Pricing of hourly exercisable electricity swing options using different price processes

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EnBW
The Power Pioneers

Agenda

- Practical importance of swing options
- Part I: Three different models for hourly power spot prices
  - Model definition
  - Comparison of results
- Part II: Least squares Monte Carlo method
  - Introduction of an efficient algorithm
  - Comparison of results for the three spot price models
- Summary & open questions

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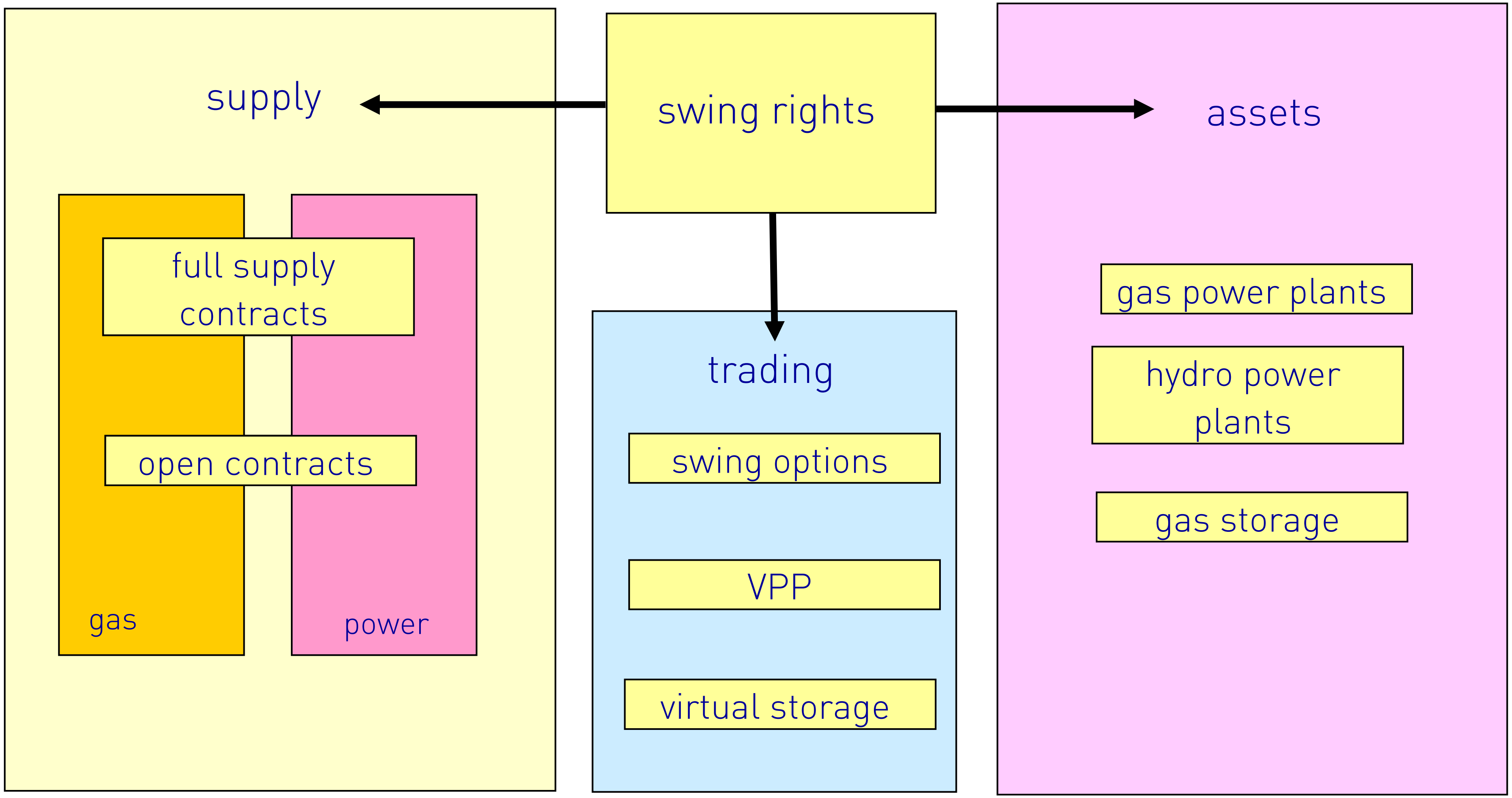
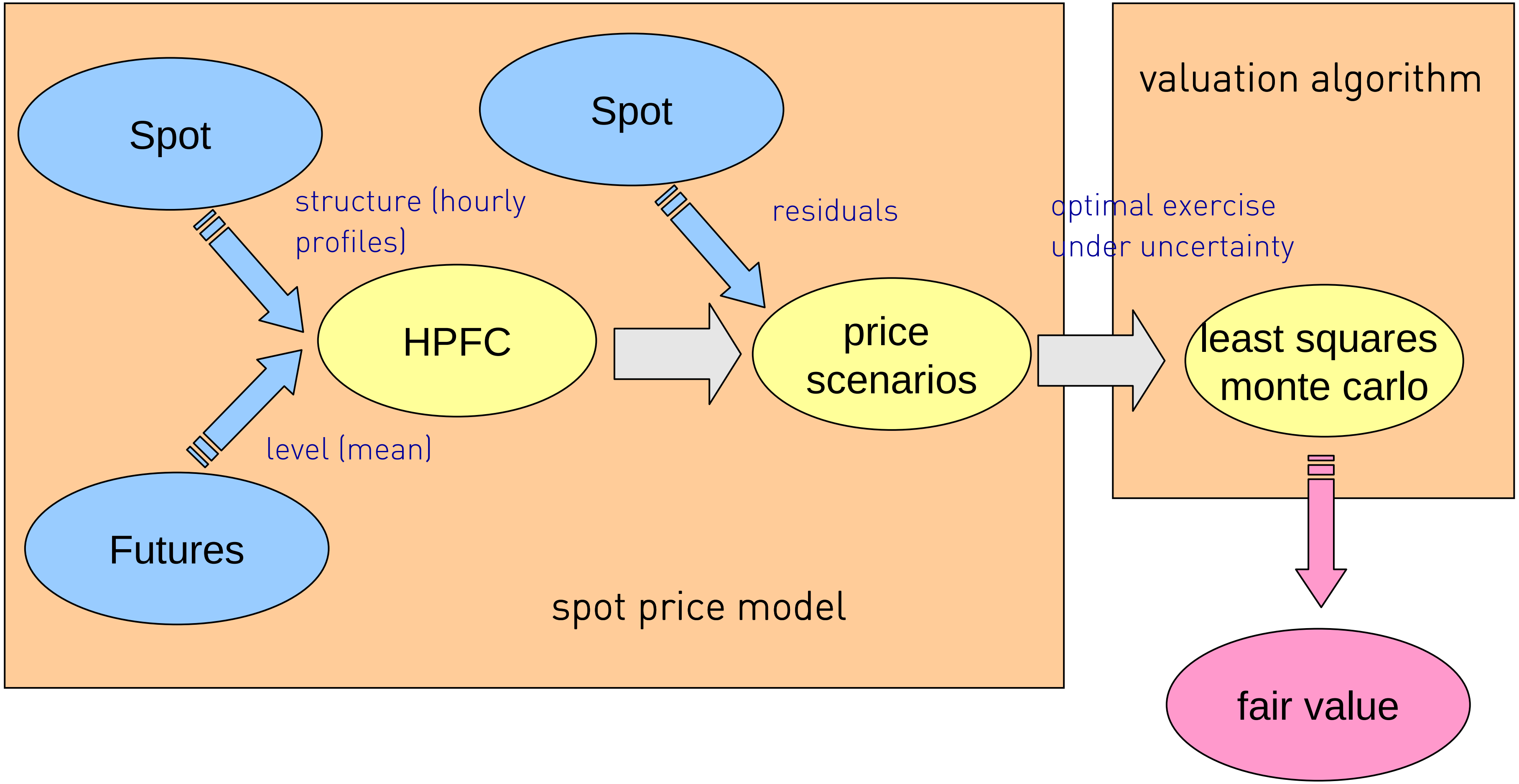
Appearance of flexibility



Valuation of swing options - overview



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Relation between HPFC and spot price model

Two factor spot price model:

$$S_T = \exp(f(T) + X_T + Y_T)$$

HPFC: hourly futures prices

$$F_{t,T} = \mathbb{E}^Q[S_T|\mathcal{F}_t]$$ with natural filtration $\mathcal{F}_t = \sigma(S_s : s \leq t)$

Practitioners approach:

- Calculate daily and weekly pattern from historical spot prices
- Construct candidate curve from these pattern and scale to quoted futures prices to get HPFC
- Scale mean of price scenarios to reproduce HPFC

Problem: consistence of HPFC and price process $\rightarrow$ tail wags the dog

Relation between HPFC and spot price model

expectation of deterministic part:

$$\mathbb{E}^Q[\exp(f(T))|\mathcal{F}_T] = \exp(f(T)) \ .$$

expectation of short term factor:

$$\mathbb{E}^Q[\exp(X_T)|\mathcal{F}_t] \approx \mathbb{E}[\exp(X_T)] \approx \exp(\mathbb{E}[X_T] + \mathrm{Var}[X_T]/2) \ .$$

expectation of long term factor:

$$\mathbb{E}^Q[\exp(Y_T)|\mathcal{F}_t] = F_t^S(T) = \sum_{i=1}^{N_F} \left(F^S_{H_{P,i}} 1_{H_{P,i}}(T) + F^S_{H_{OP,i}} 1_{H_{OP,i}}(T) \right)$$

Spot price model and HPFC:

Futures: $$F_{t,T} = F_t^S(T) \cdot \exp(\mathbb{E}[X_T] + \mathrm{Var}[X_T]/2 + f(T))$$

Spot: $$S_T = F_{t,T} \cdot \exp(-\mathbb{E}[X_T] - \mathrm{Var}[X_T]/2 + X_T + Y_T) \ .$$

Price forward curve and spikes

Do spikes have to influence the HPFC?

Two different methods possible:

1. Use **robust regression** for fitting the deterministic component $f(t)$
   Spikes end up in $X(t)$ and thereby indirectly in the HPFC via the expectation and variance of $X(t)$
2. Use **normal regression**
   Spikes end up directly in $f(t)$ and to a smaller amount indirectly via expectation and variance of $X(t)$

Anyway – the spikes should be introduced in the HPFC!

Spot price models

EEX spot prices: stochastic 24-dimensional vector process $T = (d, h)$

$$S_{d,h} = \exp(f(d,h) + X_{d,h} + Y_{d,h})$$

Log-prices split up into daily average and hourly deviations:

$$s_{d,h} = \ln S(d,h) = \bar{s}_d + \Delta s_{d,h}$$

1. **Deterministic component** $f(d,h) = \bar{f}(d) + \Delta f(d,h)$

   Regression models for daily average and deviations from daily average

2. **Long term factor** $Y_{d,h}$

   Brownian Motion with implicit volatility taken from option prices quoted at the EEX

3. **Short term factor** $X_{d,h} = \bar{X}_d + \Delta X_{d,h}$
   - Most models found in literature have only daily not hourly granularity.
   - PCA & ARMA-models for hourly deviations (cf. Schindelmayr (2005))
   - three different models for daily part.

Deterministic component – daily part

$$\bar{f}(d) = c_0 + c_1 \cdot (d - d_0) + \sum_{i=2}^{7} c_i \delta_{W(d),i} (1 - w_d) + c_8 \cdot w_d + c_9 \delta_{d,\mathrm{CN}}$$
$$+ \sum_{j=1}^{N_h} \left(c_{2j+8} \sin(2\pi j \cdot (d - d_0)/365) + c_{2j+9} \cos(2\pi j \cdot (d - d_0)/365) \right) .$$

Holidays and vacation periods:

$$w_d = \begin{cases} 0 & d \text{ is neither holiday nor bridge day} \\ p & d \text{ is holiday (p population weight)} \\ g \cdot p & d \text{ is bridge day, } g = 0.4 \end{cases}$$

$$\delta_{d,\mathrm{CN}} = \begin{cases} 1 & d \text{ business day between Christmas and New Year's Eve} \\ 0 & \text{else} \end{cases}$$

If $N_h = 3$ has been chosen, 16 coefficients $c_0, \ldots, c_{15}$ have to be fitted.

Deterministic component – hourly part

- hourly part is subdivided into four time series for the quarters of a year
- for each quarter modeled using dummy variables for each hour of the day and each weekday (Sundays and holidays are collected in a single dummy variable)

$$\Delta f(d,h) = c_1^{Q(d)} \cdot (1 - \delta_{W(d),1} \cdot \delta_{h,1}) + \sum_{i=1}^{7} \sum_{j=1}^{24} c_{24(i-1)+j}^{Q(d)} \cdot \delta_{W(d),i} \cdot \delta_{h,j} \ ,$$

hourly model contains 168 coefficients $c_1, \ldots, c_{168}$

Daily and hourly residuals result, that are split up in separate time series for business and non-business days:

$$r_d = 1_B(d) \cdot r_d^B + (1 - 1_B(t)) \cdot r_d^{NB}$$

$$\Delta r_{d,h} = 1_B(d) \cdot \Delta r_{d,h}^B + (1 - 1_B(t)) \cdot \Delta r_{d,h}^{NB} \ .$$

Examined models for short term factor

Model A: regime switching model for daily average and 24 ARMA processes for the hourly deviations from the daily average

Model B: jump diffusion model (with Bernoulli jumps) instead of regime switching. Hourly deviations treated as in Model A.

Model C: NIG-model for daily average. Hourly deviations treated as in Model A.

All models share same deterministic component and longterm factor

Model A – regime switching

AR(1) process with three regimes, transition described by Markov chain:

$$\begin{aligned} \text{Regime M with mean-reversion:} \quad & x_t = \alpha_1 \cdot x_{t-1} + \mu_1 + \sigma_1 \cdot \varepsilon_t \ , \\ \text{negative Spike-Regime S-:} \quad & x_t = \alpha_2 \cdot x_{t-1} + \mu_2 + \sigma_2 \cdot \varepsilon_t \ , \\ \text{positive Spike-Regime S+:} \quad & x_t = \alpha_3 \cdot x_{t-1} + \mu_3 + \sigma_3 \cdot \varepsilon_t \ , \end{aligned}$$

$$\text{Markov transition probability matrix:} \quad \Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} .$$

$$\varepsilon_t \ \sim N(0,1)$$

Parameter estimation: Hamilton filter

For spike days: sample hourly profile and randomly choose historical profile in simulation

Implicit definition of spikes: results from other regime, characterized by another volatility, another mean as well as another mean-reversion rate

Model B – jump diffusion

AR(1) process with up- and down-jump terms: jump height is assumed to be normally distributed, arrival rate is bernoulli distributed:

$$x_t = \alpha_{\mathrm{JD}} \cdot x_{t-1} + \mu_{\mathrm{JD}} + \sigma_{\mathrm{JD}} \cdot \varepsilon_t + \kappa^+ \cdot \nu_t^+ + \kappa^- \cdot \nu_t^-$$

$$\nu_t^+ \sim \mathrm{Bernoulli}(p_{\mathrm{JD}}^+) \ , \quad \kappa^+ \sim \mathrm{N}(\mu_{\mathrm{JD}}^+, (\sigma_{\mathrm{JD}}^+)^2) \ ,$$
$$\nu_t^- \sim \mathrm{Bernoulli}(p_{\mathrm{JD}}^-) \ , \quad \kappa^- \sim \mathrm{N}(\mu_{\mathrm{JD}}^-, (\sigma_{\mathrm{JD}}^-)^2).$$

Parameter estimation: recursive filtering for spike detection, $[-2.5 \cdot \sigma, 2.5 \cdot \sigma]$ sample historical spike profiles and randomly draw historical profile on spike days in simulation

Implicit definition of spikes: outlier characterised by mean and rareness

Model C - normal inverse Gaussian process (NIG)

- Models A and B are based on the normal distribution, heavy tails of the residuals caused by spikes and jumps are reproduced via regime switching or jump terms,
- In Model C daily average is assumed to be NIG-distributed, a generalized hyperbolic distribution that is flexible enough by itself to reproduce heavy tails:

$$\bar{x}_d \sim \mathrm{NIG}(\alpha_{\mathrm{nig}}, \beta_{\mathrm{nig}}, \delta_{\mathrm{nig}}, \mu_{\mathrm{nig}})$$

Parameter estimation: MFE Toolbox (Weron (2006, 2007))

Implicit definition of spikes: result from heavy tails of the NIG distribution

Price process for hourly deviations

- Collect all days that are not classified as spike days in time series,
- Apply orthogonal transformation of a pca
- Fit 24 ARMA(1,1) processes to resulting independent time series

$$y_t^i = C_{\mathrm{ARMA}}^i + \phi_{\mathrm{AR}}^i y_{t-1}^i + \varepsilon_t^i + \theta_{\mathrm{MA}}^i \varepsilon_{t-1}^i$$

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Comparison of estimated parameters (April 28, 2008)

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Model A

Parameter	Business days Regime 1	Business days Regime 2	Business days Regime 3	Non-business days Regime 1	Non-business days Regime 2	Non-business days Regime 3
μ	−0.000	−0.086	0.078	0.007	−0.109	−0.826
α	0.917	1.164	0.570	0.819	0.094	1.420
σ	0.101	0.245	0.154	0.173	0.318	0.833
	0.924	0.000	0.370	0.929	0.262	0.000
Π	0.076	0.186	0.516	0.056	0.592	0.926
	0.000	0.814	0.114	0.015	0.146	0.074

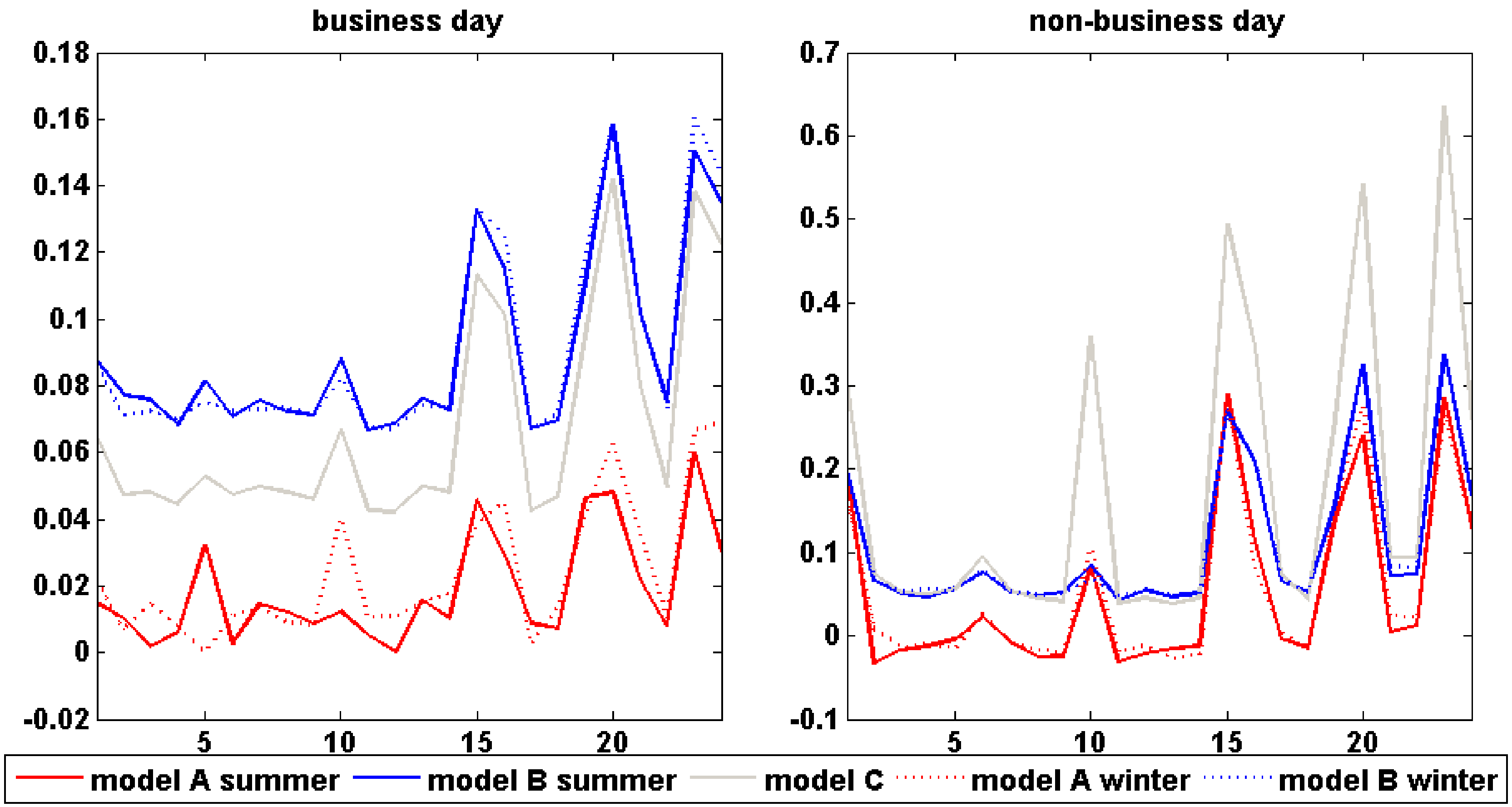
Model B

	p^+_{JD}	μ^+_{JD}	σ^+_{JD}	p^-_{JD}	μ^-_{JD}	σ^-_{JD}	μ_{JD}	α_{JD}	σ_{JD}
Business days	0.015	0.292	0.337	0.017	−0.248	0.416	0.001	0.857	0.146
Non-business days	0.000	0.000	0.000	0.035	−1.409	1.311	−0.008	0.648	0.255

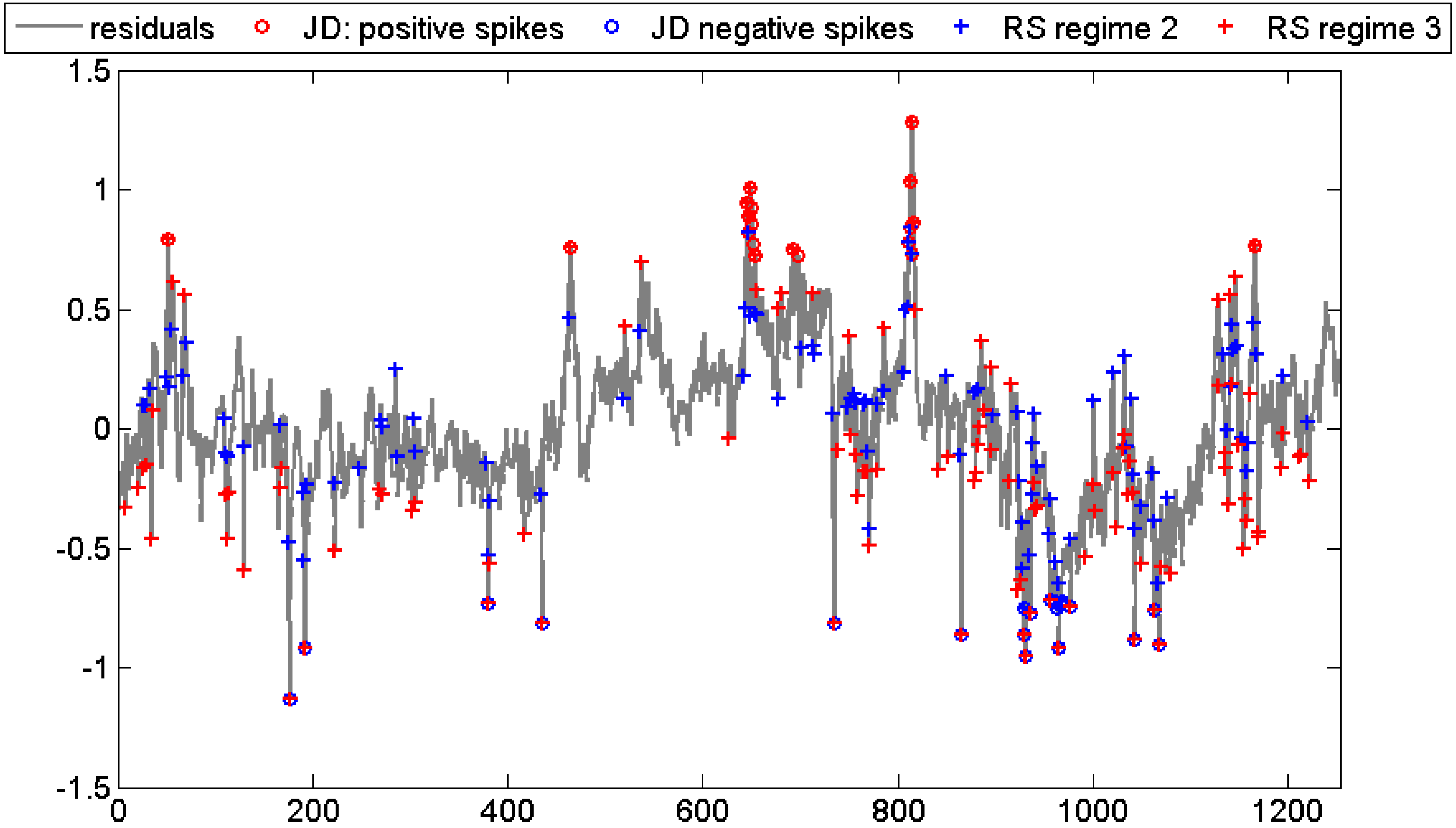
Model C

	α_{NIG}	β_{NIG}	δ_{NIG}	μ_{NIG}
Business days	5.617	0.197	0.504	−0.017
Non-business days	2.752	−1.372	0.351	0.128

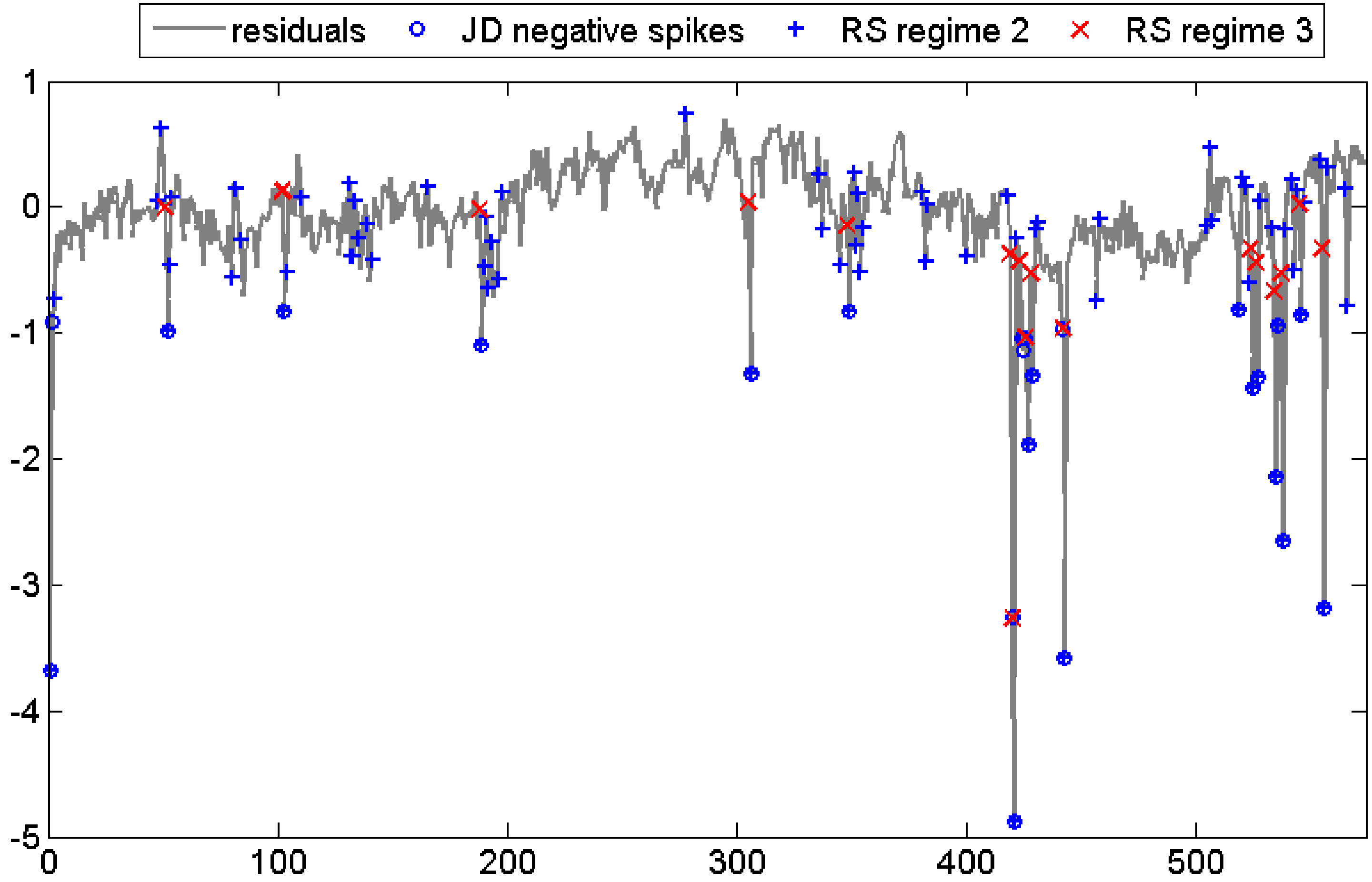
HPFC corrections resulting from short term process



Detected spikes on business days (Models A and B)

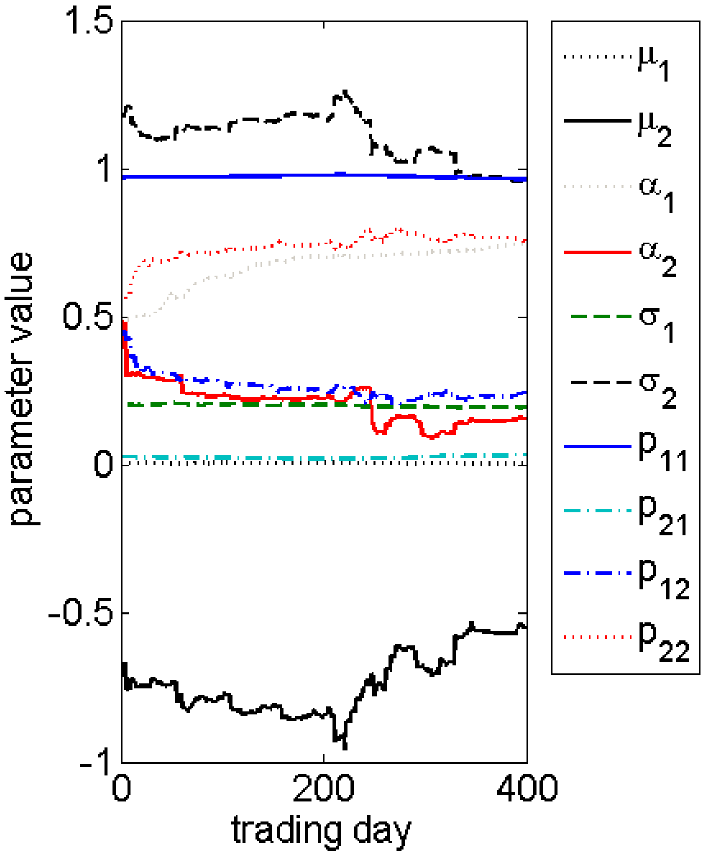


Detected spikes on non-business days (Models A and B)

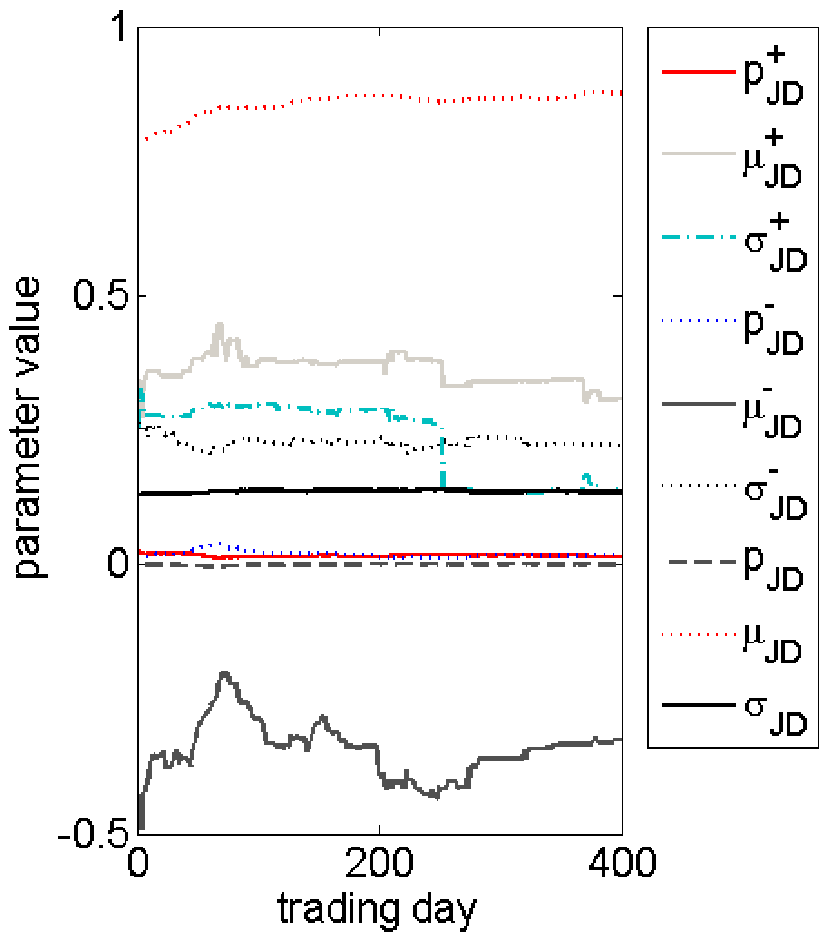


Robustness of parameter estimation

Model A: regime switching

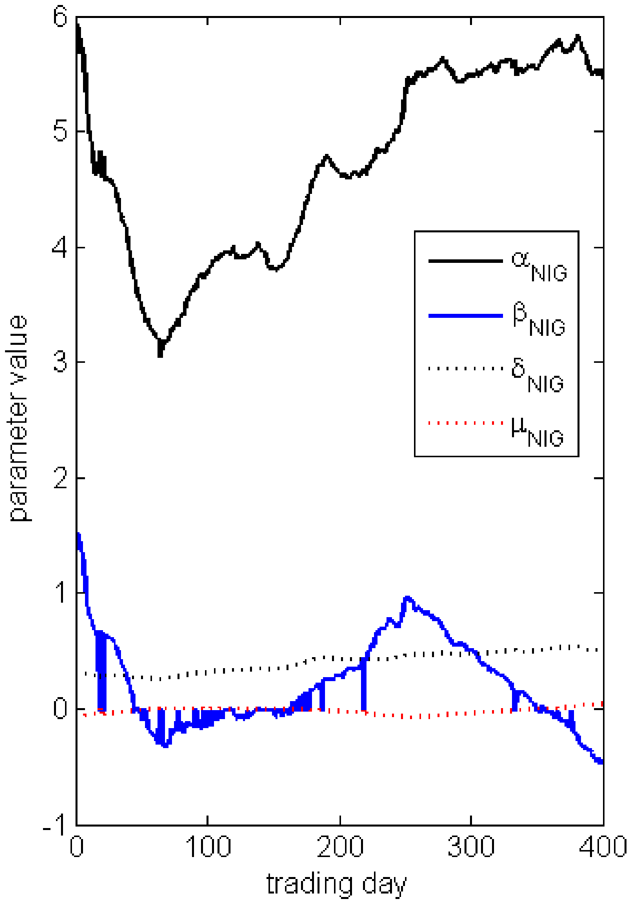


Model B: jump diffusion



Robustness of parameter estimation

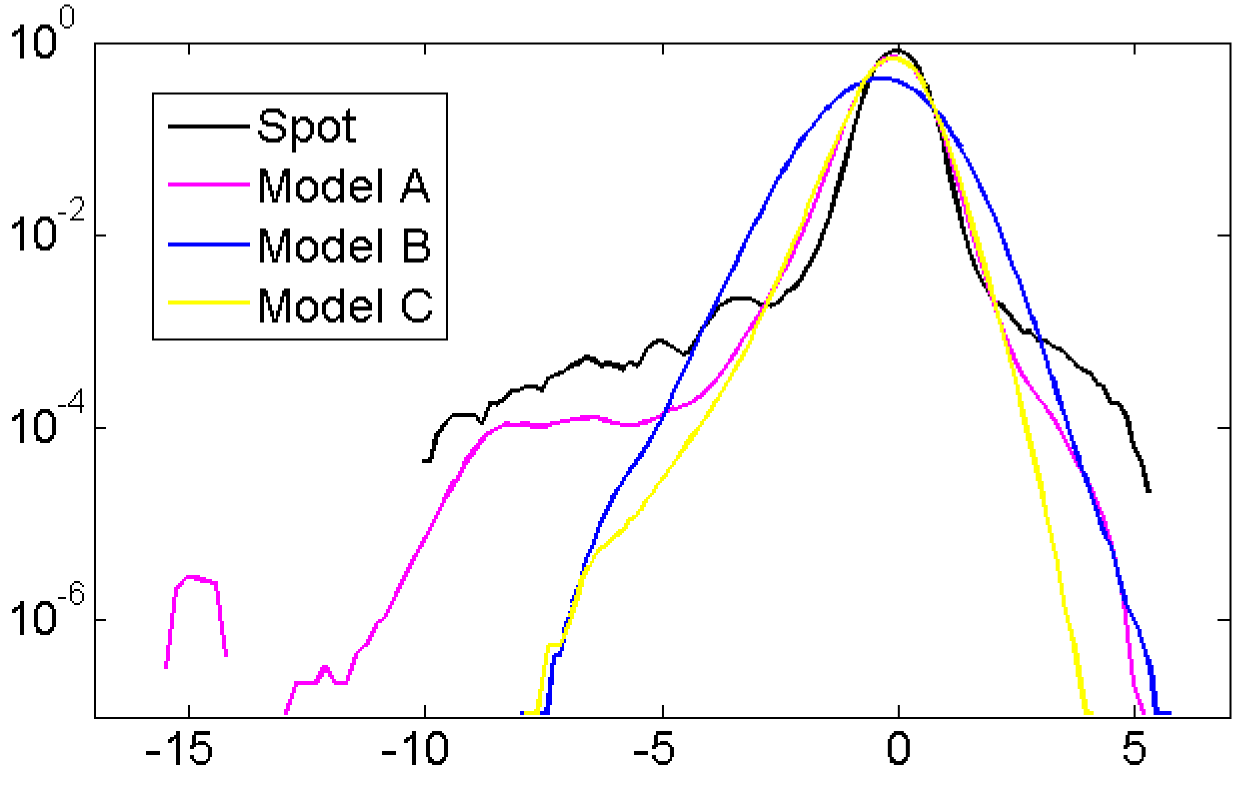
Model C: NIG



- Parameters estimated for model A and B vary in a reasonable range
- For model C variations of α_{NIG} are quite large

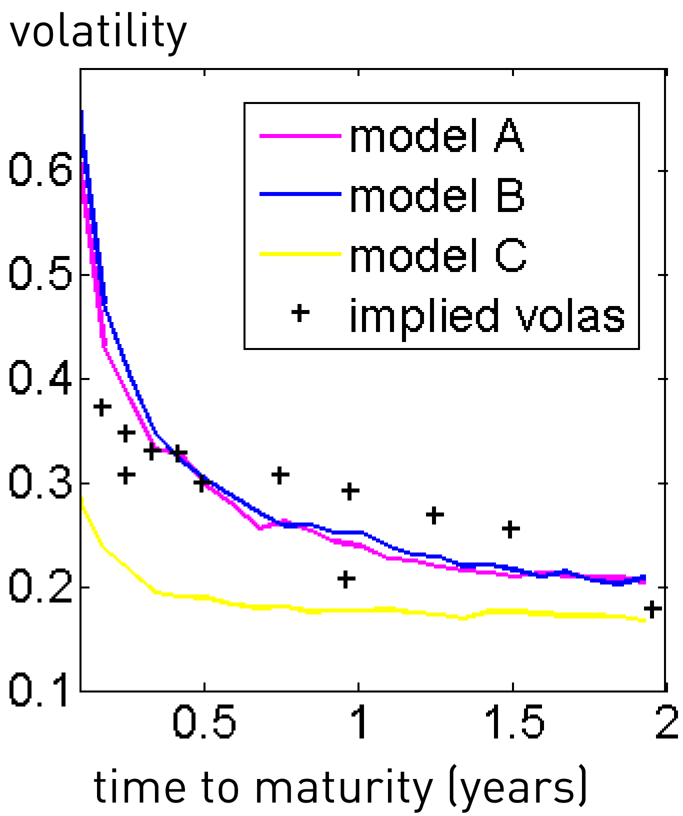
In-sample analysis

frequency of residuals

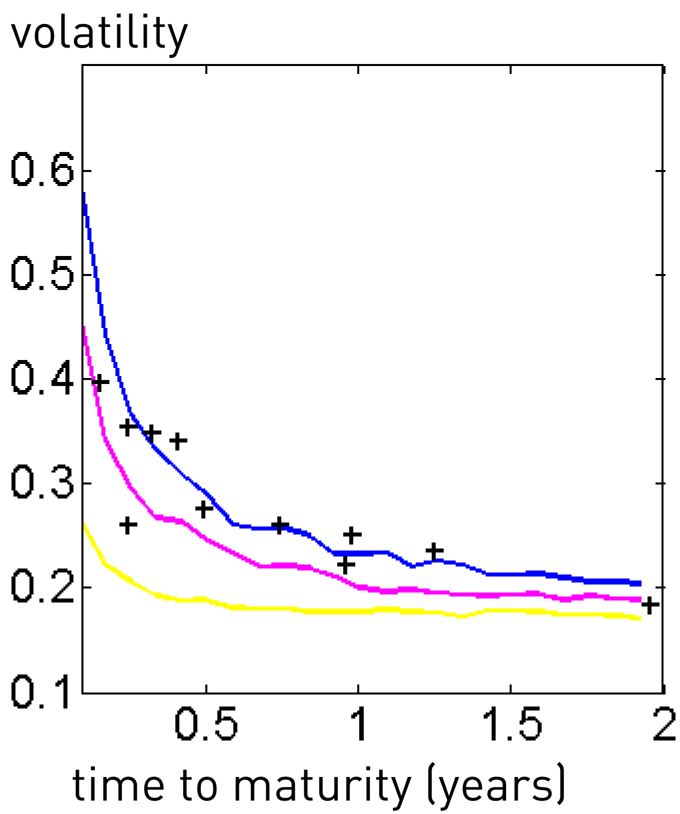


- Model A (regime switching) reproduces residuals best
- Out-of-sample Analysis shows comparable results

Volatilities of futures prices



December 28, 2007



December 28, 2006

- Quite good fit of implied volatilities by Model B and Model A
- Bad performance of Model C

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Valuation of swing options

J possible exercise times: $0 < t_1 < t_2 < \ldots < t_J = T$

Number of exercise rights: $u_{\max}$

Strike price: K

Exercise decision: $\phi_j \in \{0, 1\}$ (exercise $\phi_j = 1$ or do not exercise $\phi_j = 0$)

Payoff P: for a put option $P(t_j) = K - S(t_j)$
and for a call option $P(t_j) = S(t_j) - K$

Objective function

$$M(\phi) = \sum_{j=1}^{J} D(t_j, t_0) \cdot \phi_j \cdot P(t_j), \quad D(t_j, t_0) = \exp(-r(t_j - t_0))$$

Fair option value (risk neutral measure Q) at time t_0 : $V = \max_{\phi} \mathbb{E}_{t_0}^{Q} [M(\phi)]$

Upper and lower boundary for the fair option value

Change position of expectation-operator: starting point $V = \max_{\phi} \mathbb{E}_{t_0}^{Q}\left[M(\phi)\right]$

- Pull outwards: **Upper boundary** for fair option value
  Deterministic or ex post value: exercise of option with perfect foresight in *N* scenarios

$$EV = \mathbb{E}_{t_0}^{Q} \max_{\phi} \left[M(\phi)\right]$$

$$\approx \frac{1}{N} \sum_{k=1}^{N} \max_{\phi} \sum_{j=1}^{J} \left[D(t_j, t_0) \cdot \phi_j \cdot P(S_k(t_j))\right]$$

with $P(S_k(t_j)) = \begin{cases} S_k(t_j) - K & \text{call} \\ K - S_k(t_j) & \text{put} \end{cases}$

- Pull inwards: **Lower boundary** for fair option value
  Intrinsic value: value against price forward curve

$$IV = \max_{\phi} \sum_{j=1}^{J} \left[D(t_j, t_0) \cdot \phi_j \cdot \bar{P}(t_j)\right], \quad \text{with} \quad \bar{P}(t_j) = \begin{cases} \mathbb{E}_{t_0}^{Q}[S(t_j)] - K & \text{call} \\ K - \mathbb{E}_{t_0}^{Q}[S(t_j)] & \text{put} \end{cases}$$

could be logged in immediately in a complete market

Valuation of swing options

Option is exercised if sum of immediate payoff from exercise and value of swing option with remaining exercise rights is larger than the continuation value, i.e. the expected value of the future cashflows of the option with shorter lifetime but unchanged number of exercise rights using an optimal exercise strategy under uncertainty (exercise today or wait and hope for higher prices in future).

Continuation value: at time t_k in price path ω with u exercised swing rights

$$F(\omega, t_k, u) = \mathbb{E}^Q \left[\sum_{j=k+1}^{J} D(t_j, t_k) \cdot C_{\omega,u}(t_j) \middle| \mathcal{F}_{t_k} \right]$$

$$u = 0, 1, \ldots, u_{\max}$$

$C_{\omega,u}(t_j)$ Cashflow at time t_j in price path ω for an option with u unused exercise rights

Least squares Monte Carlo (LSM) algorithm

The $u_{\max}$ columns of the $N \times u_{\max}$ cashflow matrix are each initialized with the payoff at the last exercise opportunity $\mathbf{C}(t_J) = (P(\mathbf{S}(t_J)), \ldots, P(\mathbf{S}(t_J)))$

Iteration:

1. Discount cashflow matrix with discount factor $D(t_{k+1}, t_k)$
2. Calculate cashflow vector $P(\mathbf{S}(t_k))$ for immediate exercise at time t_k
3. For all price paths where the option is in the money (i.e. $P(S_\omega(t_k)) > 0$) the continuation value is approximated by least-squares regression

$$\mathbf{F}_u(t_k) \approx \hat{\mathbf{F}}_u(t_k) = \sum_{l=0}^{M} a_l^u B_l(\mathbf{S}(t_k)) \text{ with } \quad B_l(x) = x^l$$

$$\min \sum_{\omega=1}^{N} \left(\sum_{j=k+1}^{J} D(t_j, t_k) \cdot C_{\omega,u}(t_j) - \sum_{l=0}^{M} a_l^u B_l(S_\omega(t_k)) \right)^2$$

LSM algorithm - continued

4. Exercise option in price path ω if

$$P(S_\omega(t_k)) + \hat{F}_{u+1}(\omega, t_k) > \hat{F}_u(\omega, t_k)$$

holds and replace element $C_{\omega,u}$ of cashflow matrix

$$C_{\omega,u}(t_k) = P(S_\omega(t_k)) + C_{\omega,u+1}(t_k)$$

If the calculation starts with $u = 0$ only one matrix is needed!

5. Proceed to the next time step $t_k \to t_{k-1}$, if t_1 has not yet been reached

Calculate fair option value as average over first column of cashflow matrix after discounting:

$$V = \frac{1}{N} \sum_{\omega=1}^{N} D(t_1, t_0) C_{\omega,0}(t_1) \; .$$

Efficient implementation

1. **Efficient memory usage**: A cashflow matrix has to be used instead of a tensor
2. **Economy size decomposition**: An economy size decomposition algorithm (also called skinny QR decomposition) can be used to speed up algorithm
3. **reuse QR decomposition** to solve the $u_{\max}$ regression problems simultaneously
4. **Scaling**:

$$\hat{\mathbf{F}}_u(t_k) = \sum_{l=0}^{M} a_l^u B_l\left(\frac{\mathbf{S}(t_k)}{||\mathbf{S}(t_k)||_\infty}\right) .$$

With this scaling up to $M = 8$ basis functions can be used without rank deficiencies.

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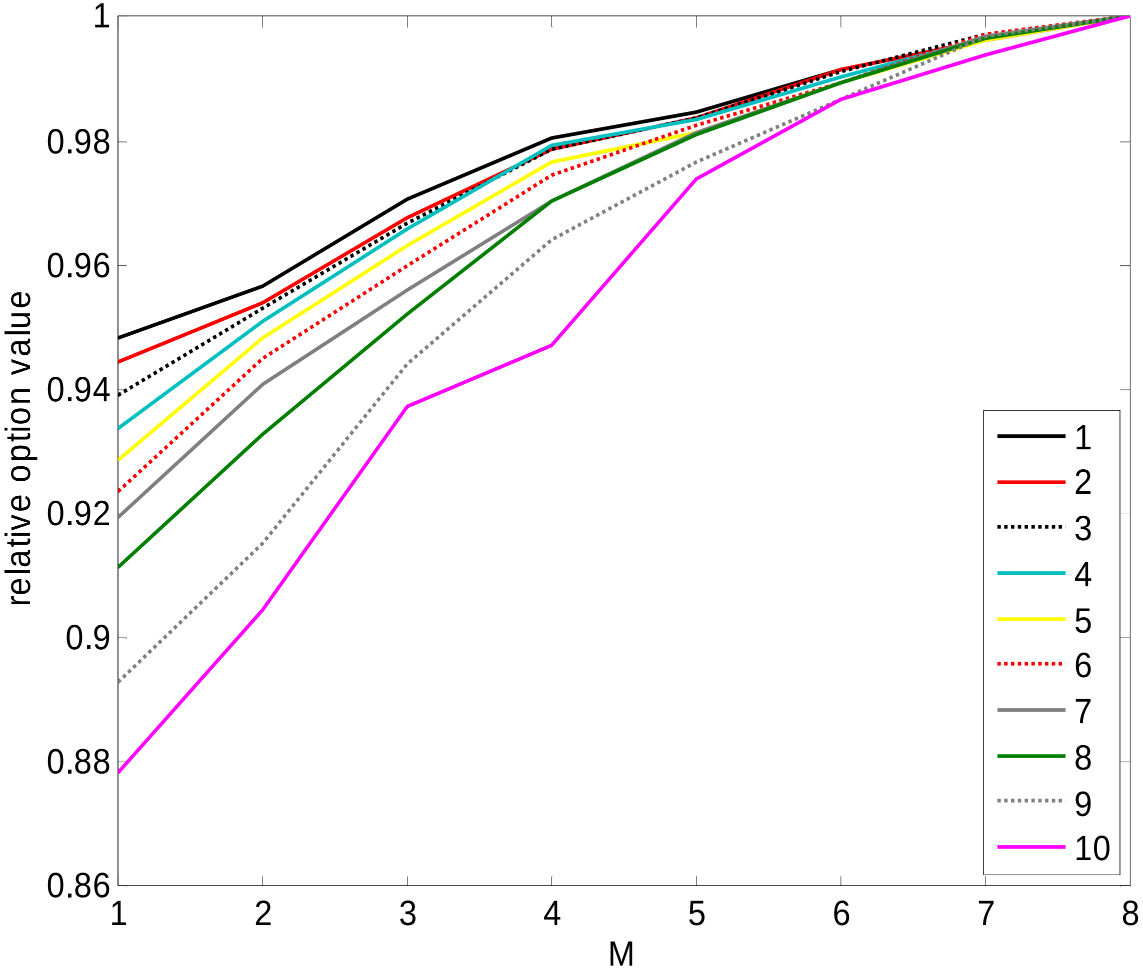
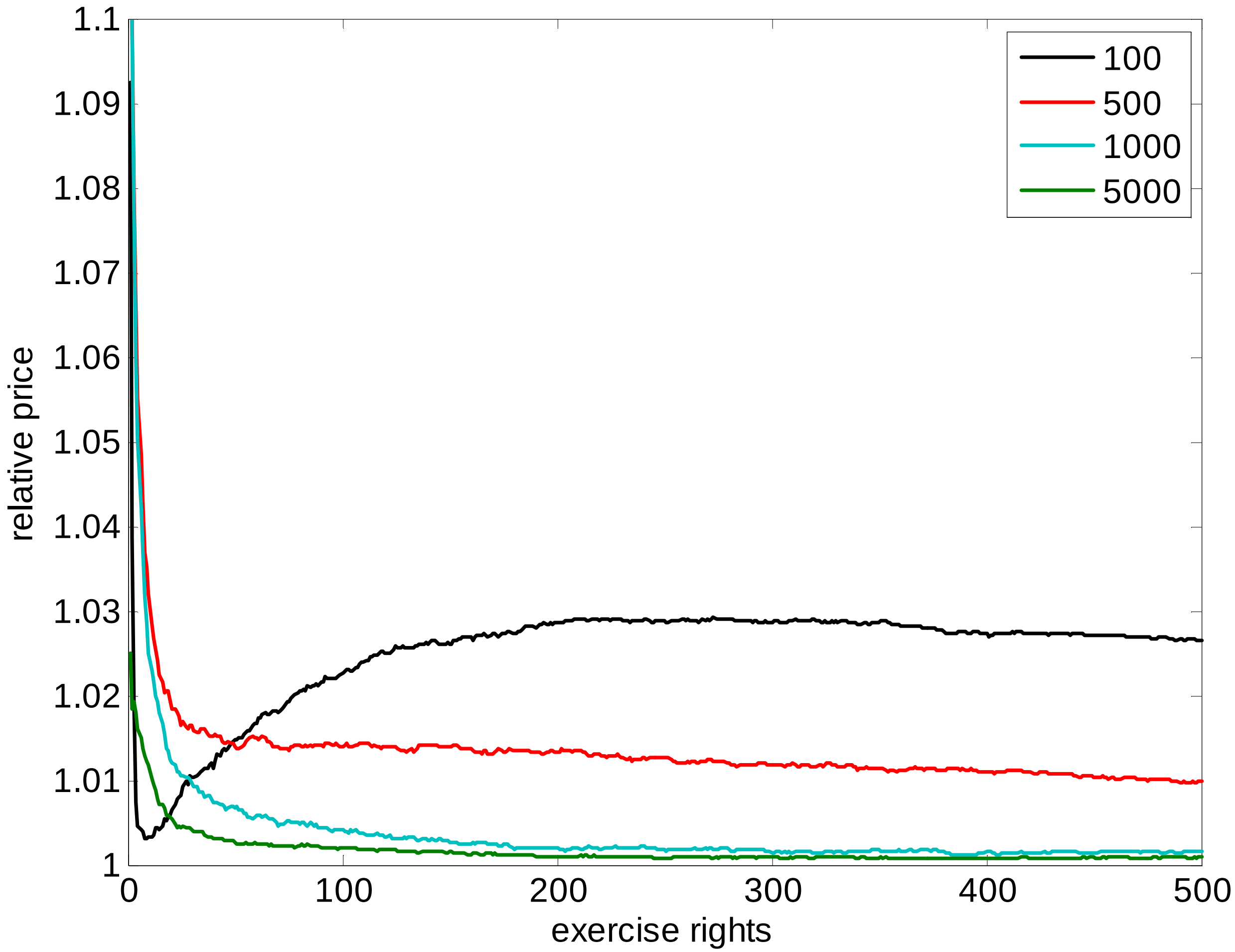
Required number of basis functions



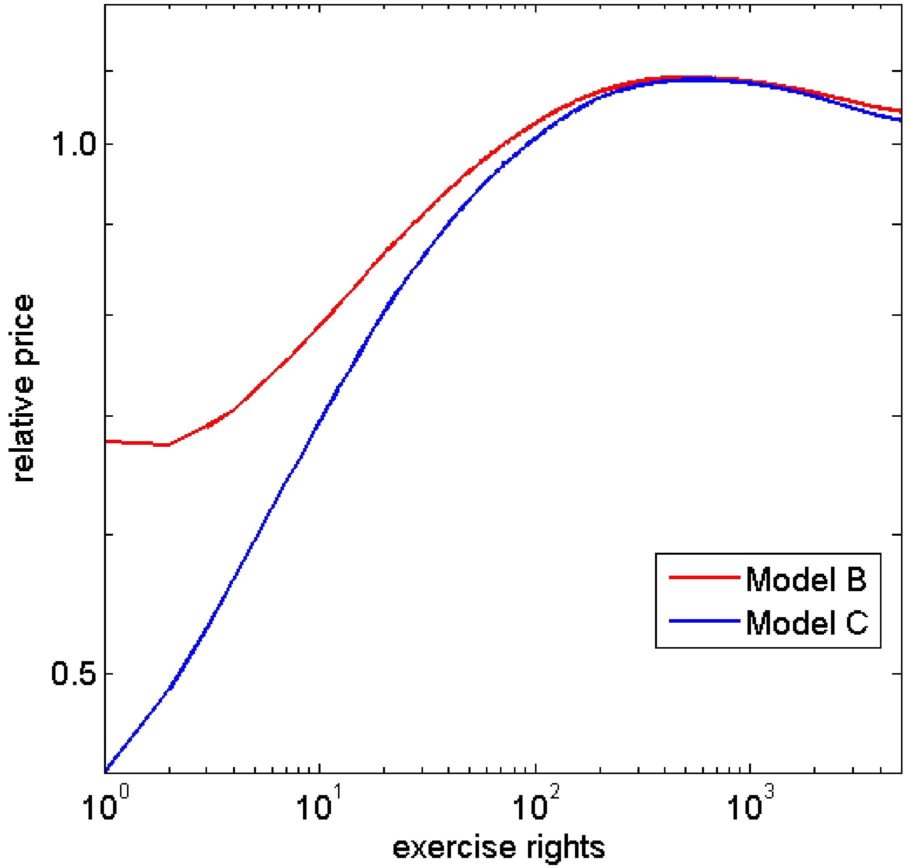
5 Basis functions are a good compromise between speed and accuracy

Required number of price scenarios



- Relative option value compared with 10,000 scenarios
- 1000 scenarios give very good results

Model risk of swing option prices

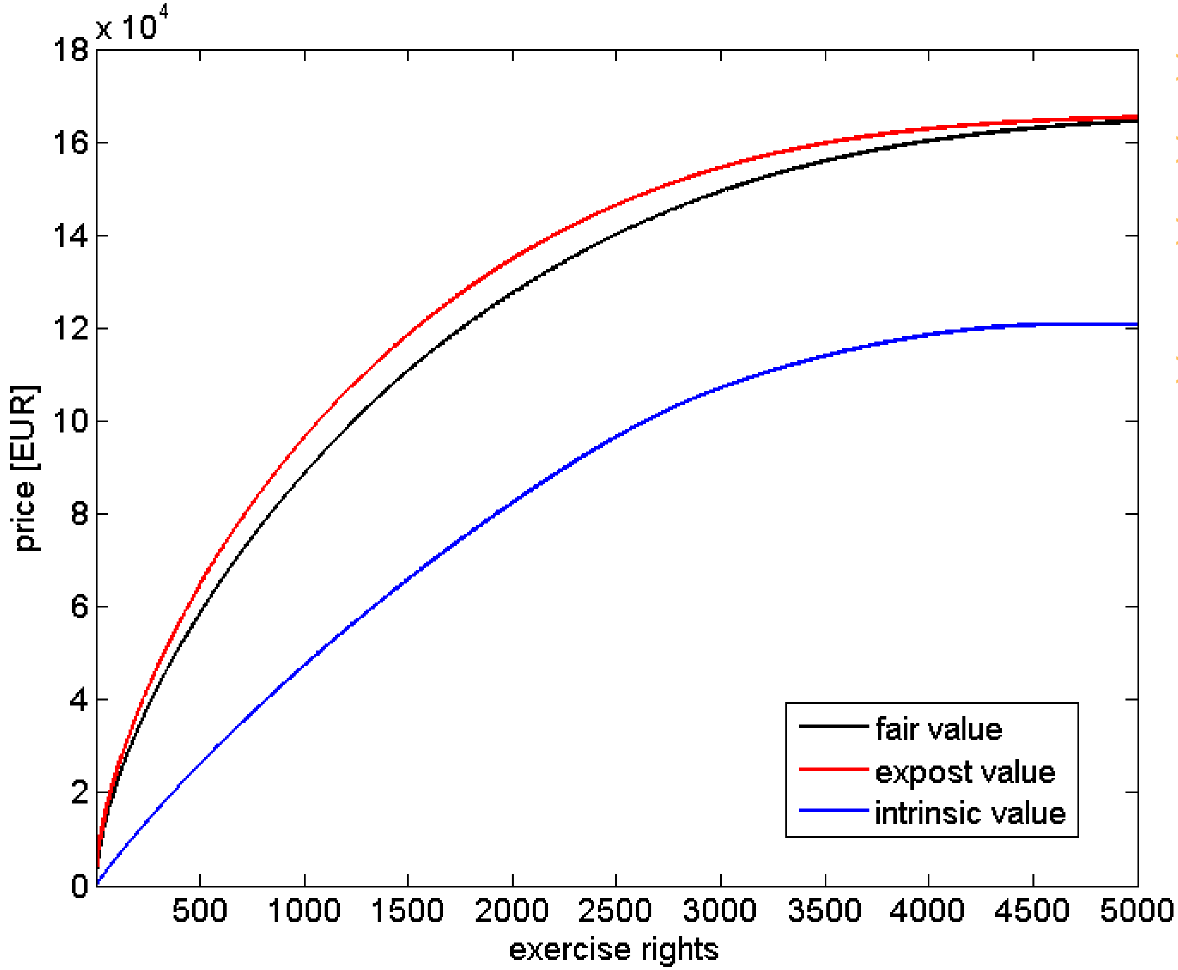


	Model B	Model C
Relative price	1.04	1.03

5000 exercise rights, strike 60 €/MWh

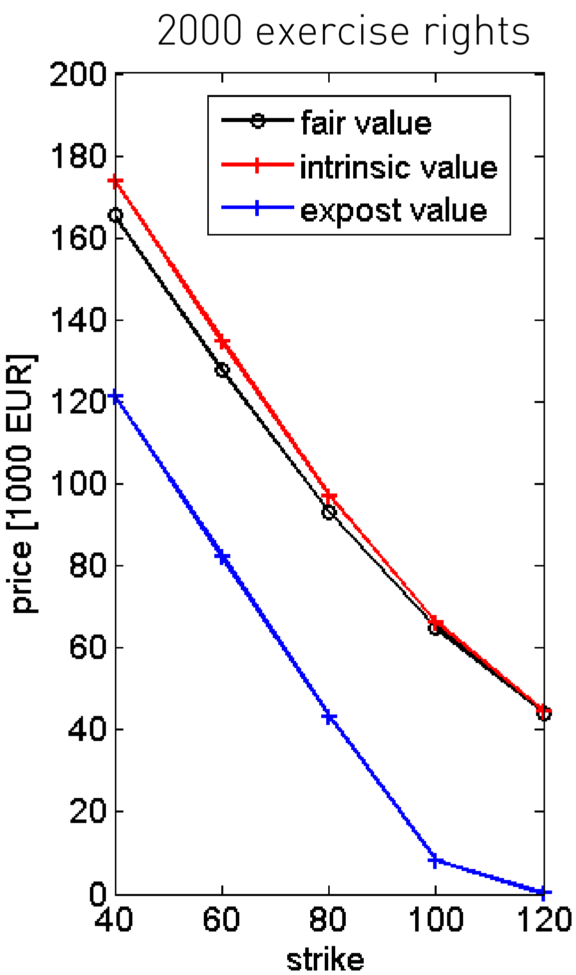
- For a small number of exercise rights the prices differ by 32 % (model B) or even 56 % (model C)
- For 500 exercise rights situation reverses: option values are 9 % higher for model B and C compared to A
- 3 - 4 % difference for larger numbers of exercise rights

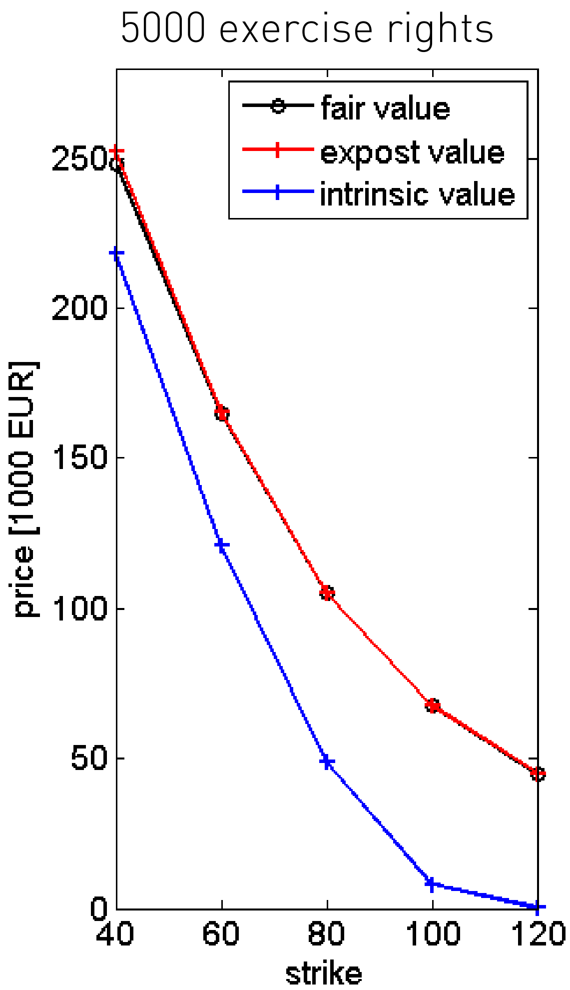
Fair, ex post and intrinsic value



- Strike price 60 €/MWh
- 1000 price scenarios
- Intrinsic value much lower than fair value
- Difference between fair and ex post value shrinks to zero as number of exercise rights approaches 5,000

Influence of the strike price





- 1,000 price scenarios
- Intrinsic value goes down to zero if strike price is above 100 €/MWh
- Difference between fair and ex post value shrinks with increasing strike price

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Summary and Open Questions

- Introduction of spikes in HPFC important
- Three different hourly spot price processes introduced: regime switching, jump diffusion, NIG process
- Regime switching is price process that best reproduces historical spot price distribution
- LSM delivers fair value of swing options and is highly versatile as far as different spot price processes are concerned
  - Spot price auction for 24 hours of the next day: What is the correct implementation?

Open questions

- Model Risk due to price process for spot prices lies between 3 and 50 %
  - What is the best price process for electricity spot prices?
    - Independent processes used for daily average and deviation from daily average – except for historical sampling of spike profiles
    - Independent processes used for business and non-business days
    - hourly price process: ARMA process is over-simplification

Literature

Least Squares Monte Carlo:

Longstaff, F. A., and Schwartz, E. S. (2001). Valuing American options by simulation: a simple least-squares approach. *Review of Financial Studies* **14**, 113–147

Dörr, U. (2003). Valuation of swing options and examination of exercise strategies by Monte Carlo techniques. Master's thesis, University of Oxford

Boogert, A., and de Jong, C. (2008). Gas storage valuation using a Monte Carlo method. *The Journal of Derivatives* **15** (3), 81–98

Hourly spot price process:

Schindelmayr, G. (2005). A regime-switching model for electricity spot prices. In *Tenth Symposium on Banking, Finance and Insurance, University of Karlsruhe*

Hirsch, G. (2009). Pricing of hourly exercisable electricity swing options using different price processes, *The Journal of Energy Markets* 2 (2), 1-44

Literature – continued

Daily Spot prices

de Jong, C. (2006). The nature of power spikes: a regime-switch approach. *Studies in Nonlinear Dynamics & Econometrics* 10(3), Article 3

Geman, H., and Roncoroni, A. (2006). Understanding the fine structure of electricity prices. *Journal of Business* 79(3), 1,225–1,262.

Benth, F. E., and Saltyte-Benth, J. (2004). The normal inverse Gaussian distribution and spot price modelling in energy markets. *International Journal of Theoretical and Applied Finance* 7(2), 177–192.

Weron, R. (2006). *Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach*. Wiley, Chichester.

Weron, R. (2007). MFE Toolbox, version 1.0.1. Available at www.im.pwr.wroc.pl/˜rweron/MFE/MFE_Toolbox.html.

Thank you for your attention!

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