Understanding the Price Dynamics of Emission Permits: A Model for Multiple Trading Periods

Steffen Hitzemann & Marliese Uhrig-Homburg
Chair of Financial Engineering and Derivatives
Motivation

- emission trading schemes designed to reduce pollution by introducing appropriated market mechanisms
- most prominent examples:
  - US Sulfur Dioxide Trading System
  - European Union Emission Trading Scheme (EU ETS)
- several other carbon market initiatives are underway or seriously under discussion
  - regulatory rules similar to EU ETS
  - e.g. Australian Carbon Pollution Reduction Scheme, New Zealand ETS, Japan Trial ETS, US, Canada
Motivation

- still no clear picture on how regulatory rules affect price dynamics
- understanding the price dynamics
  - pricing derivatives
  - sound risk management
  - energy-related investment decisions
- propose dynamic model to explain price behavior
  - take into account most important regulatory rules
    - sequence of consecutive trading periods
    - inter-period banking
    - no inter-period borrowing
    - penalty costs and later delivery of lacking permits
  - as well as abatement possibilities
Introduction to EU ETS

- EU-wide emissions trading scheme (EU ETS) on company-based level in order to reduce CO$_2$ emissions
- EU Allowances (EUAs) allow for emission of one ton of CO$_2$ each
- EUAs are traded OTC and on exchanges across Europe
  - within trading periods EUAs are storable (bankable)
  - banking and borrowing not allowed between 2007 and 2008
- meanwhile plans for indefinitely ongoing sequence of trading periods
  - third trading period until 2020
  - no inter-period borrowing but inter-period banking
  - presumable figures for permit allocation in following trading periods
- penalties for non-compliance
Literature

theoretical models

- equilibrium models considering one trading period
  - companies choose optimal trading and abatement strategies
  - companies choose optimal trading strategies only
    - e.g. Chesney/Taschini (2008)

- models considering two trading periods
  - Kijima et al (2009): either banking and borrowing or neither of them
  - Cetin, Verschuere (2009): no banking

empirical studies

- burgeoning literature
- mostly based on data from trial period
1. starting point: a simple conceptual framework
   - dynamic model for a finite trading period
   - takes into account most important features of EU ETS (first period)
     - penalty costs
     - banking and borrowing
     - trading period break
     - increasing marginal abatement costs

2. extension to a model for multiple trading periods
   - first thoughts and preliminary results
   - shed light on following questions
     - how do additional periods influence spot price dynamics?
     - how does price dynamics look like at end of trading period?
     - which part of spot price comes from different trading periods?
     - how does volatility surface evolve?
CO₂—regulated company

- stochastic emission rate (Business As Usual)
  \[dy_t = \mu(y_t)dt + \sigma(y_t)dw_t\]

- company may
  - abate \(u_t\) of CO₂ emissions with quadratic abatement costs
    \[C(u_t) = -\frac{1}{2}cu_t^2\]
  - buy or sell EUAs in market \((z_t)\)
  - pay penalty for not complying

- total expected emissions in \([0, T]\) (abatements/trading taken into account)
  \[x_t = -\int_0^t u_s ds - \int_0^t z_s ds + E_t(\int_0^T y_s ds)\]
CO₂—regulated company

- initial endowment $e_0$ of EUAs
- one finite trading period $[0, T]$, banking into next trading period prohibited
- penalty costs at end of trading period for lacking EUAs

$$P(x_T) = \min(0, p(e_0 - x_T))$$

- company’s optimization problem:

$$\max_{u_t, z_t, t \in [0, T]} E_0 \left( \int_0^T e^{-rt} C(u_t) dt - \int_0^T e^{-rt} S(t) z_t dt + e^{-rT} P(x_T) \right)$$
Market equilibrium

Consider market consisting of $N$ companies

- equilibrium consists of
  - abatement rates $u^*_i, i = 1 \ldots N$
  - trading strategies $z^*_i, i = 1 \ldots N$
  - EUA spot price $S(t)$

- solving
  - individual cost problems and
  - market clearing condition $\sum_{i=1}^{N} z_{it} = 0$ for all $t$
from first order condition:

\[ S(t) = c_i u_{it}^*, \quad i = 1 \ldots N \]

i.e. spot price \( \equiv \) marginal abatement costs

- if EUA price is above marginal abatement cost, companies may profit by abating cheap and selling higher (and vice versa)
- all companies have the same marginal abatement costs after trading

under certain conditions market equilibrium solution equivalent to least cost solution attainable by a central social planner

from optimality principle from stochastic optimal control theory

\[ V(t, x_t) = \max_{u_t} E_0 \left( e^{-rt} C(u_t) dt + V(t + dt, x_t + dx_t) \right) \]

deduce characteristic PDE with boundary conditions
Solution

- resulting spot price non-negative
- resulting discounted spot price process is a martingale, regardless of stochastic process for emissions rate
  - in particular, no mean-reversion
  - due to storability and assumption of risk-neutral agents
- if emissions rate assumed to follow white noise process then analytic solution of characteristic PDE possible (otherwise solve numerically)
Parameter values

► chosen as to remind some stylized facts in the EU ETS
  ▶ 3 year period 2005 - 2007
  ▶ allocation of about 6 billion tons
  ▶ penalty €40 plus delivering missing EUAs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value^a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penalty p</td>
<td>70</td>
</tr>
<tr>
<td>Length of trading period $T$</td>
<td>3</td>
</tr>
<tr>
<td>Initial endowment with certificates $e_0$</td>
<td>6000</td>
</tr>
<tr>
<td>Expected total emissions $x_0$</td>
<td>6240</td>
</tr>
<tr>
<td>Marginal abatement costs $c$</td>
<td>0.24</td>
</tr>
<tr>
<td>Volatility of emission rate $\sigma$</td>
<td>$500/\sqrt{T}$</td>
</tr>
</tbody>
</table>
Spot price function $S(t, x_t)$

- fixed upper bound determined by penalty costs
- EUA price never reaches zero (option value of EUA) before $T$
Volatility function $\sigma(t, S_t)$

- Volatility function goes to infinity at trading period end $t = 3$
- Volatility reaches zero at price bounds
Consistent with observed price behavior...

From theory ...  

… to reality

EUA prices first trading period
Implications for spot price process

- discounted spot prices are martingales
  - deterministic/seasonal components in emissions rate process do not influence resulting spot price process
  - verified by empirical examination (no mean reversion)
- spot prices with fixed upper bound determined by penalty costs
  - only valid for first trading period (banking allowed after 2nd trading period)
  - empirical tests seem to support this view
- volatility of spot price process
  - increases when time is coming closer to end of trading period and
  - decreases when the price is coming closer to price bounds

Do characteristics carry over to setting with more than one trading period?
### Changes of regulatory framework

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Banking into next period</td>
<td>not allowed</td>
<td>allowed</td>
</tr>
<tr>
<td>Borrowing from next period</td>
<td>not allowed</td>
<td>not allowed</td>
</tr>
<tr>
<td>Penalty costs</td>
<td>€40</td>
<td>€100</td>
</tr>
<tr>
<td>Later delivery of lacking permits</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>
2. Extension to multi-phase model

Extension to multi-phase model

trial period \([0, T]\)

multiple periods \([0, T_1], [T_1, T_2], \ldots\)

company’s new optimization problem:

\[
\max_{u_t, z_t, t \in [0, T_n]} E_0 \left( \int_0^{T_n} e^{-rt} C(u_t) dt - \int_0^{T_n} e^{-rt} S(t) z_t dt + \sum_{j=1}^{n} e^{-rT_j} P(x_{T_j}) + R(x_{T_n}) S_{\text{end}} \right)
\]
Solution: basic idea

- apply same principles as for model with one finite trading period in backwards manner
- i.e. make use of dynamic programming algorithm
  - Bellman’s principle
  - Ito’s lemma for each finite trading period
Characteristics of spot price dynamics

First results for illustrative setting:

- chosen parameter values:
  - up to four consecutive trading periods
  - first period 5 years, next periods 8 years
  - allocation according to current allocation plans

<table>
<thead>
<tr>
<th>Phase</th>
<th>Periods</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>(2008-2012)</td>
<td>10.400</td>
</tr>
<tr>
<td>III</td>
<td>(2013-2020)</td>
<td>14.775</td>
</tr>
<tr>
<td>IV</td>
<td>(2021-2028)</td>
<td>12.455</td>
</tr>
<tr>
<td>V</td>
<td>(2029-2036)</td>
<td>10.135</td>
</tr>
</tbody>
</table>

- penalty costs: \( p_j = €100 \) in each period \( j \)
- \( S_{end} = 25 \) for four periods, discounted for less than four periods

- consider spot price for first period of each setting
  - price bounds?
  - smoother transition through banking?
Spot price function $S(t, x_t, T_1)$

- additional period increases value through possible use for compliance in further period
- upper price bound depends on number of periods
Spot price function $S(t, x_t, T_1)$

time-dependent price bounds

- upper bound

$$S_{\text{upper}}(t) = \sum_{j=1}^{n} e^{-r(T_j-t)} p_j + e^{-r(T_n-t)} S_{\text{end}}$$

- lower bound

$$S_{\text{lower}}(t) = e^{-r(T_n-t)} S_{\text{end}}$$
Spot price function $S(t, x_t, T_1)$ (back)

- steepness increases as $t$ approaches $T_1$
- still discontinuity at end of each trading period although banking is allowed
Local Volatility $\sigma(t, S_{rel})$

- highest volatility for medium spot prices
- volatility surface more moderate in multi-period setting
## Value components of current spot price \( S(t, x_t, T_1) \)

<table>
<thead>
<tr>
<th>Emissions Scenario current</th>
<th>Emissions Scenario future</th>
<th>Value Component from period 1</th>
<th>Value Component from period 2</th>
<th>Value Component from period 3</th>
<th>Value Component from period 4</th>
<th>( S_{\text{end}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>medium</td>
<td>medium</td>
<td>72%</td>
<td>11%</td>
<td>2%</td>
<td>1%</td>
<td>14%</td>
</tr>
<tr>
<td>high</td>
<td>high</td>
<td>38%</td>
<td>27%</td>
<td>18%</td>
<td>10%</td>
<td>7%</td>
</tr>
<tr>
<td>high</td>
<td>low</td>
<td>65%</td>
<td>14%</td>
<td>5%</td>
<td>5%</td>
<td>11%</td>
</tr>
<tr>
<td>low</td>
<td>high</td>
<td>0%</td>
<td>47%</td>
<td>23%</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>low</td>
<td>low</td>
<td>0%</td>
<td>2%</td>
<td>22%</td>
<td>29%</td>
<td>47%</td>
</tr>
</tbody>
</table>

- substantial part of spot price attributable to future trading periods
Conclusion

- each additional trading period leads to
  - additional possible use because of banking possibility
  - additional value component in today’s spot price
  - relative share depends on current and future expected emissions
- price bounds
  - naturally depend on number of trading periods considered
  - spot prices do not decline to zero at end of a trading period
- spot price dynamics and corresponding volatility surfaces become more moderate
  \[\Rightarrow\] behavior clearly different from resulting behavior when no consecutive trading period is considered
- nevertheless overall characteristics quite similar to one period setting