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Understanding the Price Dynamics of Emission Permits: A Model for Multiple Trading Periods

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Marliese Uhrig-Homburg, Energy & Finance, INREC 2010 | October 6, 2010

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1. Introduction

- Cap-and-trade systems for greenhouse gases established in many different countries all over the world
- Emission market is characterized by a set of regulatory rules.

  $\Longrightarrow$ How does the regulatory framework affect price dynamics?
- Understanding the price dynamics crucially important for
  - pricing derivatives
  - sound risk management
  - energy-related operating and investment decisions

  $\Longrightarrow$ We propose long-term equilibrium model under uncertainty with and without abatement possibilities

Introduction

- Our equilibrium model for permit prices takes into account
  - sequence of consecutive trading periods
  - inter-period banking, no inter-period borrowing
  - penalty costs and later delivery of lacking permits

$\Longrightarrow$ How does additional consideration of consecutive trading periods change finite period view?

- We identify option analogy of emission permits
  - permit $\widehat{=}$ strip of binary options written on net cumulative emissions
  - underlying not exogenously given but derived endogenously through abatement

Stylized facts of EU ETS have changed

- initially two trading periods: 2005 - 2007 and 2008 - 2012
  - within trading periods EUAs are storable (bankable)
  - banking and borrowing not allowed between 2007 and 2008
- meanwhile plans for indefinitely ongoing sequence of trading periods
  - third trading period until 2020
  - no inter-period borrowing but inter-period banking
  - presumable figures for permit allocation in following trading periods

Literature

theoretical models

- equilibrium models considering one trading period
  - companies choose optimal trading and abatement strategies
    - Seifert et al (2008), Carmona et al (2008), Carmona/Fehr/Hinz (2009)
  - companies choose optimal trading strategies only
    - e.g. Chesney/Taschini (2008)
- models considering two trading periods
  - Kijima et al (2009): either banking and borrowing or neither of them
  - Cetin, Verschuere (2009): no banking

empirical studies

- burgeoning literature
- mostly based on data from trial period
  - Daskalakis et al (2009), Paolella, Taschini (2008), Benz, Trück (2009), Uhrig-Homburg, Wagner (2009)

Agenda

1. equilibrium model for multiple trading periods
   - takes into account most important features of EU ETS
     - penalty costs and later delivery of lacking permits
     - inter-period banking, no inter-period borrowing
   - both with and without abatement possibilities
2. properties of the EUA price dynamics
   - exploit option analogy of EUAs
3. implications for derivative pricing
   - appropriate price distributions for option pricing
   - insights into valuation of inter-period futures

2. Equilibrium model

CO_2−regulated company

- stochastic emission rate (Business As Usual)

$$dy_t = \mu(y_t)dt + \sigma(y_t)dw_t$$

- company may
  - buy or sell EUAs in market (z_t)
  - pay penalty for not complying
  - abate u_t of CO_2 emissions with abatement costs $C(u_t)$
- total expected emissions in $[0, T_k]$ (abatements/trading taken into account)

$$x_{t,T_k} = -\int_0^t u_s ds - \int_0^t z_s ds + E_t(\int_0^{T_k} y_s ds)$$

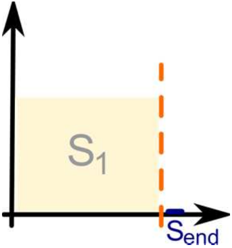
CO_2—regulated company

- n consecutive trading periods $[0, T_1], [T_1, T_2] \dots [T_{n-1}, T_n]$ with inter-period banking but no inter-period borrowing
- initial endowment e_{k-1} of EUAs at beginning of each period $[T_{k-1}, T_k]$
- penalties are incurred if
  - net realized emissions x_{T_k} from 0 until T_k exceed
  - cumulative amount $e_{T_k} = \sum_{T_j < T_k} e_j$ of permits allocated before T_k,

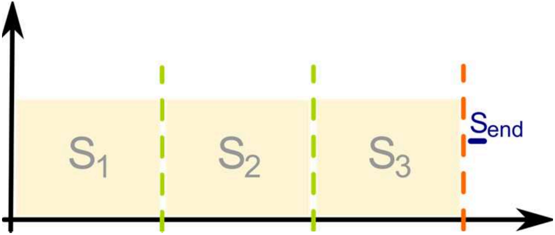
  i.e. remaining permits $R(x_{T_k}) = e_{T_k} - x_{T_k} < 0$
- penalty costs at end of each trading period T_k for lacking EUAs

$$P(x_{T_k}) = min[0, pR(x_{T_k})]$$

CO_2−regulated company



trial period $[0, T]$



multiple periods $[0, T_1], [T_1, T_2], \ldots$

- company's optimization problem:

$$\max_{u_t, z_t, t \in [0, T_n]} E_0 \left(\int_0^{T_n} e^{-rt} C(u_t) dt - \int_0^{T_n} e^{-rt} S(t) z_t dt + \sum_{j=1}^{n} e^{-rT_j} P(x_{T_j}) + R(x_{T_n}) S_{end} \right)$$

Market equilibrium

Consider market consisting of N companies

- equilibrium consists of
  - trading strategies $z_{it}^{*}, i = 1 \ldots N$
  - abatement rates $u_{it}^{*}, i = 1 \ldots N$
  - EUA spot price $S(t)$
- solving
  - individual cost problems and
  - market clearing condition $\sum_{i=1}^{N} z_{it} = 0$ for all t

Technically, we

- first consider last trading period $[T_{n-1}, T_n]$ and
- proceed backwards using dynamic programming

Solution without abatement possibilities

- Marginal value of an emission allowance consists of two components:
  1. penalty payment saved weighted by probability that penalties arise
  2. value one additional allowance can be sold for at T_n
- In equilibrium $E_t\left[1_{\{R_n(x^i_{T_n})<0\}}\right]$ is equal for all companies i
  $\Rightarrow$ take global view ($x_{T_n} = \sum x^i_{T_n}$)
  - within last trading period $[T_{n-1}, T_n]$:

$$S(t) = e^{-r(T_n - t)} E_t\left[1_{\{R(x_{T_n})<0\}}\right] p + e^{rt} S_{end}$$

  - in prior periods:

$$S(t) = \sum_{T_j > t} e^{-r(T_j - t)} E_t\left[1_{\{R(x_{T_j})<0\}}\right] p + e^{rt} S_{end}$$

Solution including abatement possibilities

- general structure still holds

$$S(t) = \sum_{T_j > t} e^{-r(T_j - t)} E_t \left[1_{\{R(x_{T_j}) < 0\}} \right] p + e^{rt} S_{end}$$

- but dynamics of cumulative net expected emissions depends on (endogenous) abatement strategies u_{it}
- from first order condition:

$$S(t) = c_i u_{it}^*, i = 1 \dots N$$

- i.e. spot price $\equiv$ marginal abatement costs
  - if EUA price is above marginal abatement cost, companies may profit by abating cheap and selling higher (and vice versa)
  - all companies have the same marginal abatement costs after trading

Solution including abatement possibilities

abatement strategies

- start with last trading period
  - deduce characteristic PDE with boundary conditions from optimality principle from stochastic optimal control theory
  - solve for strategy value V_n
- step back one period
  - deduce again characteristic PDE
  - solve for strategy value using next period's value (boundary value)
- derive abatement strategy from resulting Hamilton-Jacobi-Bellman equation

3. Properties of allowance prices

- Intra-period martingal property: Discounted spot prices are martingales within each trading period.
  - in particular, no mean-reversion or seasonal behavior
  - due to storability and assumption of risk-neutral agents
- Option characteristics: Emission allowances can be considered as a strip of binary European call options.
  - without abatement: each call is written on non-tradable underlying, the net cumulative emissions until end of given trading period
  - with abatement: market participants can influence underlying through abatement actions
- Local volatility: Local volatility is time- and state-dependent.

Properties

From option characteristics of EUA it follows:

1. each additional trading period leads to additional value component:
   - current value of binary option with non-negative payoff
2. allowance price is bounded above and below
   - lower bound: $S_{lower}(t) = e^{rt} S_{end}$
   - upper bound: $S_{upper}(t) = \sum_{j=1}^{n} e^{-r(T_j - t)} p + e^{rt} S_{end}$
3. binary part leads to discontinuity at the end of each trading period
4. induced transition from one trading period to the next

$$S(T_1^-) - S(T_1^+) = 1_{\{R(x_{T_1})<0\}} p$$

   - smooth transition if economy is in surplus
   - otherwise price decrease by amount of penalty

Properties

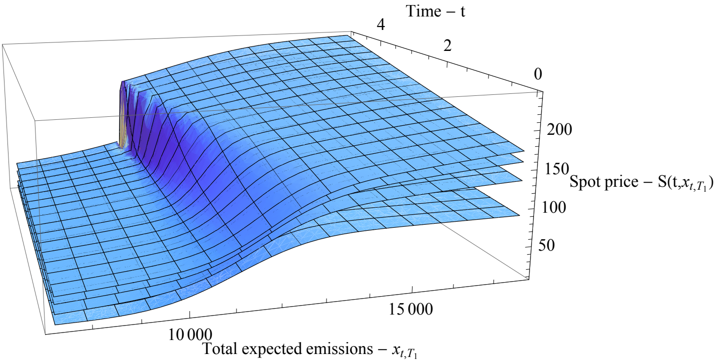
Concrete model setting in accordance with EU ETS:

- chosen parameter values:
  - up to four consecutive trading periods
  - first period 5 years, next periods 8 years
  - allocation according to current allocation plans

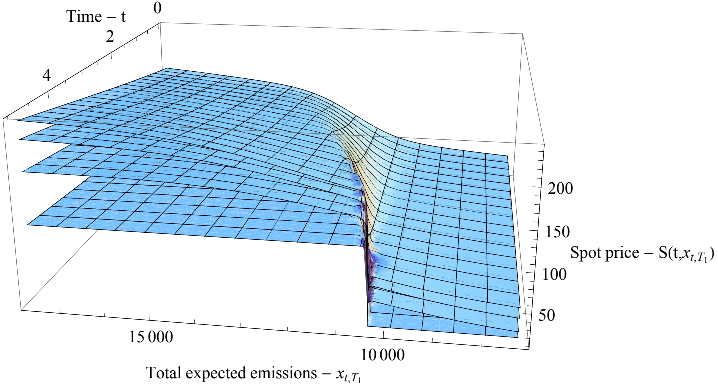
phase II (2008-2012)	10.400 billion tons
phase III (2013-2020)	14.775 billion tons
phase IV (2021-2028)	12.455 billion tons
phase V (2029-2036)	10.135 billion tons

  - penalty costs: $p_j =$ €100 in each period j
  - time-0 value $S_{end} = 14.11$
- consider spot price for first period of each setting

Spot price function $S(t, x_{t,T_1})$



Spot price function $S(t, x_{t,T_1})$ (back)



Value components of current spot price $S(t, x_{t,T_1})$

Emissions Scenario		Value Component from				
current	future	period 1	period 2	period 3	period 4	S_{end}
medium	medium	72%	11%	2%	1%	14%
high	high	38%	27%	18%	10%	7%
high	low	65%	14%	5%	5%	11%
low	high	0%	47%	23%	15%	15%
low	low	0%	2%	22%	29%	47%

- substantial part of spot price attributable to future trading periods

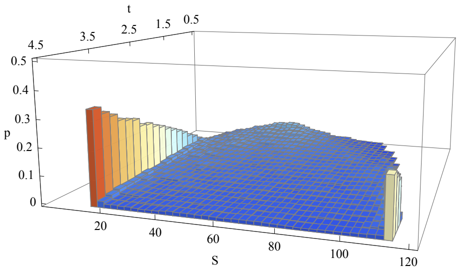
4. Price dynamics and derivative pricing

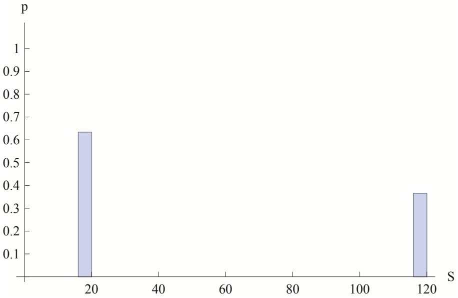
- Spot EUAs, futures, and options are traded OTC and on exchanges across Europe
  - exchange traded options typically mature in current trading period (intra-period)
  - futures with maturity in next trading period (inter-period) also available
- What do we learn from our long-term equilibrium model for derivative pricing?
  - concerning appropriate price distributions for option pricing
  - concerning valuation of inter-period futures

Intra-period options

- Pricing of non-linear carbon-related derivatives requires assumptions about probability distributions of EUA prices.
- Which kind of distribution seems appropriate according to our model?
  - for setting with one trading period (trial period)
  - for multiple period setting (current situation)
- Simulation study: consider prices at the end (and during) the first trading period conditioned on time zero information
  - for setting with only one trading period
  - for settings with two, three, and four trading periods

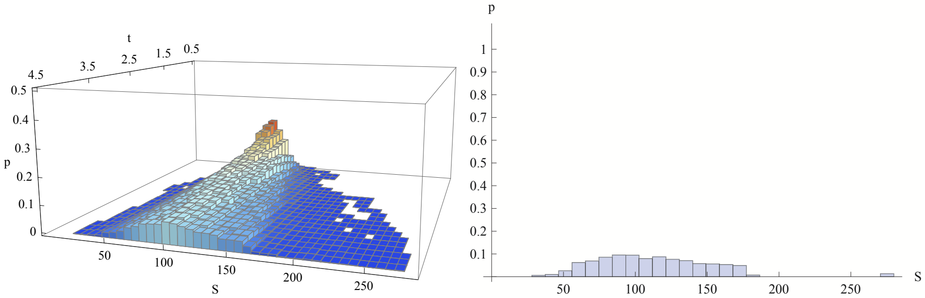
Spot price distribution (one period)





- probability distribution approaches two-point distribution
- standard models (GBM, jump-diffusion...) are obviously not able to capture this property

Spot price distribution (four periods)



- final permit price consists of two parts
  - binary part
  - value component attributed to following trading period
- within trading period standard models more appropriate than before
- at period end binary part still important

Inter-period futures

- Standard cost-of carry relation should hold for intra-period ($T < T_1$) futures (Uhrig-Homburg/Wagner (2009))

$$F(t,T) = e^{r(T-t)}S(t)$$

- Holding current permit has additional benefit compared to holding inter-period future ($T > T_1$) maturing in next trading period:

$$S(t) - e^{r(T-t)}F(t,T) = e^{-r(T_1-t)}E_t[1_{\{R(x_{T_1})<0\}}]p$$

- In commodity literature: benefit captured by convenience yield
  - but standard convenience yield models (such as in Daskalakis et al (2009)) inappropriate due to
  - cost-of carry relation for inter-period futures with different maturities

Conclusion

- each additional trading period leads to
  - additional possible use because of banking possibility
  - additional value component in today's spot price
  - relative share depends on current and future expected emissions
- EUAs $\widehat{=}$ strip of binary options written on net cumulative emissions
  - price bounds naturally depend on number of trading periods considered
  - spot prices do not decline to zero at end of a trading period
  - smooth transition into next trading period if economy is in surplus
- if at all, standard option pricing models useful for intra-period options maturing within trading period (when binary part is not too important)
- standard stochastic convenience yield models are inappropriate for inter-period futures