<0\} \right]$ is equal for all companies $i$
  ⇒ take global view ($x_{T_n} = \sum x^i_{T_n}$)

  - within last trading period $[T_{n-1}, T_n]$:
    $$S(t) = e^{-r(T_n-t)} E_t \left[ 1\{R(x_{T_n})<0\} \right] p + e^{rt} S_{end}$$

  - in prior periods:
    $$S(t) = \sum_{T_j > t} e^{-r(T_j-t)} E_t \left[ 1\{R(x_{T_j})<0\} \right] p + e^{rt} S_{end}$$
Solution including abatement possibilities

- general structure still holds

\[ S(t) = \sum_{T_j > t} e^{-r(T_j-t)} E_t \left[ 1\{R(x_{T_j})<0\} \right] p + e^{rt} S_{end} \]

- but dynamics of cumulative net expected emissions depends on (endogenous) abatement strategies \( u_{it} \)

- from first order condition:

\[ S(t) = c_i u_{it}^*, \quad i = 1 \ldots N \]

- i.e. spot price \( \equiv \) marginal abatement costs

  - if EUA price is above marginal abatement cost, companies may profit by abating cheap and selling higher (and vice versa)
  
  - all companies have the same marginal abatement costs after trading
Solution including abatement possibilities

abatement strategies

- start with last trading period
  - deduce characteristic PDE with boundary conditions from optimality principle from stochastic optimal control theory
  - solve for strategy value $V_n$
- step back one period
  - deduce again characteristic PDE
  - solve for strategy value using next period’s value (boundary value)
- derive abatement strategy from resulting Hamilton-Jacobi-Bellman equation
3. Properties of allowance prices

- **Intra-period martingal property:** Discounted spot prices are martingales within each trading period.
  - in particular, no mean-reversion or seasonal behavior
  - due to storability and assumption of risk-neutral agents

- **Option characteristics:** Emission allowances can be considered as a strip of binary European call options.
  - without abatement: each call is written on non-tradable underlying, the net cumulative emissions until end of given trading period
  - with abatement: market participants can influence underlying through abatement actions

- **Local volatility:** Local volatility is time- and state-dependent.
Properties

From option characteristics of EUA it follows:

1. each additional trading period leads to additional value component:
   ▶ current value of binary option with non-negative payoff

2. allowance price is bounded above and below
   ▶ lower bound: $S_{\text{lower}}(t) = e^{rt}S_{\text{end}}$
   ▶ upper bound: $S_{\text{upper}}(t) = \sum_{j=1}^{n} e^{-r(T_j-t)} p + e^{rt}S_{\text{end}}$

3. binary part leads to discontinuity at the end of each trading period

4. induced transition from one trading period to the next
   \[ S(T_1^-) - S(T_1^+) = 1\{R(x_{T_1}) < 0\} p \]
   ▶ smooth transition if economy is in surplus
   ▶ otherwise price decrease by amount of penalty
Properties

Concrete model setting in accordance with EU ETS:
- chosen parameter values:
  - up to four consecutive trading periods
  - first period 5 years, next periods 8 years
  - allocation according to current allocation plans

<table>
<thead>
<tr>
<th>Phase</th>
<th>Period</th>
<th>Tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase II</td>
<td>(2008-2012)</td>
<td>10.400 billion</td>
</tr>
<tr>
<td>Phase III</td>
<td>(2013-2020)</td>
<td>14.775 billion</td>
</tr>
<tr>
<td>Phase IV</td>
<td>(2021-2028)</td>
<td>12.455 billion</td>
</tr>
<tr>
<td>Phase V</td>
<td>(2029-2036)</td>
<td>10.135 billion</td>
</tr>
</tbody>
</table>

- penalty costs: $p_j = \€100$ in each period $j$
- time-0 value $S_{end} = 14.11$

- consider spot price for first period of each setting
Spot price function $S(t, x_t, T_1)$

![Graph showing the spot price function $S(t, x_t, T_1)$ with axes for total expected emissions, time, and spot price.](image)
3. Properties of allowance prices

Spot price function $S(t, x_t, T_1)$ (back)
### Value components of current spot price $S(t, x_t, T_1)$

<table>
<thead>
<tr>
<th>Emissions Scenario current</th>
<th>Emissions Scenario future</th>
<th>Value Component from period 1</th>
<th>Value Component from period 2</th>
<th>Value Component from period 3</th>
<th>Value Component from period 4</th>
<th>$S_{end}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>medium</td>
<td>medium</td>
<td>72%</td>
<td>11%</td>
<td>2%</td>
<td>1%</td>
<td>14%</td>
</tr>
<tr>
<td>high</td>
<td>high</td>
<td>38%</td>
<td>27%</td>
<td>18%</td>
<td>10%</td>
<td>7%</td>
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<tr>
<td>high</td>
<td>low</td>
<td>65%</td>
<td>14%</td>
<td>5%</td>
<td>5%</td>
<td>11%</td>
</tr>
<tr>
<td>low</td>
<td>high</td>
<td>0%</td>
<td>47%</td>
<td>23%</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>low</td>
<td>low</td>
<td>0%</td>
<td>2%</td>
<td>22%</td>
<td>29%</td>
<td>47%</td>
</tr>
</tbody>
</table>

- substantial part of spot price attributable to future trading periods
Spot EUAs, futures, and options are traded OTC and on exchanges across Europe
- exchange traded options typically mature in current trading period (intra-period)
- futures with maturity in next trading period (inter-period) also available

What do we learn from our long-term equilibrium model for derivative pricing?
- concerning appropriate price distributions for option pricing
- concerning valuation of inter-period futures
Intra-period options

- Pricing of non-linear carbon-related derivatives requires assumptions about probability distributions of EUA prices.
- Which kind of distribution seems appropriate according to our model?
  - for setting with one trading period (trial period)
  - for multiple period setting (current situation)
- Simulation study: consider prices at the end (and during) the first trading period conditioned on time zero information
  - for setting with only one trading period
  - for settings with two, three, and four trading periods
Spot price distribution (one period)

- probability distribution approaches two-point distribution
- standard models (GBM, jump-diffusion...) are obviously not able to capture this property
Spot price distribution (four periods)

- final permit price consists of two parts
  - binary part
  - value component attributed to following trading period
- within trading period standard models more appropriate than before
- at period end binary part still important
Inter-period futures

- Standard cost-of carry relation should hold for intra-period ($T < T_1$) futures (Uhrig-Homburg/Wagner (2009))

$$F(t, T) = e^{r(T-t)}S(t)$$

- Holding current permit has additional benefit compared to holding inter-period future ($T > T_1$) maturing in next trading period:

$$S(t) - e^{r(T-t)}F(t, T) = e^{-r(T_1-t)}E_t[1_{\{R(x_{T_1}) < 0\}}]p$$

- In commodity literature: benefit captured by convenience yield
  - but standard convenience yield models (such as in Daskalakis et al (2009)) inappropriate due to
  - cost-of carry relation for inter-period futures with different maturities
each additional trading period leads to
  ▶ additional possible use because of banking possibility
  ▶ additional value component in today's spot price
  ▶ relative share depends on current and future expected emissions

EUAs $\equiv$ strip of binary options written on net cumulative emissions
  ▶ price bounds naturally depend on number of trading periods considered
  ▶ spot prices do not decline to zero at end of a trading period
  ▶ smooth transition into next trading period if economy is in surplus

if at all, standard option pricing models useful for intra-period options maturing within trading period (when binary part is not too important)

standard stochastic convenience yield models are inappropriate for inter-period futures