Lévy Processes in Credit Risk

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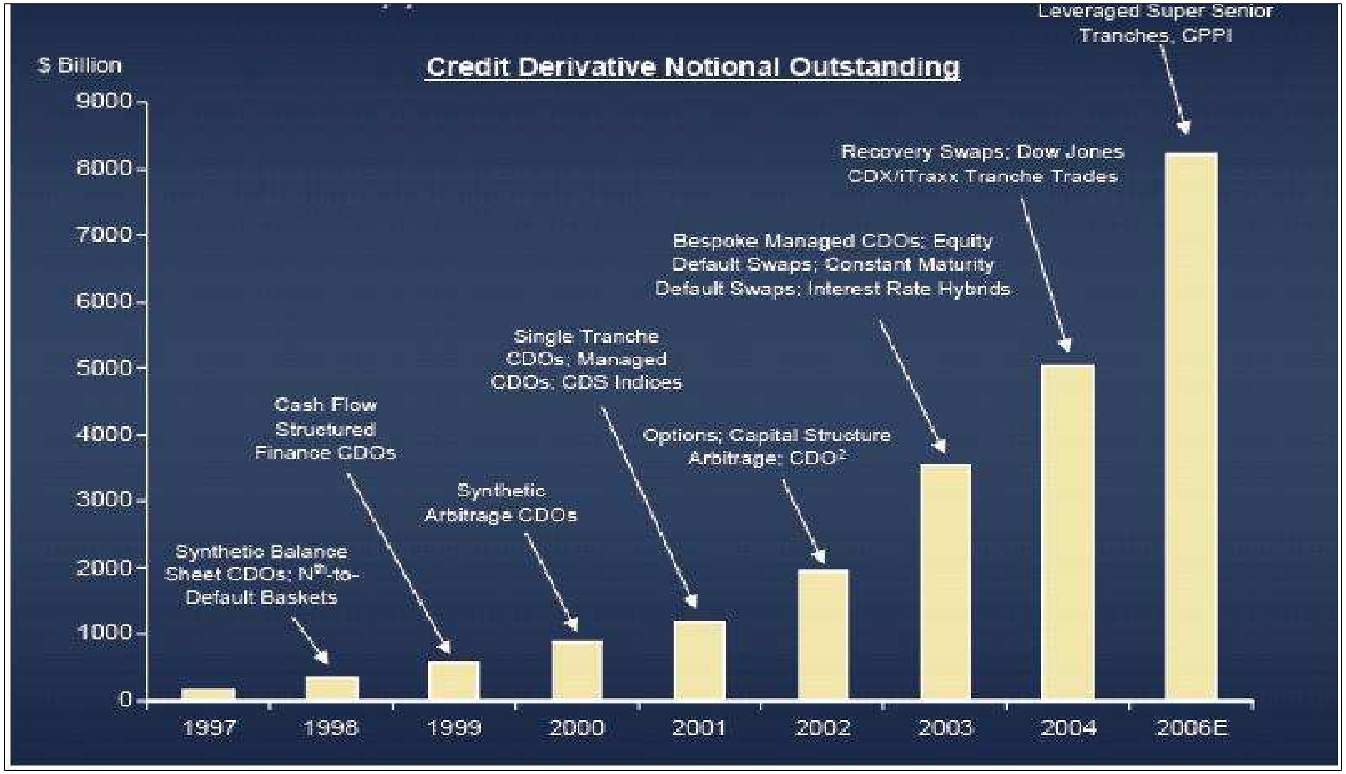
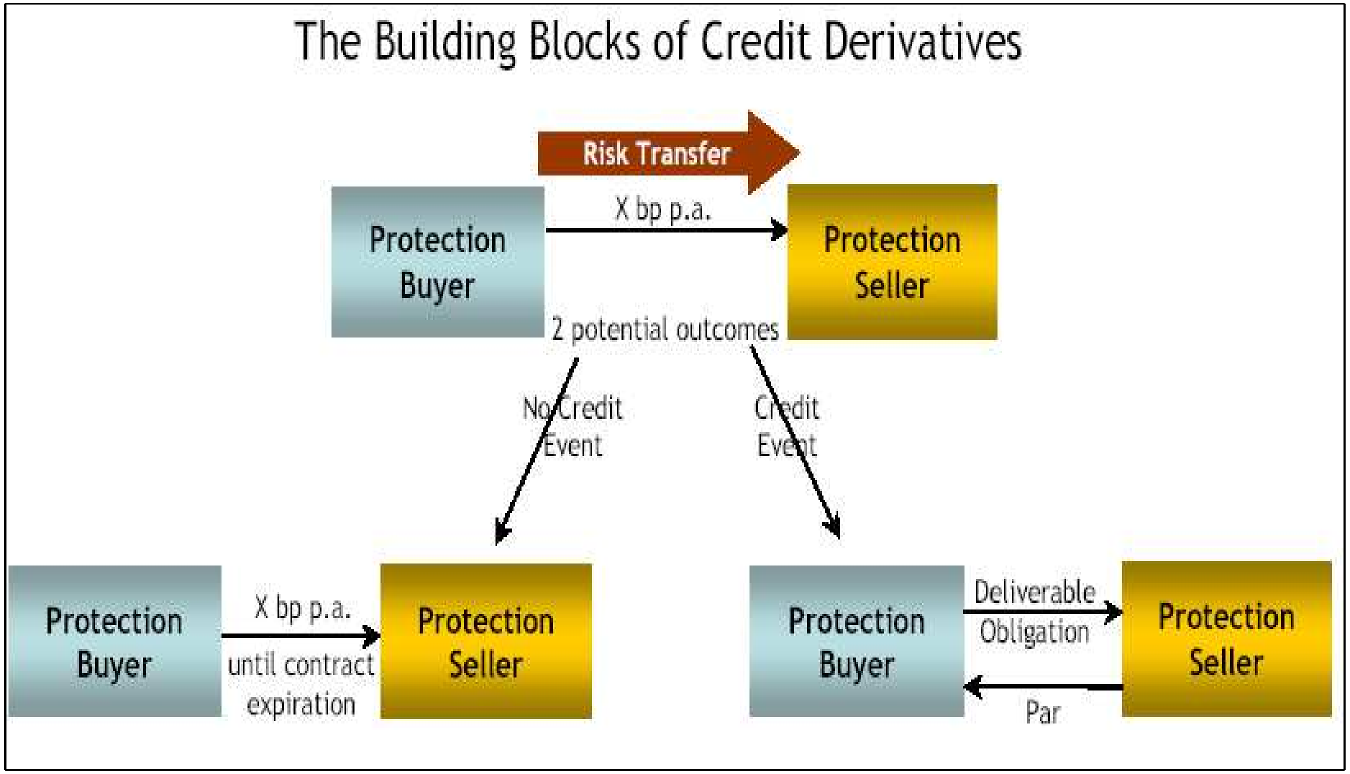
Credit Risk Market

- The credit market has seen an explosive growth the last decennium and is now touching around 30 trillion USD and triples the classical equity market.



Credit Default Swaps

- Credit Default Swaps (CDSs) are instruments that provide the buyer an insurance against the defaulting of a company (the reference entity) on its debt.



Modeling Approaches

- Firm's Value/Structural models
  - Dates back to Merton (1974), Black and Scholes (1973) and Black and Cox (1976).
  - One basically models the value of the firm's assets.
  - One often links credit risk (firm's value) with the equity market (stock's value).
  - Most models are Brownian driven, we will focus on jump driven models (Cariboni-Schoutens (2005), Madan-Schoutens (2007)).
- Intensity-based models
  - One basically models the default intensity.
  - This is for another talk : cfr. Ph.D. thesis of Jessica Cariboni.

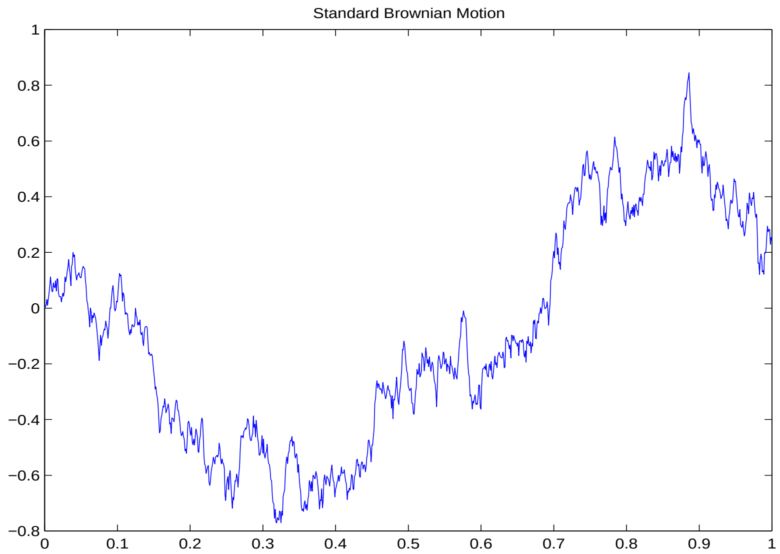
Lévy Models

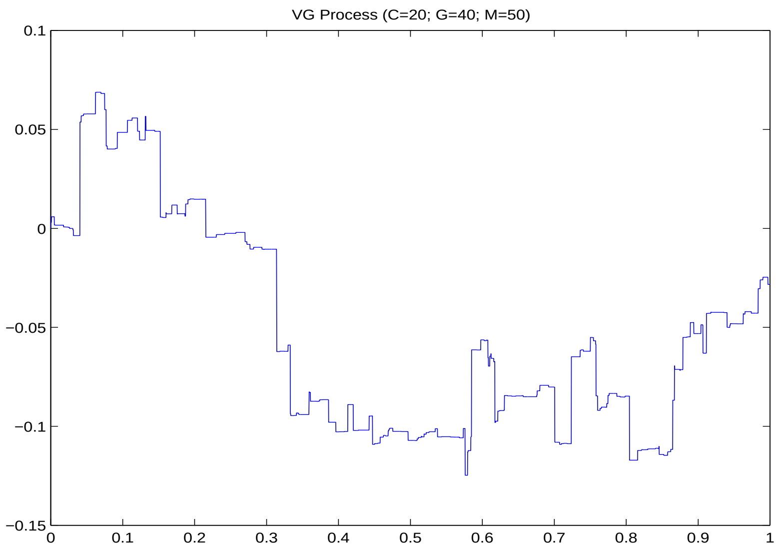
- We work under the so-called firm's value Lévy model (with jumps):

$$A_t = \exp(X_t), \quad A_0 = 1,$$

where $\{X_t, t \geq 0\}$ is a Lévy process.

- Lévy processes are generalization of Brownian Motions (stationary and independent increments) allowing for
  - non-Gaussian underlying distribution (skewness, kurtosis, more heavier tails, ...);
  - jumps.





Lévy Default Models

- Default is defined to occur the first time when

$$A_t = \exp(X_t) \leq R$$

or equivalently if

$$X_t \leq \log(R).$$

- Let us denote by $P_{surv}(t)$ the risk-neutral probability of survival between 0 and t:

$$\begin{aligned} P_{surv}(t) &= P_Q\left(X_s > \log(R), \text{for all } 0 \leq s \leq t\right); \\ &= P_Q\left(\inf_{0 \leq s \leq t} X_s > \log(R)\right); \\ &= P_Q\left(\underline{X}_t > \log(R)\right) \end{aligned}$$

- The CDS spread is then given by

$$C = \frac{(1-R)\left(-\int_0^T \exp(-rs)\mathrm{d}P_{surv}(s)\right)}{\int_0^T \exp(-rs)P_{surv}(s)\mathrm{d}s}$$

Single Sided Lévy Processes

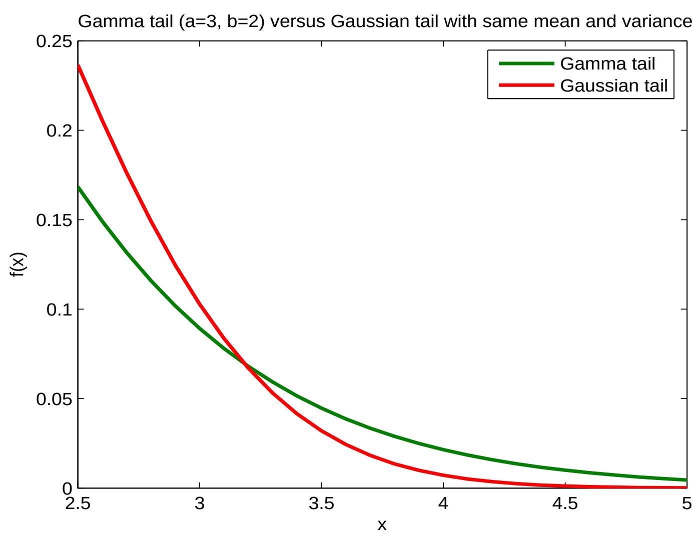
- Lévy processes have proven their modeling abilities already in many fields (Equity, FI, Volatility , FX, ...).
- Recently, they found their way into credit.
- In Cariboni-Schoutens (2005) the example of the popular VG firm's value model was worked (solving PIDE).
- We work here in the single sided setting only allowing down jumps.
- In contrast to stock price behavior where clearly up and down jumps are present, a firm tries to follow a steady growth (up trend) but is exposed to shocks (negative jumps).
- Under this setting we can calculate first passage time distributions (default probabilities) by exploiting the remarkable Wiener-Hopf factorization and performing a very fast double Laplace inversion.

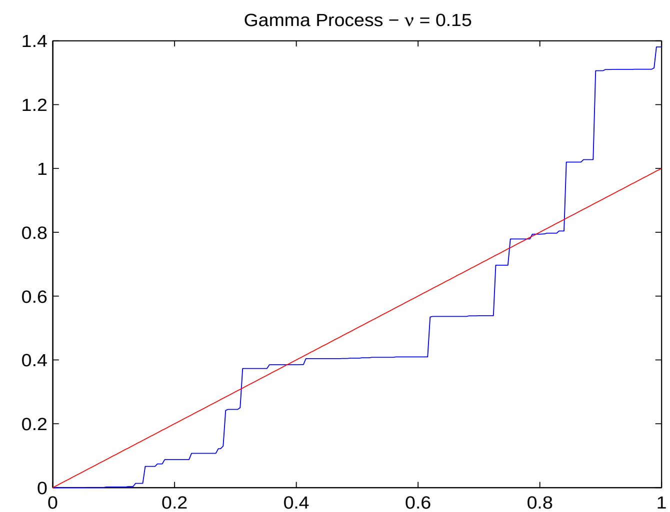
Example : Shifted Gamma Model

- The density function of the Gamma distribution Gamma(a, b) with parameters $a > 0$ and $b > 0$ is given by:

$$f_{Gamma}(x; a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp(-xb), \quad x > 0.$$

- The Gamma-process $G = \{G_t, t \geq 0\}$ with parameters $a, b > 0$ is a stochastic process which starts at zero and has stationary, independent Gamma-distributed increments and G_t follows a Gamma(at, b) distribution.



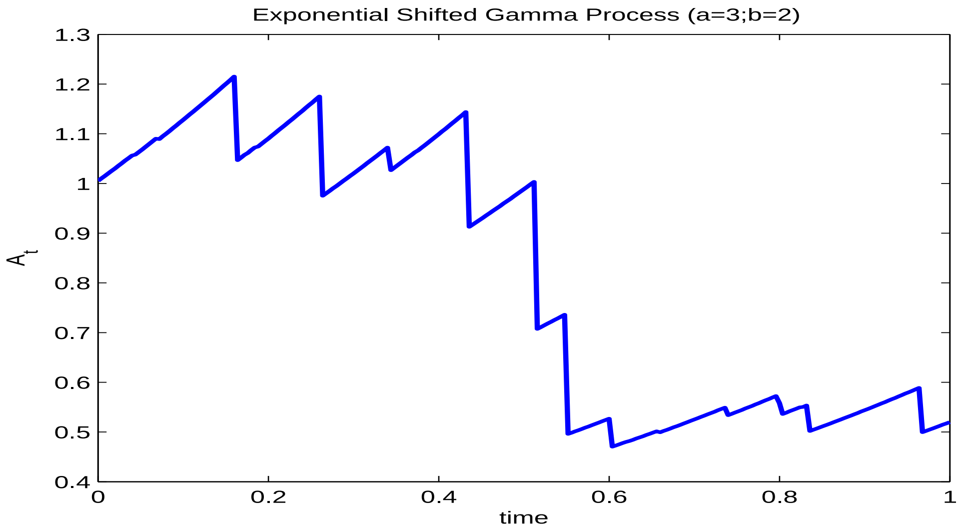


Examples

- As driving Lévy process (in a risk neutral setting), we then take:

$$X_t = \mu t - G_t, \quad t \in [0,1],$$

where in this case $\mu = r + a \log\left(1 + b^{-1}\right)$.



- Other examples : Shifted IG Model (SIG) and Shifted CMY Model (SCMY)

First Passage Time

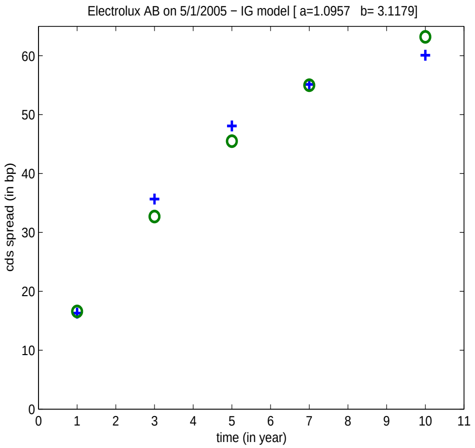
- Define

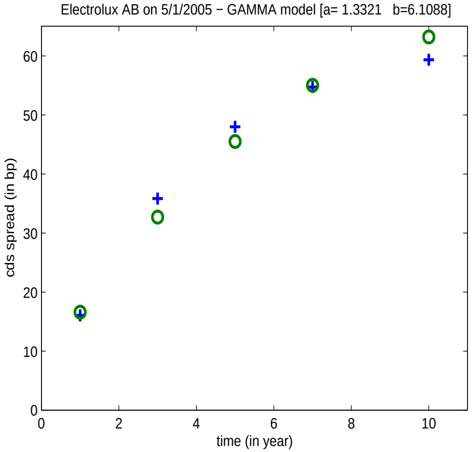
$$h(\lambda) = \psi(\lambda/\mu), \text{ where } E\left[\exp\left(zX_t\right)\right] = \exp(t\psi_X(z)).$$

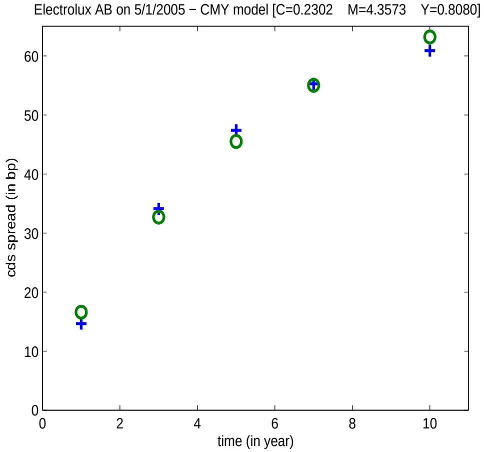
- We have

$$P\left(\underline{X}_t > -x\right) =$$

$$-\frac{-1}{(2\pi)^2}\int_{\Gamma_1}\int_{\Gamma_2} h'(\lambda)\exp(h(\lambda)t + zx)\frac{(\lambda/\mu) - z}{\left(h(\lambda) - \psi(z)\right)(\lambda/\mu)z}\mathrm{d}\lambda\mathrm{d}z.$$

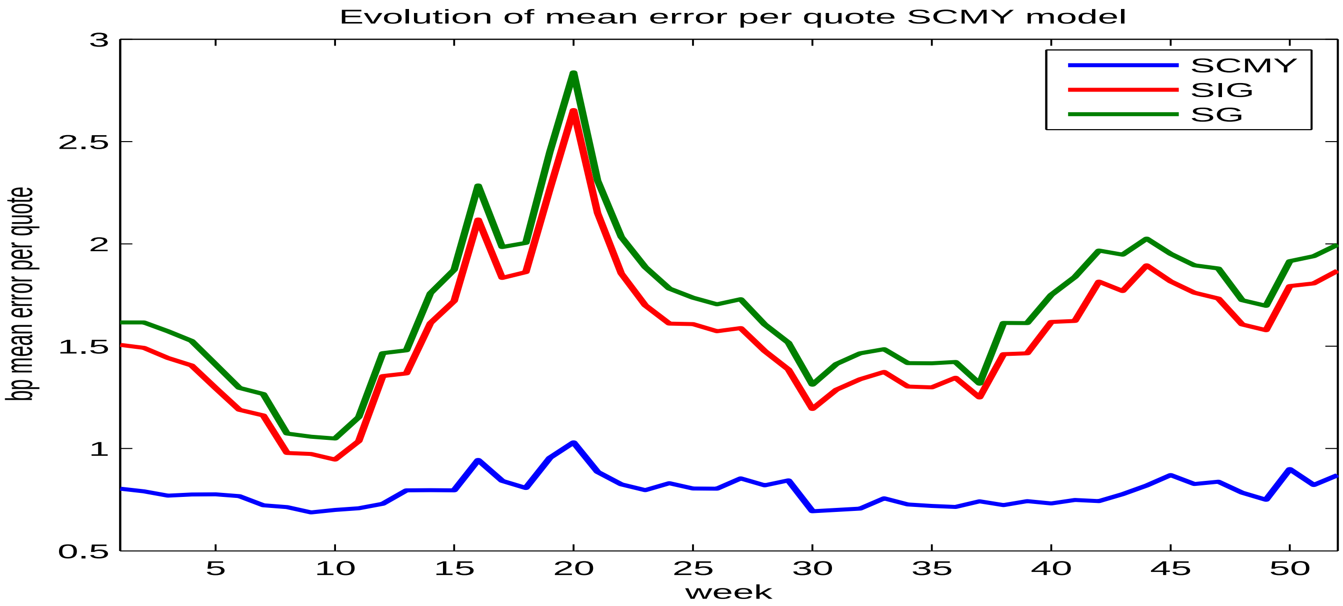






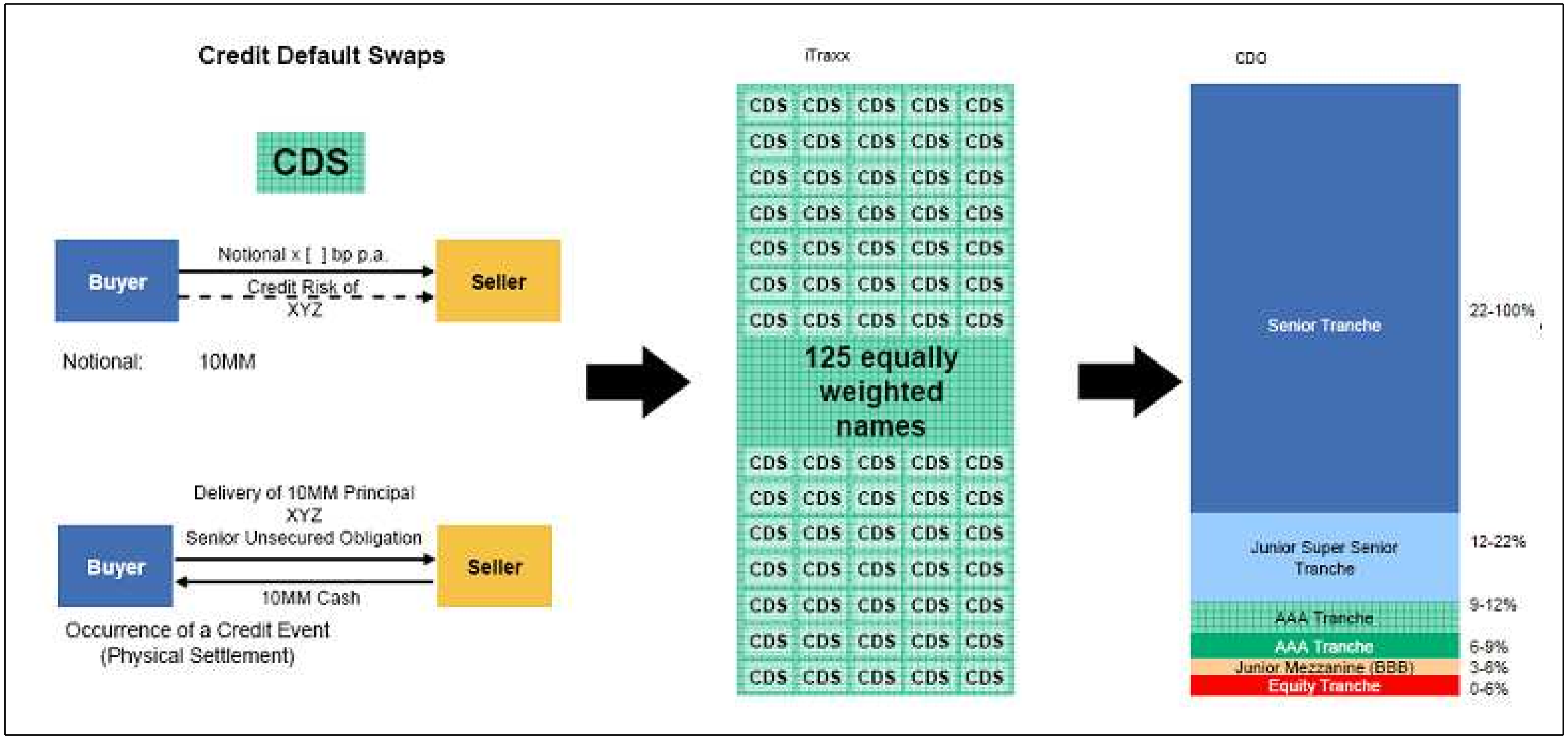
Calibration

- Calibration results on 6500 CDS curves (iTraxx 2005-2006) gave a very satisfactory fit



CDOs

- (Synthetic) Collateralized Credit Obligations (CDOs) are complex multivariate credit risk derivatives.
- A CDO transfers the credit risk on a reference portfolio (typically 125 CDSs) of assets in a tranched way.



Gaussian Copula Model

- The Gaussain one-factor model (Vasicek, Li) assumes the following "dynamics" for the standardized firm's value:
  - $A_i(T) = \sqrt{\rho}\,Y + \sqrt{1-\rho}\,\epsilon_i,\ i = 1, \ldots, n;$
  - Y and ϵ_i, $i = 1, \ldots, n$ are i.i.d. standard normal with cdf Φ.
- The ith obligor defaults at time T if $A_i(T)$ falls below some preset barrier $K_i(T)$ (extracted from CDS quotes to match individual default probabilities).
- This model is actual based on the Gaussian Copula with its known problems (cfr. correlation smile).
- The underlying reason is the too light tail-behavior of the standard normal rv's (a large number of joint defaults will be caused by a very negative common factor Y).
- Therefore we look for models where the distribution of the factors has more heavy tails.

Correlation via Running Time

Outline

Single Name Modeling

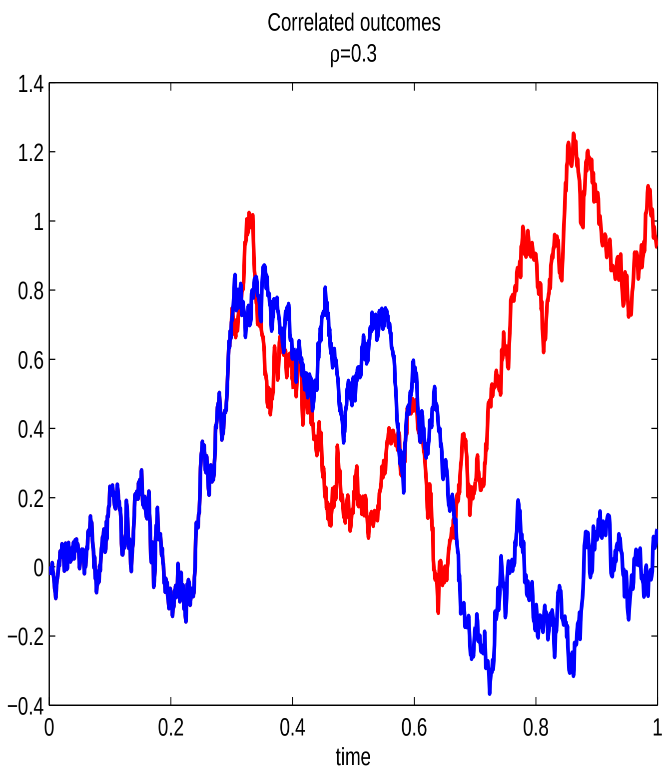
CDO Modeling
- CDOs
- Gaussian Copula Model
- Correlation via Running Time
- Standardized Lévy Processes
- Lévy Copula Model
- The Generic Lévy Model
- Loss Distribution
- Example: Shifted Gamma CDO Model
- Lévy Base Correlation

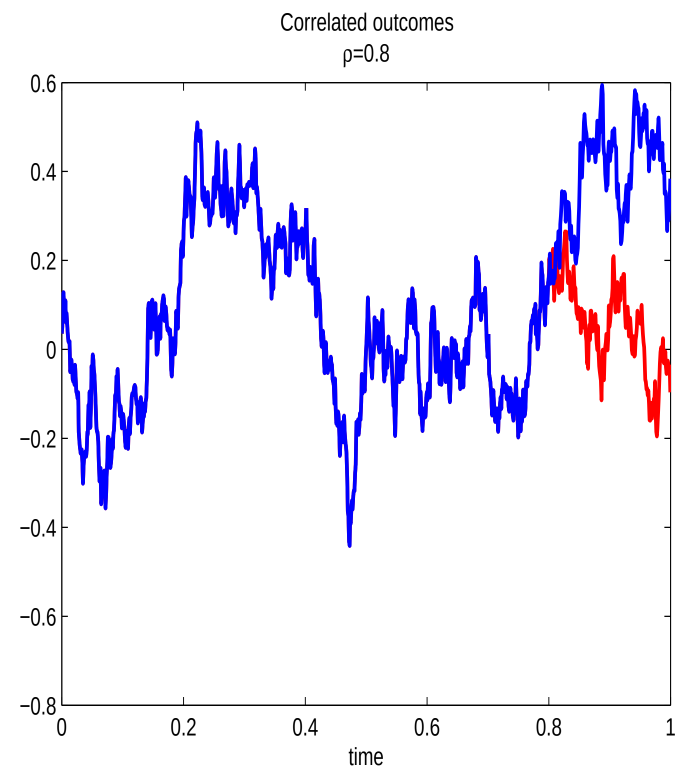
Credit Index Modeling

Portfolio Modeling

Conclusion

- We want to generate standardized (zero mean, variance one) multivariate random non-normal vectors with a prescribed correlation.
- Basic idea: correlate by letting Lévy processes run some time together and then let them free (independence).





Standardized Lévy Processes

- Let $X = \{X_t, t \in [0,1]\}$ be a Lévy process based on an infinitely divisible distribution: $X_1 \sim L$.
- Denote the cdf of X_t by $H_t(x)$, $t \in [0,1]$, and assume it is continuous.
- Assume the distribution is standardized: $E[X_1] = 0$ and Var$[X_1] = 1$.
- Let $X = \{X_t, t \in [0,1]\}$ and $X^{(i)} = \{X_t^{(i)}, t \in [0,1]\}$, $i = 1, 2, \ldots, n$ be independent and identically distributed Lévy processes (so all processes are independent of each other and are based on the same mother infinitely divisible distribution L).
- Let $0 < \rho < 1$, be the correlation that we assume between the defaults of the obligors.

Lévy Copula Model

- We propose the generic one-factor Lévy model.
- We assume that the asset value of obligor $i = 1, \ldots, n$ is of the form:

$$A_i(T) = X_\rho + X^{(i)}_{1-\rho}, \quad i = 1, \ldots, n.$$

- Each $A_i = A_i(T)$ has by the stationary and independent increments property the same distribution as the mother distribution L with distribution function $H_1(x)$. Hence $E[A_i] = 0$ and $\mathrm{Var}[A_i] = 1$.
- Further we have

$$\mathrm{Corr}[A_i, A_j] = \frac{E[A_i A_j] - E[A_i]E[A_j]}{\sqrt{\mathrm{Var}[A_i]}\sqrt{\mathrm{Var}[A_j]}} = E[A_i A_j] = E[X_\rho^2] = \rho.$$

The Generic Lévy Model

- Starting from any mother standardized infinitely divisible law, we can set up a one-factor model with the required correlation.
- The ith obligor defaults at time T if the firm's value $A_i(T)$ falls below some preset barrier $K_i(T)$: $A_i(T) \leq K_i(T)$.
- In order to match default probabilities under this model with default probabilities $p_i(T)$ observed in the market, set $K_i(T) := H_1^{[-1]}(p_i(T))$.
- Then, the probability of having k defaults until time T equals:

$$P(M = k) = \int_{-\infty}^{+\infty} P(M = k | X_\rho = y)\, \mathrm{d}H_\rho(y), \quad k = 0, \ldots, n.$$

Loss Distribution

- Denote by $p_i(y;t)$ is the probability that the firm's value A_i is below the barrier $K_i(t)$, given that the systematic factor X_ρ takes the value y:

$$\begin{aligned} p_i(y;t) &= \mathbb{P}(X_\rho + X^{(i)}_{1-\rho} \leq K_i(t))|X_\rho = y) \\ &= H_{1-\rho}(K_i(t) - y). \end{aligned}$$

- Denote with by $\Pi^k_{n,y}(t)$ the probability to have k defaults out of n firms conditional on the market factor y at time t. Then we have the classical recursive loss distribution formula

$$\begin{aligned} \Pi^0_{n+1,y}(t) &= \Pi^0_{n,y}(t)(1 - p_{n+1}(y;t)) \\ \Pi^k_{n+1,y}(t) &= \Pi^k_{n,y}(t)(1 - p_{n+1}(y;t)) + \Pi^{k-1}_{n,y}(t)p_{n+1}(y;t), \quad k = 1, \ldots, n \\ \Pi^{n+1}_{n+1,y}(t) &= \Pi^n_{n,y}(t)p_{n+1}(y;t), \end{aligned}$$

- This leads to the unconditional probability to have k defaults out of a group of n firms

$$P(M = k) = \int_{-\infty}^{+\infty} \Pi^k_{n,y} \mathrm{d}H_\rho(y)$$

Example: Shifted Gamma CDO Model

- Let us start with a Gamma process $G = \{G_t, t \in [0,1]\}$ with parameters $a > 0$ and $b = \sqrt{a}$, such that Var$[G_1] = 1$ and $E[G_1] = \sqrt{a}$.
- As driving Lévy process, we take the shifted Gamma process :

$$X_t = \sqrt{a}t - G_t, \quad t \in [0,1].$$

- The interpretation in terms of firm's value is again that there is a deterministic up trend $(\sqrt{a}t)$ with random downward shocks (G_t).
- The one-factor shifted Gamma-Lévy model is:

$$A_i = X_\rho + X^{(i)}_{1-\rho},$$

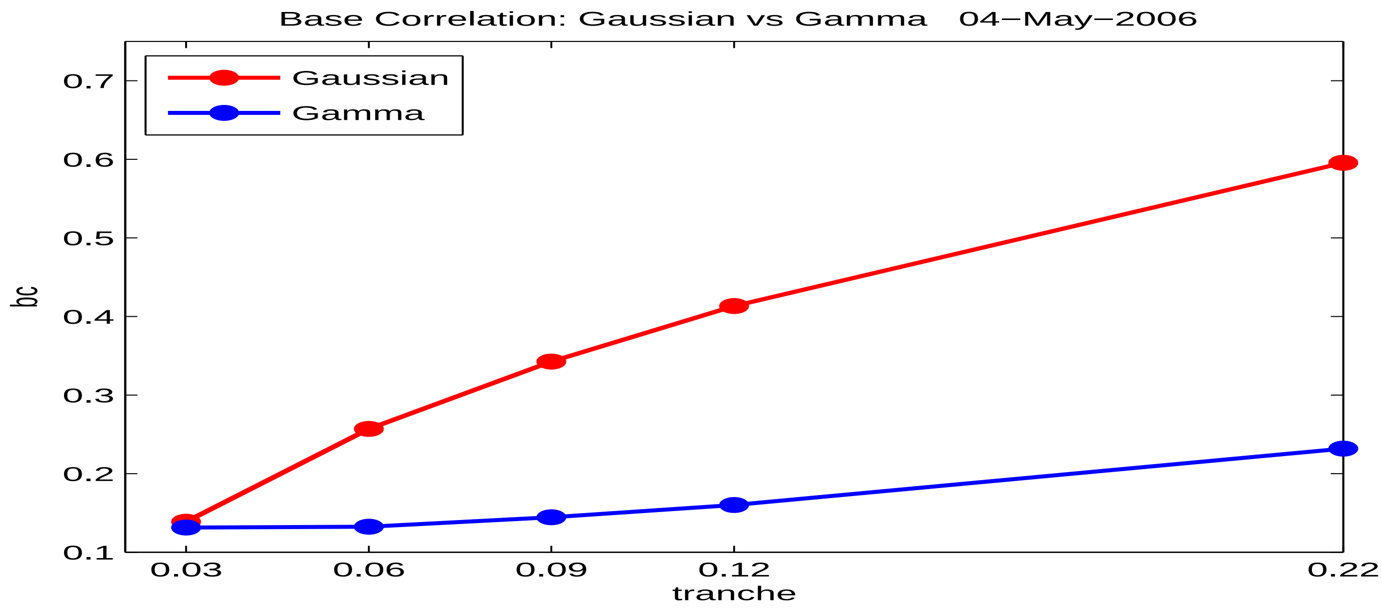
where $X_\rho, X^{(i)}_{1-\rho}, i = 1, \ldots, n$ are independent shifted Gamma-processes.

- The cumulative distribution function $H_t(x;a)$ of X_t, $t \in [0,1]$, can easily be obtained from the Gamma cumulative distribution function:

$$H_t(x;a) = P(\sqrt{a}t - G_t \leq x) = 1 - P(G_t \leq \sqrt{a}t - x), \quad x \in (-\infty, \sqrt{a}t).$$

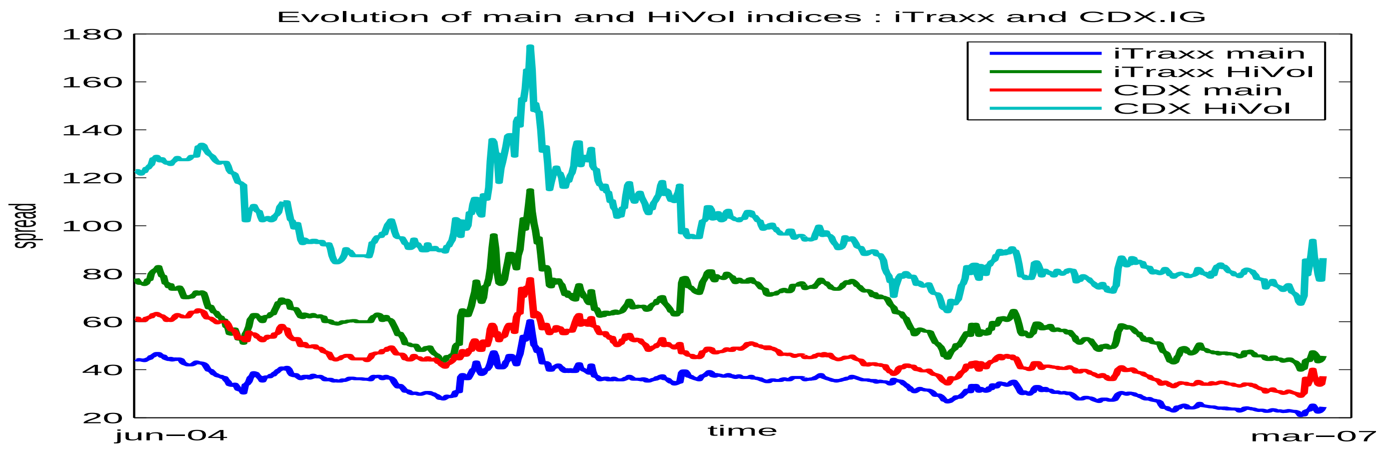
Lévy Base Correlation

- The global fit of the Shifted Gamma model is typically much better than for the Gaussian model.
- Base correlation curve is much more flatter (important for bespoke CDO pricing).
- Delta's are higher for Shifted Gamma compared with Gaussian.
- Bespoke tranches prices differ significantly: e.g. the 5-10 tranche is priced 12.21 bp under the Gaussian and 14.67 bp under the Gamma model.



Credit Indices

- Positions in and derivatives on credit indices have gained a lot in popularity the last years.
- Some of the most known and liquid indices are :
  - iTraxx Main : 125 names.
  - iTraxx HiVol : 30 names
  - CDX.NA.IG Main : 125 names.
  - CDX.NA.HiVol : 30 names
- The spread of the indices (giving protection to all components) are highly correlated with each other.



Black's Model

- The market standard for modeling credit spreads (Pederson (2004)) and pricing swaptions is a modification of the Black's model for interest rate.
- It models spread dynamics in a Black-Scholes' fashion:

$$S_t = S_0 \exp(-\sigma^2 t/2 + \sigma W_t)$$

- Denoting with T the swapoption maturity date, Black's formula simplifies to

$$\begin{aligned}\text{Payer}(T,K) &= \text{forward annuity } \times (F_0^{(\text{adj})}\text{N}(d_1) - K\text{N}(d_2))\\ \text{Receiver}(T,K) &= \text{forward annuity } \times (K\text{N}(-d_2) - F_0^{(\text{adj})}\text{N}(-d_1)).\end{aligned}$$

where $F_0^{(\text{adj})}$ is the adjusted (for no-knockout) forward spread and

$$d_1 = \frac{\log(S_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

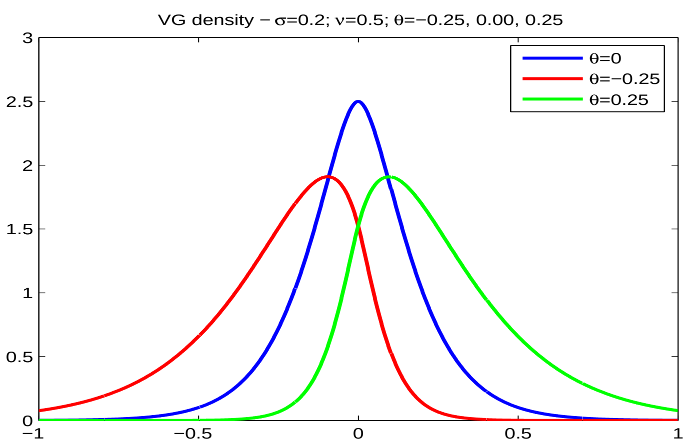
- There is a striking connection with vanilla option prices in equity.
- However, the model has all the deficiencies of the Black-Scholes model.

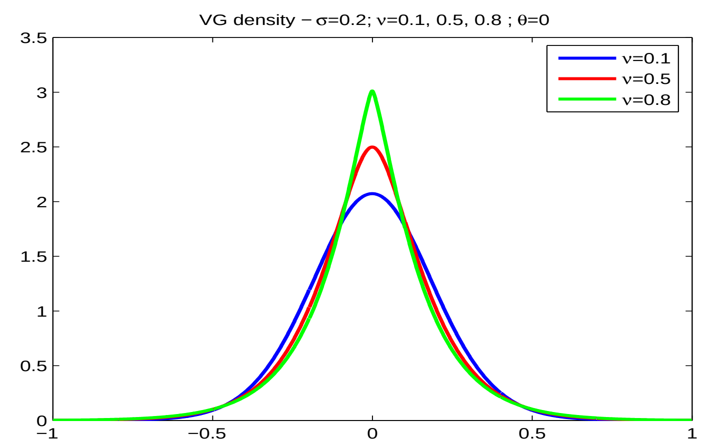
VG Distribution

- To overcome the shortcomings of the Normal distribution in the Black-Scholes setting, we use instead the Variance Gamma $VG(\sigma, \nu, \theta)$ distribution, which is defined by its characteristic function

$$\phi(u) = \left(1 - \mathrm{i}u\theta\nu + \frac{1}{2}\sigma^2\nu u^2\right)^{-1/\nu}.$$

- This distribution has already proven it's modeling abilities in the many settings, because the underlying distribution can taken into account, in contrast to the Normal distribution, skewness and excess-kurtosis.



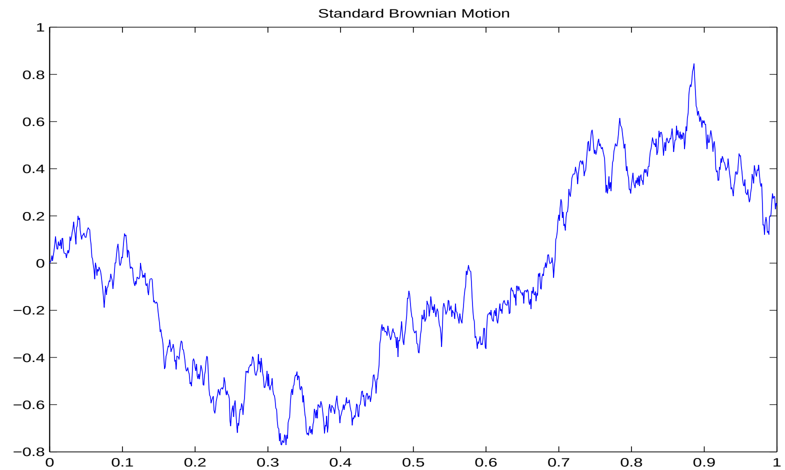


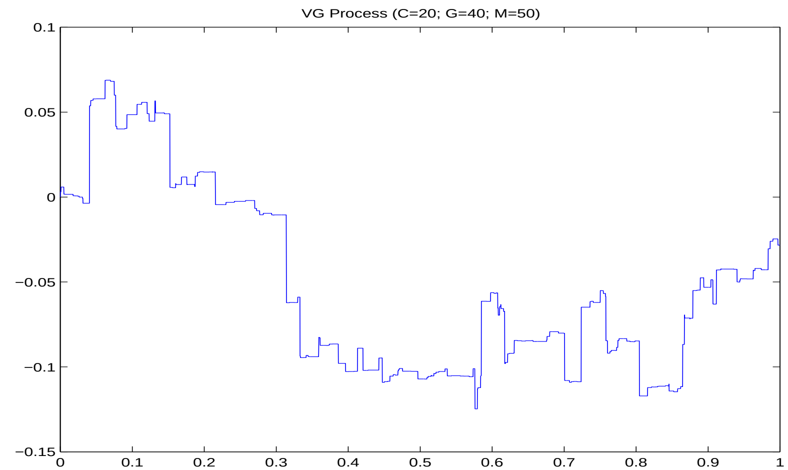
VG Process

- A VG process can be constructed by time-changing a Brownian Motion with drift:

$$X_t = \theta G_t + \sigma W_{G_t}, t \geq 0,$$

  where $W = \{W_t, t \geq 0\}$ is a standard Brownian motion independent from the Gamma process $G = \{G_t, t \geq 0\}$.
- Standard Lévy process theory learns then that X_1 follows a VG(σ, ν, θ) distribution.
- We now look at Black-Scholes according to a new business clock (Gamma time).
- Moreover, the process turns out to be a pure jump process.





VG Model

- We propose a jump driven Lévy model for index spread dynamics.
- Completely similar as in the equity setting. We will replace the Black-Scholes dynamics with the better performing jump dynamics of VG. We now model the spread dynamics as

$$S_t = S_0 \exp(\omega t + \theta G_t + \sigma W_{G_t}) = S_0 \exp(\omega t + X_t),$$

  where $\omega = \nu^{-1} \log\left(1 - \frac{1}{2}\sigma^2\nu - \theta\nu\right)$ assures that $E[S_t] = S_0$..
- The model is the analogue of the VG equity model.
- The pricing of vanilla's (using the Carr-Madan formula in combination with FFT methods) has already worked out in full detail in equity settings.

VG Swaption Pricing

- The main ingredient in the pricing formula is the characteristic function of the log price process of the spread at maturity T.

$$\phi(u;T) = E\left[\exp\left(\mathrm{i}u\left(\log F_0^{(\text{adj})} + \omega T + X_T\right)\right)\right],$$

which is known analytically in the VG case (and in many other Lévy dynamics).

- Swaptions are priced using the Carr-Madan formula:

$$\text{Payer}(T,K) = \text{forward annuity} \times \frac{\exp(-\alpha\log(K))}{\pi}$$

$$\times \int_0^{+\infty} \exp(-\mathrm{i}v\log(K))\frac{\phi(v-(\alpha+1)\mathrm{i};T)}{\alpha^2+\alpha-v^2+\mathrm{i}(2\alpha+1)v}\mathrm{d}v.$$

Multivariate VG Processes I

- To build a multivariate Variance Gamma model (MVG), we need several ingredients:
  - a common Gamma process $G = \{G_t, t \geq 0\}$ with parameters $a = b = 1/\nu$.
  - a N-dimensional Brownian motion $\vec{W} = \{(W_t^{(1)}, \ldots, W_t^{(N)}), t \geq 0\}$.
  - assume that $\vec{W}$ is independent of G.
  - the Brownian motions have a correlation matrix:

$$\rho_{ij}^W = E[W_1^{(i)} W_1^{(j)}].$$

- A multivariate VG process $\vec{X} = \{(X_t^{(1)}, \ldots, X_t^{(N)}), t \geq 0\}$ is defined as:

$$X_t^{(i)} = \theta_i G_t + \sigma_i W_{G_t}^{(i)}, \quad t \geq 0.$$

Multivariate VG Processes II

- There is dependency between the $X_t^{(i)}$'s due to two causes:
  - the processes are all constructed by time-changing with a common Gamma time.
  - there is dependency also built in via the Brownian motions.
- The correlation between two components is given by:

$$
\begin{aligned}
\rho_{ij} &= \frac{E[X_1^{(i)}X_1^{(j)}] - E[X_1^{(i)}]E[X_1^{(j)}]}{\sqrt{\text{Var}[X_1^{(i)}]}\sqrt{\text{Var}[X_1^{(j)}]}} \\
&= \frac{\theta_i\theta_j\nu + \sigma_i\sigma_j\rho_{ij}^W}{\sqrt{\sigma_i^2 + \theta_i^2\nu}\sqrt{\sigma_j^2 + \theta_j^2\nu}}
\end{aligned}
$$

MVG Spread Dynamics

- We use the MVG processes to describe the evolution of N correlated spreads:

$$S_t^{(i)} = S_0^{(i)} \exp(\omega_i t + \theta_i G_t + \sigma_i W_{G_t}^{(i)}), \quad i = 1, \ldots, N,$$

where

$$\omega_i = \nu^{-1} \log \left(1 - \frac{1}{2}\sigma_i^2 \nu - \theta_i \nu \right).$$

- The parameters ν, θ_i and σ_i are coming from a (joint) calibration on swaptions on the individual spreads.
- Then ρ_{ij}^W is set to match a pre-specified (e.g. historical) correlation ρ_{ij} between the spreads:

$$\rho_{ij}^W = \frac{\rho_{ij}\sqrt{\sigma_i^2 + \theta_i^2 \nu}\sqrt{\sigma_j^2 + \theta_j^2 \nu} - \theta_i \theta_j \nu}{\sigma_i \sigma_j}.$$

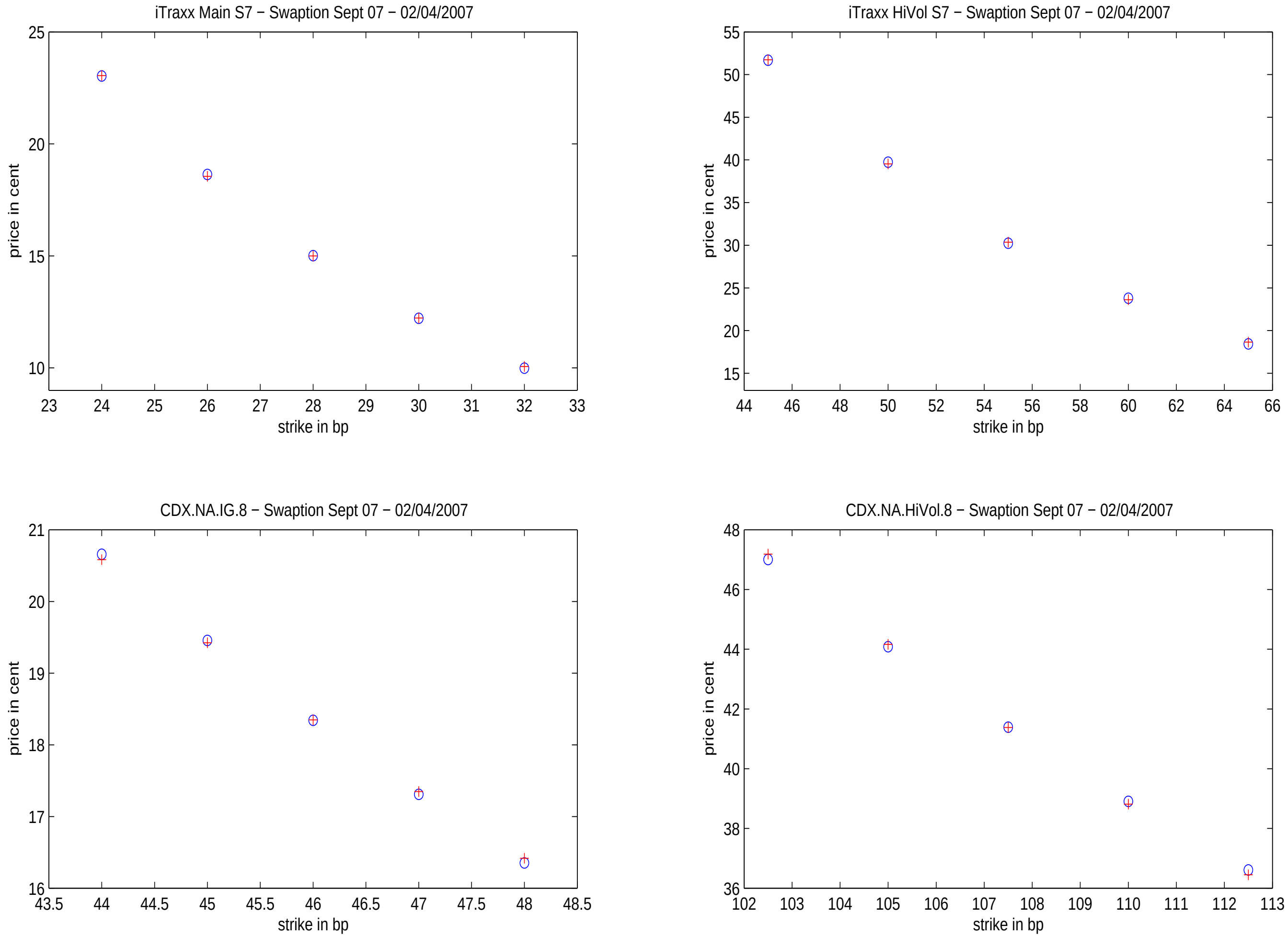
Correlated Spread Dynamics

- We assume the following correlated VG dynamics for the spreads:

$$
\begin{aligned}
S_t^{\text{(iTraxx main)}} &= S_0^{\text{(iTraxx main)}} \exp(\omega_1 t + \theta_1 G_t + \sigma_1 W_{G_t}^{(1)}) \\
S_t^{\text{(iTraxx HiVol)}} &= S_0^{\text{(iTraxx HiVol)}} \exp(\omega_2 t + \theta_2 G_t + \sigma_2 W_{G_t}^{(2)}) \\
S_t^{\text{(CDX main)}} &= S_0^{\text{(CDX main)}} \exp(\omega_3 t + \theta_3 G_t + \sigma_3 W_{G_t}^{(3)}) \\
S_t^{\text{(CDX HiVol)}} &= S_0^{\text{(CDX HiVol)}} \exp(\omega_4 t + \theta_4 G_t + \sigma_4 W_{G_t}^{(4)}),
\end{aligned}
$$

  where G_t is a common Gamma Process, such that $G_t \sim$ Gamma$(t/\nu, 1/\nu)$, and the $W_t^{(i)}$'s are correlated standard Brownian motions with a given correlation matrix ρ^W.
- First calibrate (with a common ν parameter) the individual spread dynamics on a series of swaptions.

Joint Calibration on Swaptions



Matching Correlation

- Then match with the required correlation (log-return historical correlation from 21/06/2004 - 13/03/2007):

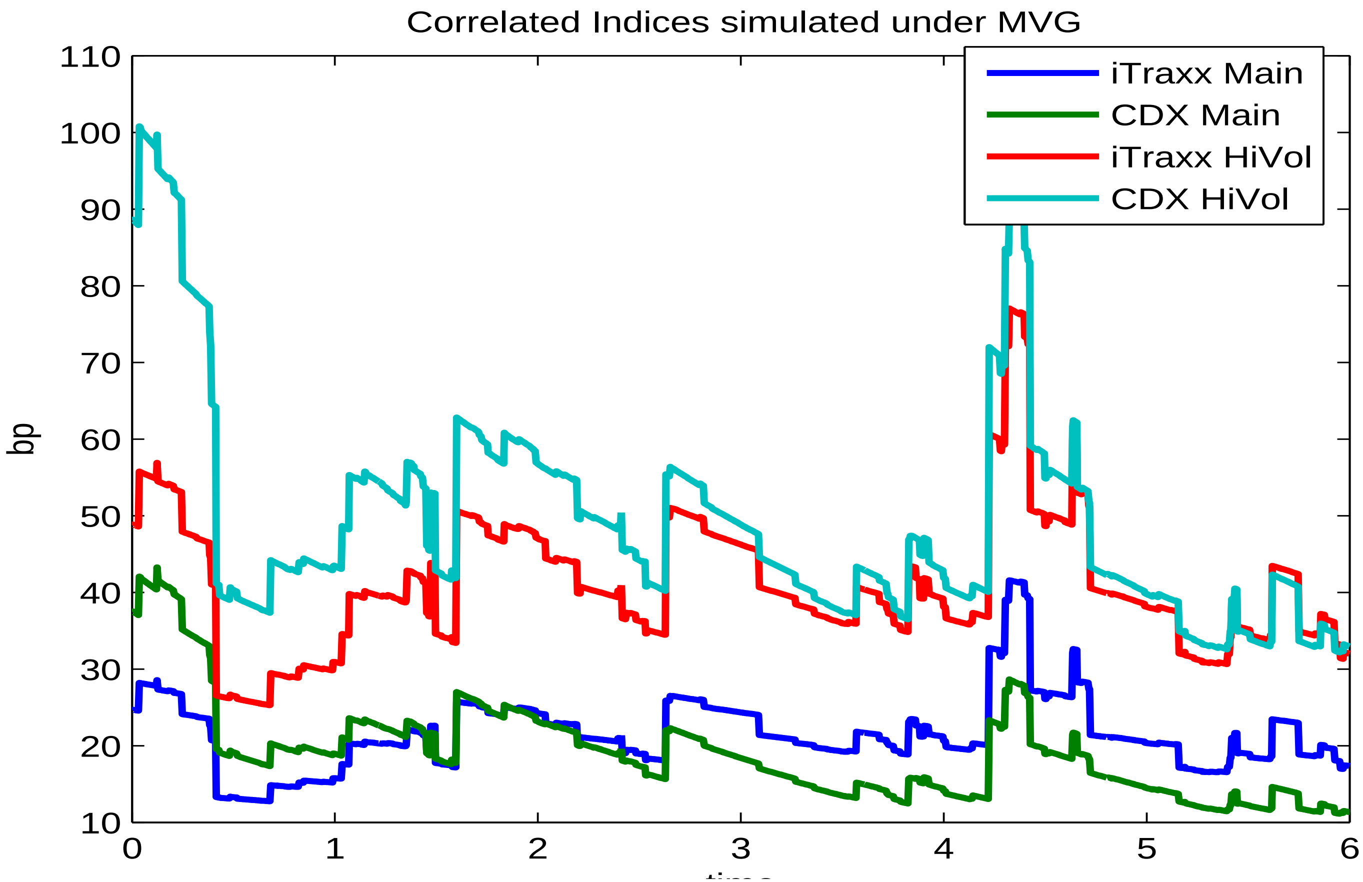
correlation	iTtraxx main	iTraxx Hivol	CDX main	CDX HiVol
iTtraxx main	1.0000	0.9258	0.4719	0.3339
iTraxx HiVol	0.9258	1.0000	0.4398	0.3281
CDX main	0.4719	0.4398	1.0000	0.8580
CDX HiVol	0.3339	0.3281	0.8580	1.0000

- To do that, set the Brownian correlation matrix equal to:

$$\rho^W = \begin{bmatrix} 1.0000 & 0.9265 & 0.4814 & 0.3354 \\ 0.9265 & 1.0000 & 0.4379 & 0.3218 \\ 0.4814 & 0.4379 & 1.0000 & 0.8525 \\ 0.3354 & 0.3218 & 0.8525 & 1.0000 \end{bmatrix}$$

Simulating Correlated Indices

- Simulation of Gamma processes and correlated Brownian motions is easy and opens the way to exotic basket option pricing on credit indices.



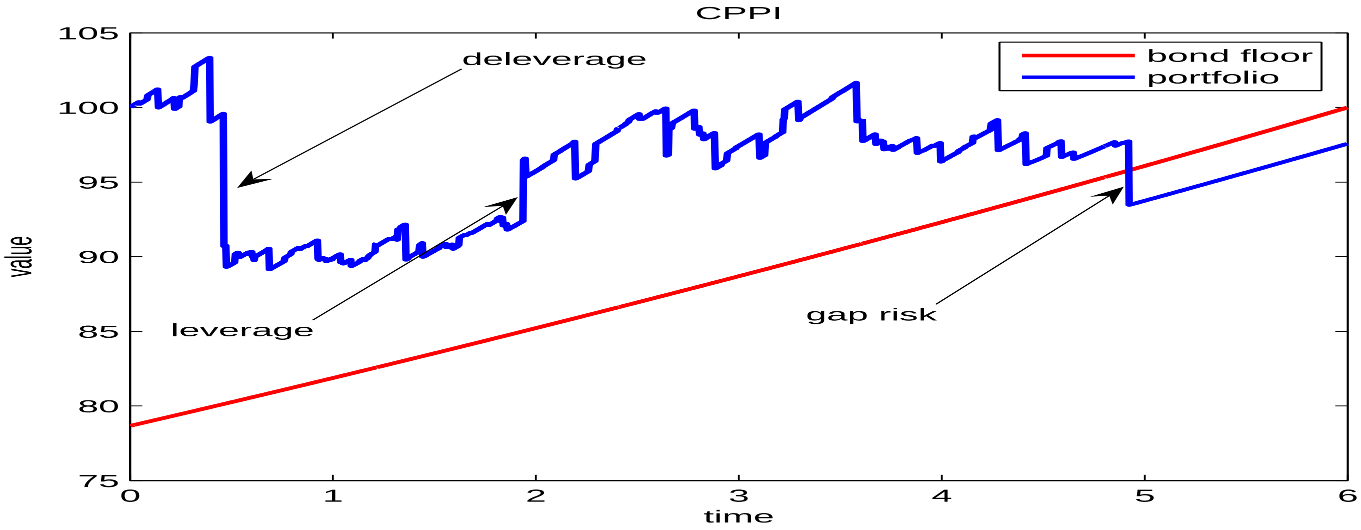
CPPIs

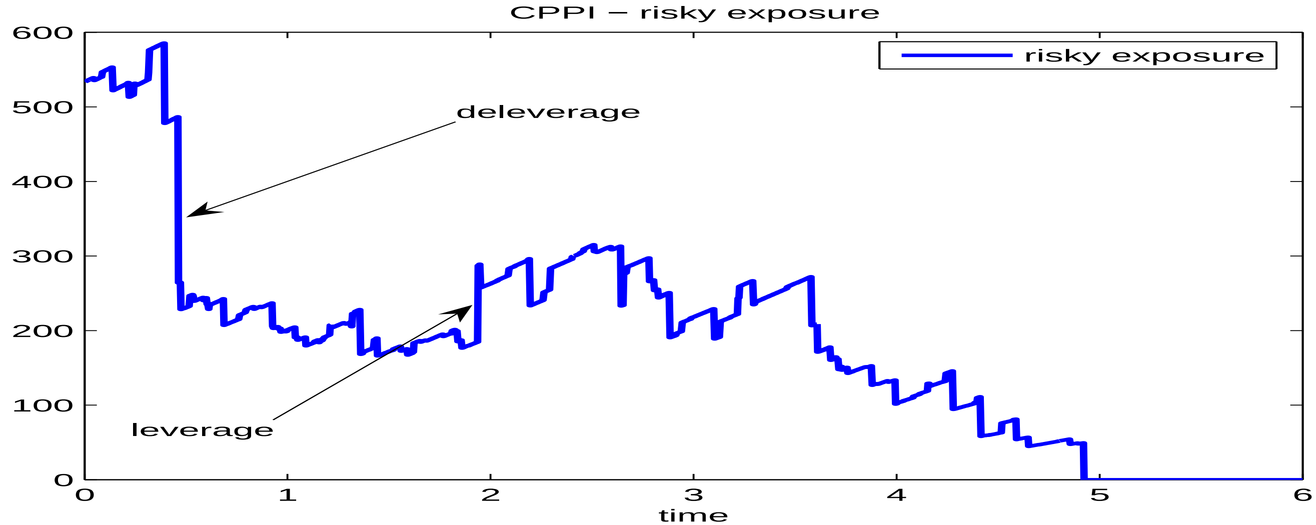
- The Constant Proportion Portfolio Insurance (CPPI) was first introduced by Fisher Black and Robert Jones in 1986.
- Recently, credit-linked CPPIs have become popular to create capital protected credit-linked notes.
- CPPI products are leveraged investments whose return depends on the performance of an underlying trading strategy. Quite often positions are taken into the available credit indices (iTraxx, CDX, ...).
- Credit CPPIs combine dynamic leverage with principal protection.
- Leverage is increased when the strategy performs well and is reduced when it performs poorly.

CPPI Example

- We start with a portfolio of 100M EUR and an investment horizon of 6 y.
- The principal of the initial investment is protected (bond floor).
- We set the leverage at 25.
- We call the cushion the difference between the portfolio value and the bond-floor.
- Multiplying the cushion with the constant leverage factor of 25, gives the risky exposure that we are going to take.
- We are taking the following positions:
  - sell protection on iTraxx Europe Main On the run (5Y) for half of the risky exposure;
  - buy protection on iTraxx Europe HiVol On the run (5Y) for $\frac{1}{2}\frac{30}{125}$ the risky exposure;
  - sell protection on DJ CDX.NA.IG Main On the run (5Y) for half of the risky exposure;
  - buy protection on DJ CDX.NA.HiVol On the run (5Y) for $\frac{1}{2}\frac{30}{125}$ of the risky exposure.
- We rebalance daily and roll every six months.

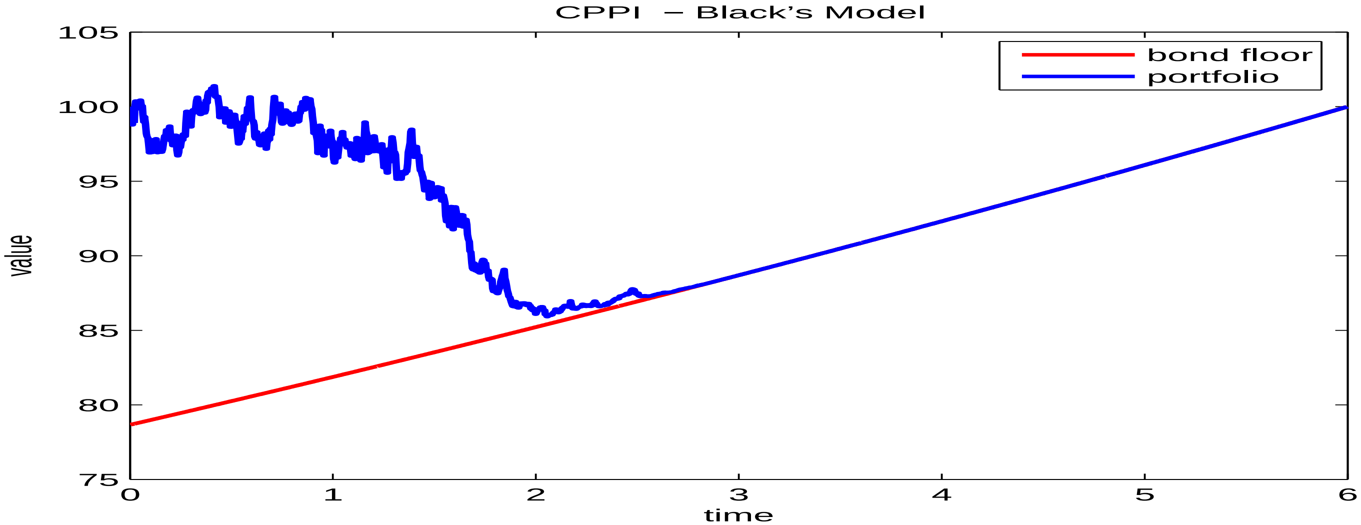
CPPI Gap Risk





CPPI No Black Gap

- The gap risk under Black's model is ...zero !!!
- Indeed, because of the continuous paths of the Brownian Motion, the bond floor is never crossed but always hit.

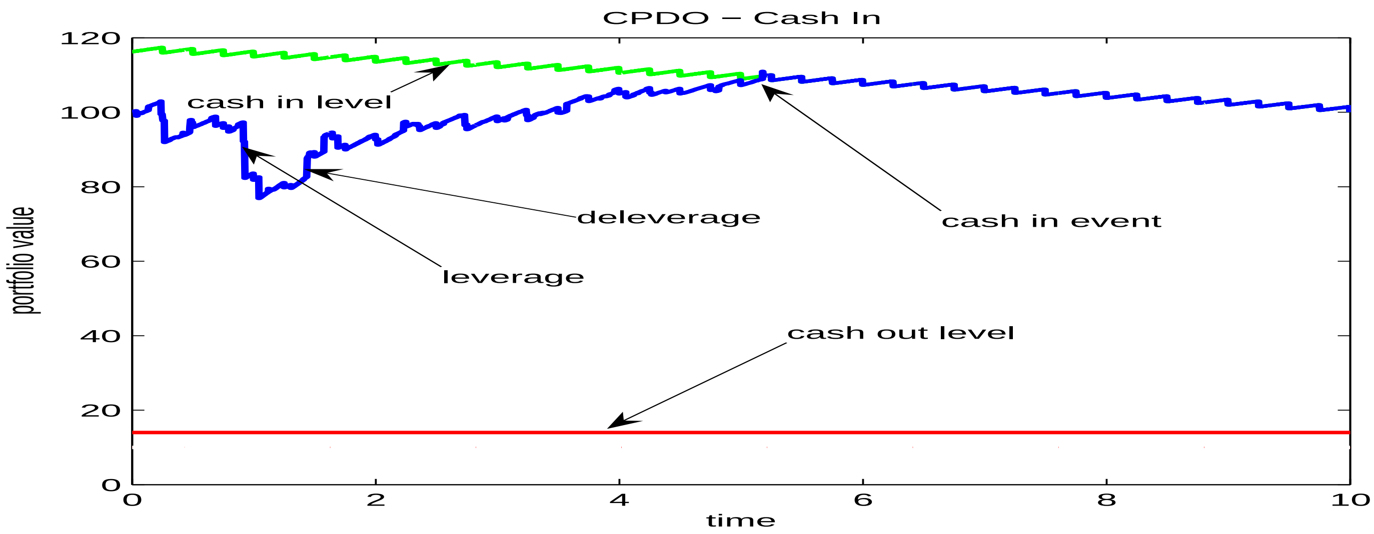


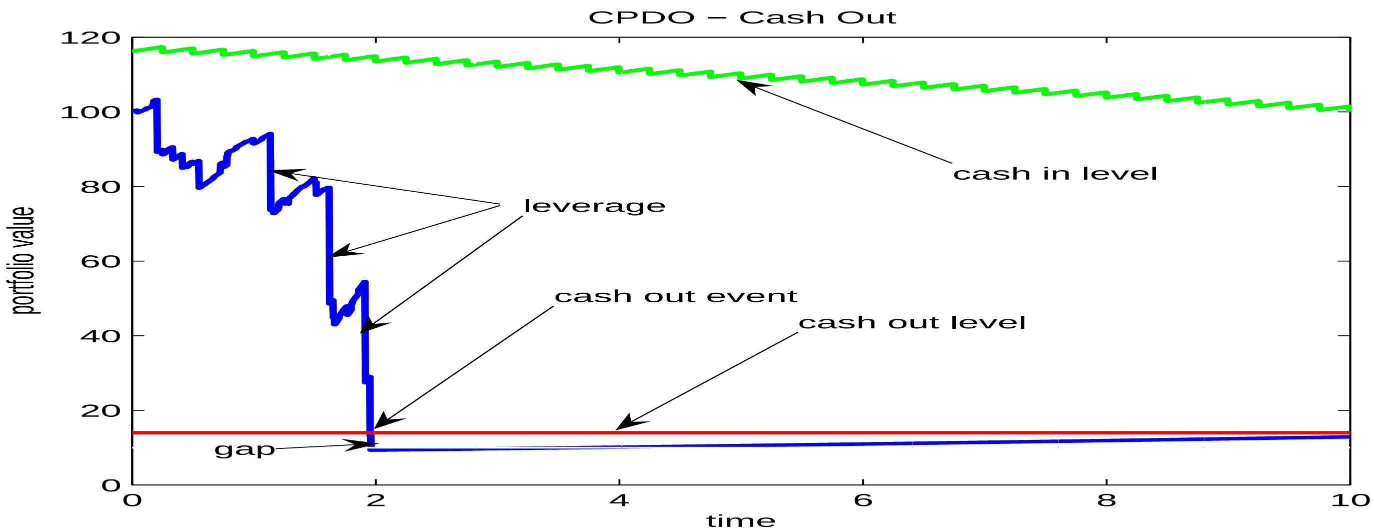
- The gap risk estimate based on 100000 MC simulations (<15 min) for VG model is around 1.5 bp per year.

CPDOs

- The Constant Proportion Debt Obligation (CPDO) is another kind of leveraged investments strategy.
- Here the aim is to obtain a preset return (e.g. LIBOR plus 100 bp).
- If this return can be guaranteed the risky positions are closed (cash-in).
- If one comes close to the target one deleverages.
- If one underperforms one increase leverages (cfr. a gambler chasing losses).
- If the portfolio really performs bad, we have a cash-out event (cfr. default).

CPDO: Cash In - Cash Out





CPDO Details

- We have a prefixed goal of return: say LIBOR + 200 bps pa.
- We fix a target loading factor: $\alpha = 1.05$, maximum leverage factor $M = 15$, cash-out level $C_O = 15\%$ and a risky income fraction (fudge factor): $\beta = 0.75$.
- For each evaluation date
  - the portfolio value V_t is composed out of the cash account, the fee income and the MtM of the positions;
  - the target T_t is the PV of all future liabilities (coupons and par value);
  - if V_t is above the target all positions are closed and we have a cash-in;
  - if V_t is below the cash-out level $V_0 C_O$, we have a cash-out, all positions are closed and we have a gap of $V_0 C_O - V_t$;
  - the shortfall is target minus value: $S_t = \alpha T_t - V_t$;
  - the PV risky income is spread times risky annuity: $I_t = \beta S_t A_t$;
  - the exposure is $E_t = \min \{S_t / I_t, M V_0\}$ which is equally divided (sell protection) into iTraxx main and CDX Main.
  - note leverage $m = E_t / V_0$ is capped at M;

Rating CPDOs I

- The target return of the first CPDO (ABN Amro's SURF) was LIBOR + 200 bps pa and it was rated AAA.
- This means that a return of at least LIBOR + 200 bps pa (cash in) is achieved in at least 99.272 % of the cases.

Standard and Poor's Target Probabilities

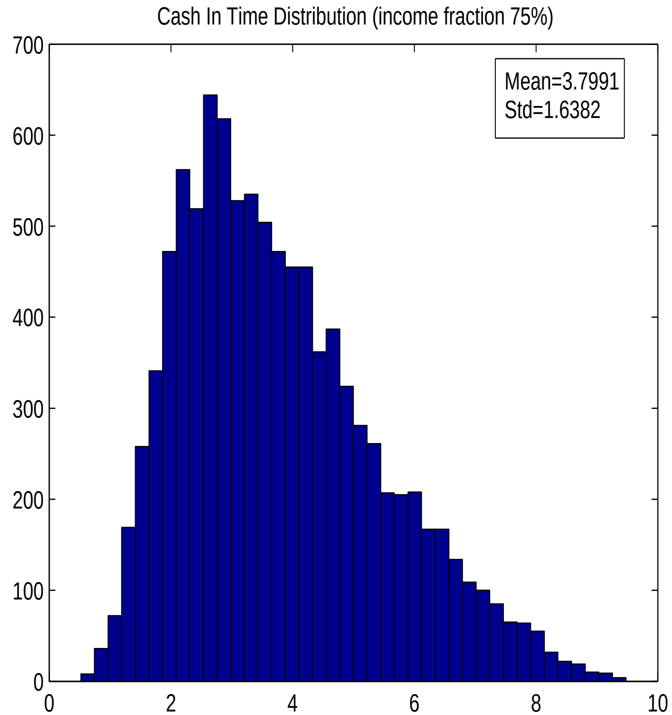
	AAA	AA+	AA	AA-	A+	A	A-	BBB+
1	0.000%	0.002%	0.013%	0.024%	0.027%	0.033%	0.049%	0.234%
2	0.009%	0.017%	0.062%	0.078%	0.097%	0.121%	0.185%	0.514%
3	0.030%	0.050%	0.135%	0.166%	0.212%	0.263%	0.396%	0.850%
4	0.065%	0.104%	0.232%	0.290%	0.372%	0.459%	0.676%	1.246%
5	0.118%	0.182%	0.356%	0.452%	0.578%	0.709%	1.020%	1.704%
6	0.190%	0.287%	0.512%	0.654%	0.830%	1.013%	1.424%	2.221%
7	0.285%	0.420%	0.701%	0.897%	1.128%	1.368%	1.883%	2.792%
8	0.405%	0.584%	0.927%	1.182%	1.472%	1.774%	2.395%	3.413%
9	0.552%	0.781%	1.191%	1.509%	1.859%	2.226%	2.954%	4.076%
10	0.728%	1.013%	1.493%	1.876%	2.290%	2.724%	3.557%	4.777%

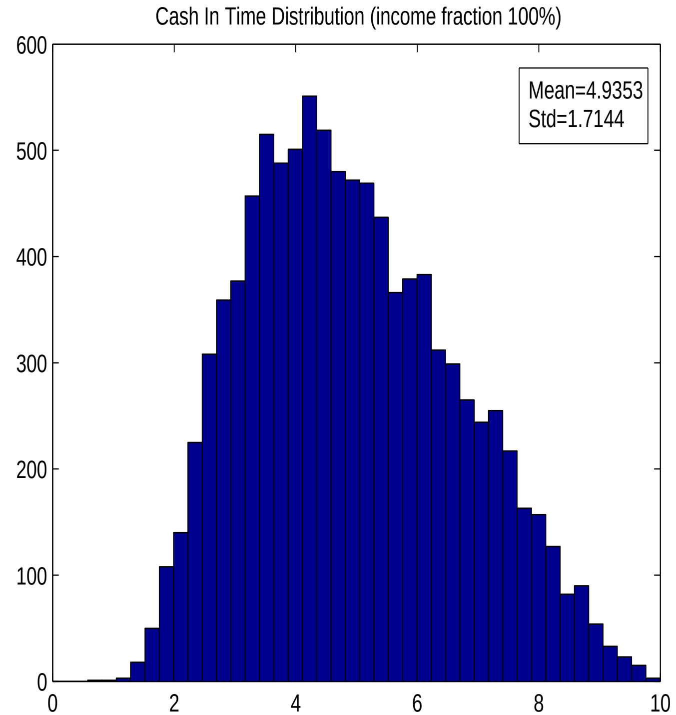
Cash in Time Distribution

- Cash-in-time distribution properties vary for different fudge factors.
- For LIBOR + 150bps pa, we have:

$$\text{mean}_{0.75} = 3.80, \qquad \text{std}_{0.75} = 1.64$$

$$\text{mean}_{1.00} = 4.94, \qquad \text{std}_{1.00} = 1.71$$





Rating CPDOs II

- Cash-in and ratings of the CPDO under BS and the MVG model (fudge factor: $\beta = 0.75$).

	MVG	B_{35}	B_{50}
100	99.02% (AA+)	99.79% (AAA)	99.24% (AA+)
150	97.86% (A+)	99.29% (AAA)	98.35% (AA-)
200	97.43% (A)	98.53% (AA)	97.62% (A)

Conclusion

- The Credit Risk market has seen an explosive growth.
- Contracts of an unprecedented complexity have found their way into the business.
- Jump models are essential to assess all the risks involved.
- Lévy Models (VG, Gamma, SCMY, ...) are capable of describing in more realistic way the risks present.
- Many techniques from Equity can readily be applied in the credit setting.
- Model can not only be used for pure credit setting, but can be pimped to hybrids easily.

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