Understanding the Price Dynamics of Emission Permits: A Model for Multiple Trading Periods

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Motivation

- emission trading schemes designed to reduce pollution by introducing appropriated market mechanisms
- most prominent examples:
  - US Sulfur Dioxide Trading System
  - European Union Emission Trading Scheme (EU ETS)
- several other carbon market initiatives are underway or seriously under discussion
  - regulatory rules similar to EU ETS
  - e.g. Australian Carbon Pollution Reduction Scheme, New Zealand ETS, Japan Trial ETS, US, Canada

Motivation

- still no clear picture on how regulatory rules affect price dynamics
- understanding the price dynamics
  - pricing derivatives
  - sound risk management
  - energy-related investment decisions
- propose dynamic model to explain price behavior
  - take into account most important regulatory rules
    - sequence of consecutive trading periods
    - inter-period banking
    - no inter-period borrowing
    - penalty costs and later delivery of lacking permits
  - as well as abatement possibilities

Introduction to EU ETS

- EU-wide emissions trading scheme (EU ETS) on company-based level in order to reduce CO_2 emissions
- EU Allowances (EUAs) allow for emission of one ton of CO_2 each
- EUAs are traded OTC and on exchanges across Europe
- initially two trading periods: 2005 - 2007 and 2008 - 2012
  - within trading periods EUAs are storable (bankable)
  - banking and borrowing not allowed between 2007 and 2008
- meanwhile plans for indefinitely ongoing sequence of trading periods
  - third trading period until 2020
  - no inter-period borrowing but inter-period banking
  - presumable figures for permit allocation in following trading periods
- penalties for non-compliance

Literature

theoretical models

- equilibrium models considering one trading period
  - companies choose optimal trading and abatement strategies
    - Fehr/Hinz (2006), Seifert et al (2008), Carmona et al (2009)
  - companies choose optimal trading strategies only
    - e.g. Chesney/Taschini (2008)
- models considering two trading periods
  - Kijima et al (2009): either banking and borrowing or neither of them
  - Cetin, Verschuere (2009): no banking

empirical studies

- burgeoning literatue
- mostly based on data from trial period
  - Daskalakis et al (2009), Paolella, Taschini (2008), Benz, Trück (2009), Uhrig-Homburg, Wagner (2009)

Agenda

1. starting point: a simple conceptual framework
    - dynamic model for a finite trading period
    - takes into account most important features of EU ETS (first period)
        - penalty costs
        - banking and borrowing
        - trading period break
        - increasing marginal abatement costs
2. extension to a model for multiple trading periods
    - first thoughts and preliminary results
    - shed light on following questions
        - how do additional periods influence spot price dynamics?
        - how does price dynamics look like at end of trading period?
        - which part of spot price comes from different trading periods?
        - how does volatility surface evolve?

CO_2–regulated company

- stochastic emission rate (Business As Usual)

$$dy_t = \mu(y_t)dt + \sigma(y_t)dw_t$$

- company may
  - abate u_t of CO_2 emissions with quadratic abatement costs

$$C(u_t) = -\frac{1}{2}cu_t^2$$

  - buy or sell EUAs in market (z_t)
  - pay penalty for not complying
- total expected emissions in $[0, T]$ (abatements/trading taken into account)

$$x_t = -\int_0^t u_s ds - \int_0^t z_s ds + E_t(\int_0^T y_s ds)$$

CO_2−regulated company

- initial endowment e_0 of EUAs
- one finite trading period $[0, T]$, banking into next trading period prohibited
- penalty costs at end of trading period for lacking EUAs

$$P(x_T) = min(0, p(e_0 - x_T))$$

- company's optimization problem:

$$\max_{u_t, z_t, t \in [0,T]} E_0\Big(\int_0^T e^{-rt} C(u_t) dt - \int_0^T e^{-rt} S(t) z_t dt + e^{-rT} P(x_T) \Big)$$

Market equilibrium

Consider market consisting of N companies

- equilibrium consists of
  - abatement rates $u^*_{it}, i = 1 \dots N$
  - trading strategies $z^*_{it}, i = 1 \dots N$
  - EUA spot price $S(t)$
- solving
  - individual cost problems and
  - market clearing condition $\sum_{i=1}^{N} z_{it} = 0$ for all t

Solution

- from first order condition:

$$S(t) = c_i u^*_{it}, i = 1 \dots N$$

- i.e. spot price $\equiv$ marginal abatement costs
  - if EUA price is above marginal abatement cost, companies may profit by abating cheap and selling higher (and vice versa)
  - all companies have the same marginal abatement costs after trading
- under certain conditions market equilibrium solution equivalent to least cost solution attainable by a central social planner
- from optimality principle from stochastic optimal control theory

$$V(t, x_t) = \max_{u_t} E_0\Big(e^{-rt} C(u_t) dt + V(t + dt, x_t + dx_t)\Big)$$

deduce characteristic PDE with boundary conditions

Solution

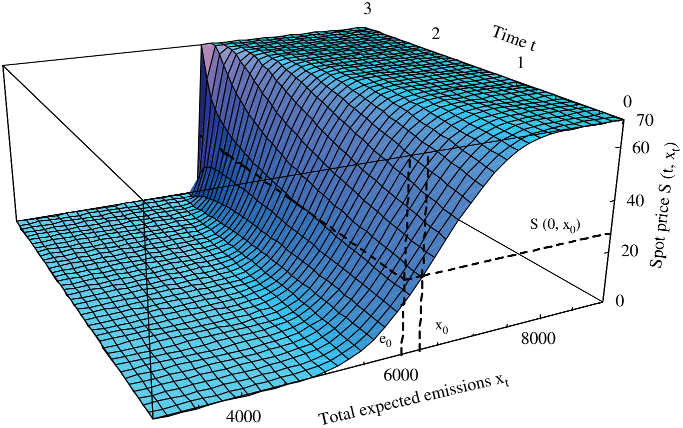
- resulting spot price non-negative
- resulting discounted spot price process is a martingale, regardless of stochastic process for emissions rate
  - in particular, no mean-reversion
  - due to storability and assumption of risk-neutral agents
- if emissions rate assumed to follow white noise process then analytic solution of characteristic PDE possible (otherwise solve numerically)

Parameter values

- chosen as to remind some stylized facts in the EU ETS
  - 3 year period 2005 - 2007
  - allocation of about 6 billion tons
  - penalty €40 plus delivering missing EUAs

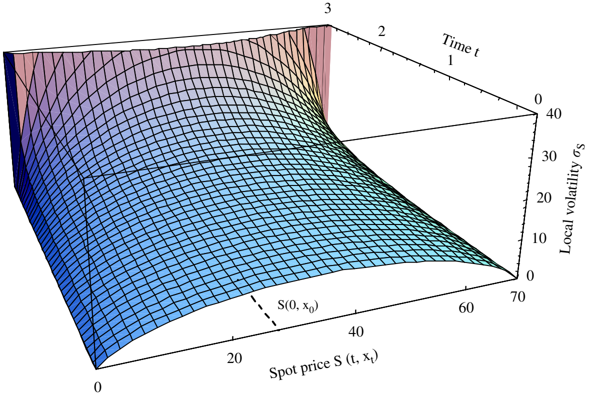
Parameter	Value[a]
Penalty p	70
Length of trading period T	3
Initial endowment with certificates e_0	6000
Expected total emissions x_0	6240
Marginal abatement costs c	0.24
Volatility of emission rate σ	$500/\sqrt{T}$

Spot price function $S(t, x_t)$



- fixed upper bound determined by penalty costs
- EUA price never reaches zero (option value of EUA) before T

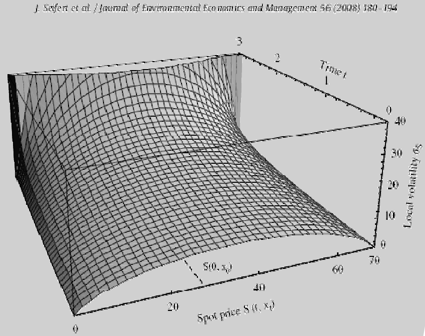
Volatility function $\sigma(t, S_t)$



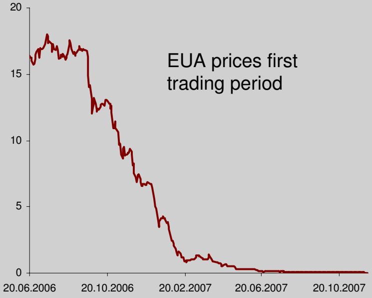
- volatility function goes to infinity at trading period end $t = 3$
- volatility reaches zero at price bounds

Consistent with observed price behavior...

From theory ...



... to reality



Implications for spot price process

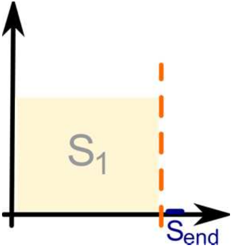
- discounted spot prices are martingales
  - deterministic/seasonal components in emissions rate process do not influence resulting spot price process
  - verified by empirical examination (no mean reversion)
- spot prices with fixed upper bound determined by penalty costs
  - only valid for first trading period (banking allowed after 2nd trading period)
  - empirical tests seem to support this view
- volatility of spot price process
  - increases when time is coming closer to end of trading period and
  - decreases when the price is coming closer to price bounds

Do characteristics carry over to setting with more than one trading period?

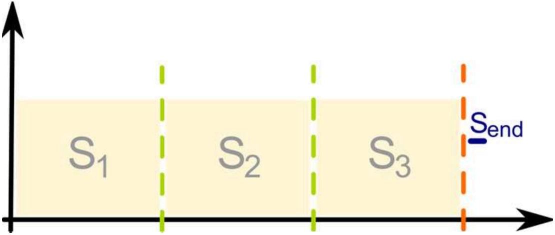
Changes of regulatory framework

	Period I (2005-2007)	Period II, Period III ... (2008-2012, 2013-2020, ...)
Banking into next period	not allowed	allowed
Borrowing from next period	not allowed	not allowed
Penalty costs	€40	€100
Later delivery of lacking permits	yes	yes

Extension to multi-phase model



trial period $[0, T]$



multiple periods $[0, T_1], [T_1, T_2], \ldots$

- company's new optimization problem:

$$\max_{u_t, z_t, t \in [0, T_n]} E_0 \Big(\int_0^{T_n} e^{-rt} C(u_t) dt - \int_0^{T_n} e^{-rt} S(t) z_t dt + \sum_{j=1}^{n} e^{-rT_j} P(x_{T_j}) + R(x_{T_n}) S_{end} \Big)$$

Solution: basic idea

- apply same principles as for model with one finite trading period in backwards manner
- i.e. make use of dynamic programming algorithm
  - Bellman's principle
  - Ito's lemma for each finite trading period

Characteristics of spot price dynamics

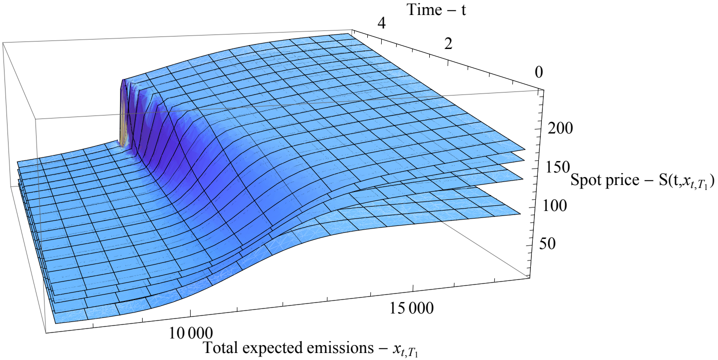
First results for illustrative setting:

- chosen parameter values:
  - up to four consecutive trading periods
  - first period 5 years, next periods 8 years
  - allocation according to current allocation plans

phase II (2008-2012)	10.400 billion tons
phase III (2013-2020)	14.775 billion tons
phase IV (2021-2028)	12.455 billion tons
phase V (2029-2036)	10.135 billion tons

  - penalty costs: $p_j =$ €100 in each period j
  - $S_{end} = 25$ for four periods, discounted for less than four periods
- consider spot price for first period of each setting
  - price bounds?
  - smoother transition through banking?

Spot price function $S(t, x_{t,T_1})$



- additional period increases value through possible use for compliance in further period
- upper price bound depends on number of periods

Spot price function $S(t, x_{t,T_1})$

time-dependent price bounds

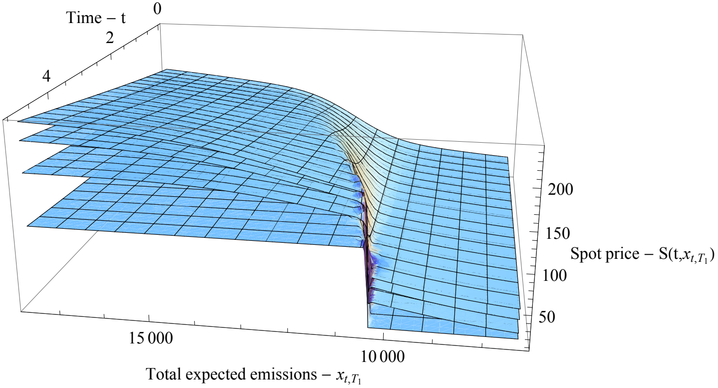
- upper bound

$$S_{upper}(t) = \sum_{j=1}^{n} e^{-r(T_j - t)} p_j + e^{-r(T_n - t)} S_{end}$$

- lower bound

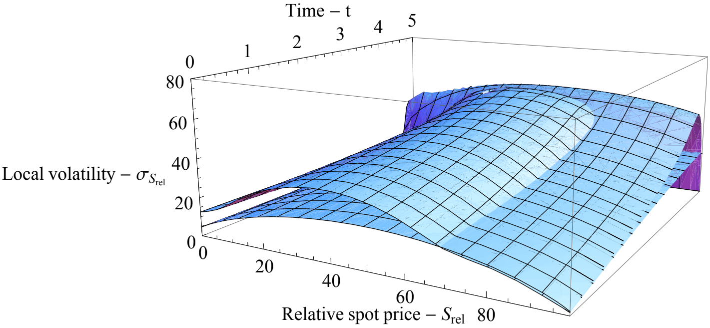
$$S_{lower}(t) = e^{-r(T_n - t)} S_{end}$$

Spot price function $S(t, x_{t,T_1})$ (back)



- steepness increases as t approaches T_1
- still discontinuity at end of each trading period although banking is allowed

Local Volatility $\sigma(t, S_{rel})$



- highest volatility for medium spot prices
- volatility surface more moderate in multi-period setting

Value components of current spot price $S(t, x_{t,T_1})$

Emissions Scenario		Value Component from				
current	future	period 1	period 2	period 3	period 4	S_{end}
medium	medium	72%	11%	2%	1%	14%
high	high	38%	27%	18%	10%	7%
high	low	65%	14%	5%	5%	11%
low	high	0%	47%	23%	15%	15%
low	low	0%	2%	22%	29%	47%

- substantial part of spot price attributable to future trading periods

Conclusion

- each additional trading period leads to
  - additional possible use because of banking possibility
  - additional value component in today's spot price
  - relative share depends on current and future expected emissions
- price bounds
  - naturally depend on number of trading periods considered
  - spot prices do not decline to zero at end of a trading period
- spot price dynamics and corresponding volatility surfaces become more moderate
  $\Longrightarrow$ behavior clearly different from resulting behavior when no consecutive trading period is considered
- nevertheless overall characteristics quite similar to one period setting