Risk premia in energy markets

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Introduction
Aim of the Project

We are interested in risk premia between spot and forward prices in energy markets.

We derive analytic expressions for such risk premia when spot prices are modelled by Lévy semistationary processes.

Today, we will study the relation between spot and forward prices using classical no-arbitrage arguments.

Empirical study of electricity risk premia.
Introduction

Outline of Talk

1. Introduction
2. Literature review
3. Lévy semistationary processes
4. The spot price model
5. Risk premia
6. Pricing with a risk premium
7. Analytical results
8. Empirical results
9. Conclusion
Several approaches have been proposed to explain the behaviour, sign and magnitude of ex-ante and ex-post risk premia.

- Bessembinder & Lemmon (2002) suggest an equilibrium model that provides a link between spot and forward prices.

- If $F(t, T) = \mathbb{E}_t^Q(S(T))$, then the ex-ante risk premium is just given by $\mathbb{E}_t^Q(S(T)) - \mathbb{E}_t^P(S(T))$ and can be studied analytically:

- E.g. Benth & Sgarra (2012) provide a theoretical justification via a change of measure argument for the sign change observed in empirical data on risk premia.

- The empirical literature has mainly focused on the ex-post risk premia, see e.g. Lucia & Torró (2011) and the references therein.
Benth, Cartea and Kiesel (2008) provide an explanation for the sign and the magnitude of the market risk premium by modelling market players and their risk preferences directly by applying the certainty equivalence principle.

Benth & Meyer-Brandis (2009) model forward prices as conditional expectations where forward-looking information are incorporated into the conditional expectation. This leads to an information based approach to explaining the risk-premium.
Lévy semistationary processes
Lévy processes

- Let $L = (L(t))_{t \geq 0}$ denote a càdlàg modification of a Lévy process with characteristic triplet given by $(\gamma, c, \nu)$, where $\gamma \in \mathbb{R}$, $c \geq 0$ and $\nu$ denotes the Lévy measure on $\mathbb{R}$ with $\nu(\{0\}) = 0$, and $\int_{\mathbb{R}} (|x|^2 \wedge 1) \nu(dx) < \infty$.

- Lévy-Khintchine representation of $L$:

$$
\mathbb{E}^P(\exp(i\theta L(t))) = \exp(t\Psi^P_L(\theta)), \quad \text{for } \theta \in \mathbb{R},
$$

where

$$
\Psi^P_L(\theta) = i\gamma \theta - \frac{1}{2}c\theta^2 + \int_{\mathbb{R}} \left(e^{i\theta x} - 1 - i\theta x \mathbb{1}_{\{|x| \leq 1\}}\right) \nu(dx)
$$

denotes the characteristic exponent of $L$ under $P$.

- Also, let $\psi^P_L(\theta) := \Psi^P_L(-i\theta)$ for $\theta \in \mathbb{R}$ be the logarithm of the moment generating function of $L$, provided it exists.
Extend the definition of a Lévy process defined on $\mathbb{R}_+$ to a two-sided Lévy process defined on $\mathbb{R}$.

Let $L'$ denote another Lévy process (and as before we consider its càdlàg modification) that is independent of $L$ and has the same characteristic triplet as $L$. We define a new process $L^*$ by setting

$$L^*(t) := \begin{cases} L(t), & \text{for } t \geq 0, \\ -L'(-(t-)), & \text{for } t < 0. \end{cases}$$

(1)

The process $L^* = (L^*(t))_{t \in \mathbb{R}}$ is called a two-sided Lévy process, see e.g. Brockwell (2009), which is càdlàg. Note, however, that in the following, we will always write $L$ rather than $L^*$ to simplify notation.

Lévy semistationary (LSS) processes have been introduced and studied in the context of energy markets in Barndorff-Nielsen et al. (2013); Veraart & Veraart (2014).
Lévy semistationary processes

**Definition 1**

An \( \mathcal{LSS} \) process on \( \mathbb{R} \) is defined as

\[
Y(t) := \int_{-\infty}^{t} g(t - s) \sigma(s-) dL(s),
\]

where \( L \) denotes the two–sided Lévy process. Further, \( g : \mathbb{R} \to \mathbb{R} \) denotes a deterministic, measurable function satisfying \( g(t - s) = 0 \) whenever \( t < s \), and \( \sigma = (\sigma(t))_{t \in \mathbb{R}} \) denotes a càdlàg, adapted and positive stochastic process, which we typically refer to as *stochastic volatility*.

- We assume that the stochastic processes \( L \) and \( \sigma \) are independent.
- The precise integrability conditions needed are stated in Veraart & Veraart (2013).
- \( \mathcal{LSS} \) processes are in general not semimartingales. Under additional smoothness condition on the kernel function \( g \) we obtain a subclass of \( \mathcal{LSS} \) processes which are semimartingales.
Lévy semistationary processes

Notation

- Sometimes we will write
  \[ Y(t) = X_0(t) + X(t), \quad \text{for } t \geq 0, \]
  where \( X_0(t) = \int_{-\infty}^{0} g(t - s)\sigma(s-)dL(s) \) is \( F_0 \)-measurable for all \( t \geq 0 \), and \( X(t) = \int_{0}^{t} g(t - s)\sigma(s-)dL(s) \).

- To shorten the notation, write \( dM(s) = \sigma(s-)dL(s) \).

- Let \( i \in \{1, 2\} \). Then
  \[ Y^{(i)}(t) = X_0^{(i)}(t) + X^{(i)}(t), \quad t \geq 0, \]
  where \( X^{(i)}(t) = \int_{0}^{t} g(t - s)dM^{(i)}(s), \)
  with
  \[ dM^{(1)}(s) = \sigma(s-) (dB(s) + \gamma ds), \]
  \[ dM^{(2)}(s) = dL(s). \]
Energy spot prices typically exhibit seasonalities.

We therefore incorporate seasonal behaviour and possibly a trend in our model in terms of a deterministic function $\Lambda : \mathbb{R}_+ \mapsto \mathbb{R}$.

We denote by $S = (S(t))_{t \geq 0}$ the energy spot/day-ahead price.
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We therefore incorporate seasonal behaviour and possibly a trend in our model in terms of a deterministic function $\Lambda : \mathbb{R}_+ \mapsto \mathbb{R}$.

We denote by $S = (S(t))_{t \geq 0}$ the energy spot/day-ahead price.

We define the geometric spot price model by

$$S(t) = \Lambda(t) \exp(Y(t)), \quad t \geq 0,$$

and the arithmetic spot price model by

$$S(t) = \Lambda(t) + Y(t), \quad t \geq 0.$$

It should be noted that in a concrete application the seasonality and trend function $\Lambda$ is typically not the same.
Consider a market with finite time horizon denoted by $T^*$, where $0 < T^* < \infty$.

**Definition 2**

Let $S = (S(t))_{t \geq 0}$ denote the spot price and let $(F(t, T))_{0 \leq t \leq T}$ denote the forward price with time of delivery $0 < T < T^*$. In the following, let $0 \leq t \leq T$.

1. The *ex-ante risk premium* at time $t$ is defined as
   \[
   R(t, T) = F(t, T) - \mathbb{E}_t^P(S(T)).
   \]

2. The *ex-post risk premium* at time $t$ is defined as
   \[
   r(t, T) = F(t, T) - S(T). \tag{6}
   \]

3. The *basis* at time $t$ is defined as
   \[
   B(t, T) = F(t, T) - S(t).
   \]
Lemma 3

The ex-ante risk premium, the ex-post risk premium and the basis are related by the following identity.

\[ r(t, T) = R(t, T) + \left( \mathbb{E}^P(S(T)|\mathcal{F}_t) - S(T) \right). \]

Also,

\[ B(t, T) = R(t, T) + \mathbb{E}^P \left[ (S(T) - S(t)) | \mathcal{F}_t \right]. \]
Risk premia

Delivery period \([T_1, T_2]\) for \(0 \leq T_1 \leq T_2 < T^*\). Swap contracts are denoted by \(F(t, T_1, T_2)\).

Definition 4

In the following, let \(0 \leq t \leq T_1\).

1. The *ex-ante swap risk premium* at time \(t\) is defined as

\[
R(t, T_1, T_2) = F(t, T_1, T_2) - \mathbb{E}_t^P \left( \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(T) dT \right).
\]

2. The *ex-post swap risk premium* at time \(t\) is defined as

\[
r(t, T_1, T_2) = F(t, T_1, T_2) - \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(T) dT. \tag{7}
\]

3. The *swap basis* at time \(t\) is defined as

\[
B(t, T_1, T_2) = F(t, T_1, T_2) - S(t).
\]
Pricing with a risk premium

- Classical no-arbitrage theory and hedging arguments lead to forward prices:

\[ F(t, T) = \mathbb{E}^Q_t(S(T)), \]

where \( Q \) is a risk-neutral probability measure.

In energy markets, not all assets are directly tradable, and hence the relationship between their forward and spot prices is far less clear. In particular, we cannot hold a portfolio of electricity spot prices, and hence one needs to be very cautious when applying hedging arguments to assets such as electricity spot prices.

An alternative approach: Define the time-\( t \) forward price for delivery at time \( T \) by \( F(t, T) \):

\[ F(t, T) = \mathbb{E}^P_t(S(T)) + R(t, T), \]

for some stochastic process \( R(t, T) \) for \( 0 \leq t \leq T \).
Pricing with a risk premium

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for some stochastic process \((R(t, T))_{0 \leq t \leq T}\).
Analytical results

Explicit results for $E^P_t(S(T))$ are available in Veraart & Veraart (2013).

Today we focus on the results based on classical no-arbitrage theory:

Assume that the forward price is given by

$$F(t, T) = E^Q_t(S(T)),$$

where $Q$ is a risk-neutral probability measure. Then, the (ex-ante) risk premium is given by

$$R(t, T) = E^Q_t(S(T)) - E^P_t(S(T)).$$
The spot price in the geometric model is given by \( S(t) = \Lambda(t) \exp(Y(t)) \).

**Proposition 1**

Assume that \( P \) and \( Q \) are equivalent probability measures. Let \( Z \) denote the Radon-Nikodým derivative of \( Q \) relative to \( P \). Then the risk premium is given by

\[
R(t, T) = \Lambda(T) \exp \left( \int_{-\infty}^{t} g(T - s) \sigma(s-) \, dL(s) \right) \xi(t; T),
\]

where

\[
\xi(t; T) = \mathbb{E}_t^P \left[ \exp \left( \int_{t}^{T} g(T - s) \sigma(s-) \, dL(s) \right) \left( \frac{Z(T)}{Z(t)} - 1 \right) \right].
\]

Independent of the choice of \( Q \), there is a stochastic component in \( R(t, T) \)!
Corollary 5

Assume that $P$ and $Q$ are equivalent probability measures. Let $Z$ denote the Radon-Nikodým derivative of $Q$ relative to $P$. Assume that the kernel function is given by $g(T - s) = \exp(-\lambda(T - s))$, for $\lambda > 0$, corresponding to an Ornstein-Uhlenbeck process. Then the risk premium is given by

$$R(t, T) = \Lambda(T) \exp \left( e^{-\lambda(T-t)} Y(t) \right) \bar{\zeta}(t; T),$$

where $\bar{\zeta}(t; T)$ is defined as in (9).
Analytical results
Arithmetic model

The spot price in the arithmetic model is given by $S(t) = \Lambda(t) + Y(t)$.

**Proposition 2**

Assume that $P$ and $Q$ are equivalent probability measures. Let $Z$ denote the Radon-Nikodým derivative of $Q$ relative to $P$. Then the risk premium is given by

$$R(t, T) = \mathbb{E}_t^P \left[ \int_t^T g(T - s)\sigma(s-)dL(s) \left( \frac{Z(T)}{Z(t)} - 1 \right) \right]. \quad (10)$$
Analytical results

Arithmetic model

The spot price in the arithmetic model is given by $S(t) = \Lambda(t) + Y(t)$.

### Proposition 2

Assume that $P$ and $Q$ are equivalent probability measures. Let $Z$ denote the Radon-Nikodým derivative of $Q$ relative to $P$. Then the risk premium is given by

$$R(t, T) = \mathbb{E}_t^P \left[ \int_t^T g(T - s)\sigma(s-)dL(s) \left( \frac{Z(T)}{Z(t)} - 1 \right) \right]. \quad (10)$$

### Proposition 3

Assume that $P$ and $Q$ are related through a structure preserving change of measure. Then the risk premium is given by

$$R(t, T) = \int_t^T g(T - s) \left( \mathbb{E}^Q(L_1)\mathbb{E}_t^Q(\sigma(s-)) - \mathbb{E}^P(L_1)\mathbb{E}_t^P(\sigma(s-)) \right) ds.$$
Analytical results

Arithmetic model

Assumption 1

Under the measure $P$, suppose the stochastic volatility process $\sigma$ satisfies either

$$d\sigma(u) = -\alpha \sigma(u) du + dJ(\alpha u),$$  \hspace{1cm} (11)

for $\alpha > 0$ and a (two-sided) Lévy subordinator $J$, which is independent of $L$, and $\beta := \mathbb{E}(J(1))$, or

$$d\sigma(u) = -\alpha (\sigma(u) - \beta) du + \nu \sqrt{\sigma(u)} dW(u),$$  \hspace{1cm} (12)

for $\alpha, \beta, \nu > 0$ and where $W$ is a (two-sided) standard Brownian motion independent of $L$. 

Proposition 4

Assume that \( P \) and \( Q \) are related through a structure preserving change of measure. In addition, suppose that Assumption 1 holds. Then the risk premium is given by

\[
R(t, T) = \sigma(t) h_1(t, T) + h_2(t, T),
\]

where \( h_1 \) and \( h_2 \) are deterministic functions, which in the case of model (11) are given by

\[
\begin{align*}
h_1(t, T) &= \left( \mathbb{E}^Q(L(1)) - \mathbb{E}^P(L(1)) \right) \int_t^T g(T - s) \exp(-\alpha(s - t)) \, ds, \\
h_2(t, T) &= \left[ \mathbb{E}^Q(L(1)) \mathbb{E}^Q(J(1)) - \mathbb{E}^P(L(1)) \mathbb{E}^P(J(1)) \right] \int_t^T g(T - s) (1 - \exp(-\alpha(s - t))) \, ds,
\end{align*}
\]

and in the case of model (12) are given by

\[
\begin{align*}
h_1(t, T) &= \int_t^T g(T - s) \left\{ \mathbb{E}^Q(L(1)) \exp(-\alpha^Q(s - t)) - \mathbb{E}^P(L(1)) \exp(-\alpha^P(s - t)) \right\} \, ds, \\
h_2(t, T) &= \int_t^T g(T - s) \left\{ \mathbb{E}^Q(L(1)) \beta^Q(1 - \exp(-\alpha^Q(s - t))) \\
&\quad - \mathbb{E}^P(L(1)) \beta^P(1 - \exp(-\alpha^P(s - t))) \right\} \, ds.
\end{align*}
\]
Empirical results

The data

- Data from the European Energy Exchange (EEX).

- **Phelix Day Peakload**: The Phelix Day Peakload consists of the arithmetic mean of the 12 hourly prices between 08:00 am (CET) to 08:00 pm (CET) derived on the weekdays of the daily auction on the spot market of EPEX Spot for the German/Austrian market area. All prices are specified in Euro per MWh.

- **Phelix Peakload Futures**: Futures contracts for which the Phelix Day Peakload is the underlying. The delivery rate of these future contracts is 1MW electricity per hour. Focus on **Front Month** Futures Peakload contracts.

- Time period: 01/10/2009 until 28/9/2012 excluding weekends. 782 observations over three years which corresponds to approximately 261 observations per year.
Empirical results
EEX Phelix peakload spot and futures, 01/10/2009 – 28/9/2012
(excl. weekends)
Empirical results
Ex-post risk premium

The empirical risk-premium for delivery during one month $M$ for the peakload futures contract is given by

$$r_{\text{emp.}}(t; M) = F(t; M) - \frac{1}{\#\{\text{weekdays in month } M\}} \sum_{T \in \{\text{weekday in month } M\}} S(T).$$
In March 2011 the German government announced major changes to their energy policy due to the nuclear catastrophe that occurred in Japan as a consequence of a devastating earthquake on 11th March 2011 and a tsunami that hit the Fukushima Daiichi Nuclear Power Plant.

On 14th March the German government announced a (temporary) moratorium on the lifespan extension of nuclear power plants.

On 15th March, it was announced that several nuclear power plants in Germany would be temporarily shut, see Market Surveillance of European Energy Exchange AG (2011) for further discussion.

These developments clearly affected the evolution of the futures prices during this time.

We observe a large upward jump of the futures prices on 14th and 15th March.
Empirical results
Ex-post risk premium: ‘Fukushima effect’ in March 2011

- The spot price was unaffected by this, which resulted in a large ex-post risk premium.
- Shut-down of several power plants ⇒ decreased future supply of electricity for the German market ⇒ futures prices increased significantly over this short period.
- This price jump can be observed in all futures traded on the Phelix on that day.
- In addition to this upward price jump, also the trading volume increased significantly, see Market Surveillance of European Energy Exchange AG (2011).
- This example clearly shows that it will always be difficult to link futures prices on electricity to current spot prices.
- Including an information premium in the modelling of futures prices along the lines of Benth & Meyer-Brandis (2009) could be promising in such a situation.
Empirical results

Ex-post risk premium: ‘Fukushima effect’ in March 2011
Empirical results
Towards the ex-ante risk premium: Fitting the spot model

- Arithmetic model based on an $LSS$ process.

- Deseasonalise the data using the following seasonality function:

$$\Lambda(t) = c_0 + c_1 t + c_2 t^2 + c_3 \cos\left(\frac{\tau_1 + 2\pi t}{5}\right) + c_4 \cos\left(\frac{\tau_2 + 2\pi t}{261}\right),$$

using a robust estimation method, where the coefficients were estimated to be $c_0 = 45.45$, $c_1 = 0.07$, $c_2 = -8.57 e^{-5}$, $c_3 = 1.08$, $c_4 = 3.02$, $\tau_1 = 35.45$, $\tau_2 = -5103.71$.

- Work with deseasonalised and demeaned time series from now on.
Empirical results
Towards the ex-ante risk premium: Fitting the spot model

- Preliminary data analysis suggests a CAR(3) model.

- Such a model corresponds to choosing

\[ g(t - s) = \mathbf{b}_1' \exp(\mathbf{A}(t - s))\mathbf{b}_p, \]

where \( \mathbf{b}_1 \) and \( \mathbf{b}_p \) denote the first and \( p \)th unit vector in \( \mathbb{R}^p \), respectively, and \( \mathbf{A} \) is a \( p \times p \) matrix given by

\[
\mathbf{A} = \begin{bmatrix}
0 & \mathbf{I}_{p-1} \\
-\alpha_1 & -\alpha_{p-1} & \ldots & -\alpha_1
\end{bmatrix},
\]

where \( \mathbf{I}_{p-1} \) denotes the \( (p - 1) \times (p - 1) \)-identity matrix, \( \mathbf{0} \) is a \( p - 1 \)-dimensional vector consisting of zeros, and \( \alpha_1, \ldots, \alpha_p \) are constants which are typically assumed to be positive.

- The Euler-discretisation of a CAR(3) process leads to an AR(3) process \( (\Xi_n)_{n \in \mathbb{N}} \) of the form

\[
\Xi(n) = b_1 \Xi(n - 1) + b_2 \Xi(n - 2) + b_3 \Xi(n - 3) + \epsilon(n),
\]

where \( \epsilon(n) \) corresponds to the Euler-discretisation of the process \( M \).
Empirical results
Model fit of the CAR(3) process

- Empirical and fitted ACF of the detrended and deseasonalised spot prices,
- ACF of the AR(3) residuals,
- ACF of the squared AR(3) residuals,
- Quantile-Quantile plot comparing the empirical distribution of the AR(3) residuals with the symmetric Student-t distribution.
Ex-ante swap risk premium: Based on a Brownian-driven CAR(3) model, which is modulated by a stationary stochastic volatility process $\sigma$ such that $\sigma^2$ has inverse Gamma stationary distribution, we compute the conditional expectation of the future averaged spot price over each month and compare it to the corresponding forward price.
Empirical results
Ex-ante risk premium

➤ Does the predicted spot price under the measure $P$, i.e. the market expectation, has any predictive power for the corresponding forward prices?

➤ Only a little bit! Prediction of seasonality and trend seems more important than predictions from the stochastic model.

➤ Linear regression analysis: futures $\sim$ predicted average spot price. We computed the residual sum of squares (RSS) and divided it by the total sum of squares (TSS) and obtain $\text{RSS/TSS} = 53\%$. 
Main contributions

➤ Study of risk premia in energy markets when energy spot prices are modelled by Lévy semistationary (\( \text{LSS} \)) processes.

➤ We have derived explicit analytical results for the corresponding risk premia based on no-arbitrage arguments.

➤ Structural difference between geometric and arithmetic models based on \( \text{LSS} \) processes.

➤ Stochastic volatility plays a key role in describing the dynamics of the risk premium.

➤ Extended empirical work on EEX electricity futures and spot.
   - Despite the fact that our \( \text{LSS} \)-based model fits the spot prices very well, we find that the corresponding conditional expectation has only some explanatory power for the futures.
   - There is still a significant amount of variability (\( > 50\% \)) in the futures which cannot be explained by our predicted average spot prices.


