Managing Temperature Driven Volume Risks

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The European Market

Trading hubs in Europe:
- the National Balacing Point (NBP) in UK
- Zeebruegge in Belgium (ZEE)
- Title Transfer Facility (TTF) in the Netherlands
- NCG and GASPOOL in Germany
- PEGn, PEGs in France

Hubs are connected:
- UK market and continental Europe are connected by the interconnector
- TTF and Zeebruegge are connected by a network of pipelines

Figure 1: European gas hubs and gas exchanges
Gas Futures Market

- monthly, quarterly, seasonally, yearly contracts
- seasonal contracts are *summer* (Apr-Sep) and *winter* (Oct-Mar)
- **cascading** of fwd contracts: on their last day of trading these futures are replaced with equivalent futures with shorter delivery periods
- day-ahead forwards, weekend ahead, ...

**TTF and NCG year ahead prices**
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*NBP and ZEE year ahead prices*
Gas Spot Market

- You can trade day-ahead products OTC or on exchange (eg EEX, ENDEX, POWERNEXT).
- Spot market permits anonymous, transparent and non-discriminatory 24/7 trading of quality-specific gas products.
- Typical delivery periods are Day Ahead, Weekend, Individual Days.
Gas demand is a function of local temperature.

- The gas demand is a function of temperature.
- The normalized demand curve can be modeled by a sigmoid function.
- The sigmoid function depends on customer specific parameter and the stochastic temperature $T$

\[
\text{demand} = \delta + \frac{\alpha}{1 + \left( \frac{\beta}{T - \theta} \right)^\gamma}
\]
Full supply gas contracts of retail costumers are temperature dependent

- A common contract type offered to retail customers is based on a full supply gas delivery. The customer pays a fixed price (per MWh) for its individual consumption with no constraints.
- The gas supplier takes the volume risk of deviations from the projected load profile of the costumer. There is a deterministic dependency of gas volumes and temperature.

![Diagram showing temperature-driven volume risks]

- Customer’s excess volumes have to be sold by supplier to balance the position. Market price is lower than fixed customer’s price.
- To meet customer’s high demand supplier has to buy gas at market at high prices and gets paid fixed price by customer.
Temperature Swaps as Hedge Products

We need products to hedge temperature driven fluctuations of gas prices.

Spot versus M1 prices are influenced by temperature deviations.

Cummulative payoff of daily legs given by:

\[ \sum_{i \in I} N_i \cdot (S(\tau_i) - \text{GasStrike}_i) \cdot (\text{TempStrike}_i - T(\tau_i)) \]

- \( N_i \) is the notional for day \( i \), \( \text{GasStrike}_i \) is the strike for the spot price and \( \text{TempStrike}_i \) is the temperature strike (typically the historical mean).
Modeling Idea

Goal

*Define an analytical treatable model for gas and temperature dynamics which allows for implied and/or historical calibration to market data (forwards and options).*

The model specification follows a modular framework:

- First, the commodity-leg is specified, considering the commodity as an outright asset (gas model).
- Second, the temperature leg is specified, again as an outright model (temperature model).
- Last, both dynamical systems are joint together.
The General Framework

Basic Model

\[ \log S(t) = h(t) + X(t) + Y(t) \]
\[ T(t) = \theta(t) + Z(t) + W(t) \]

- \( S \) denotes the spot price and \( T \) the temperature index.
- The processes \( X, Y \) and \( Z, W \) determine the stochastic dynamics of spot and temperature.
- \( h(t) \) and \( \theta(t) \) are deterministic functions.
The General Framework

Basic Model

\[ \log S(t) = h(t) + X(t) + Y(t) \]

\[ T(t) = \theta(t) + Z(t) + W(t) \]

Spot Processes

\[ dX = -\kappa_X X dt + \sigma_X(t) dB_X, \quad X(0) = x \]

and a stochastic mean-reversion level process \( Y(t) \) defined by

\[ dY = -\frac{\sigma_Y(t)^2}{2} dt + \sigma_Y(t) dB_Y, \quad Y(0) = y. \]

- The deterministic volatility functions \( \sigma_X(t), \sigma_Y(t) \) and the mean-reversion speed \( \kappa_X \) are model parameters.
- Processes \( B_X \) and \( B_Y \) are not correlated (for simplicity).
The General Framework

Basic Model

\[ \log S(t) = h(t) + X(t) + Y(t) \]
\[ T(t) = \theta(t) + Z(t) + W(t) \]

Risk-Neutral Measure

- If a risk-neutral measure \( Q \) is chosen, the function \( h(t) \) has to be chosen, such that
  \[ E^Q(S(t) | \mathcal{F}_0) = f(t) \]
  is satisfied, where \( f(t) \) is the daily forward curve.
- Expectation \( \mu(t) \) and variance \( \nu(t) \) of the log price can easily be computed.
- The risk-neutral condition requires
  \[ h(t) = \log f(t) - \frac{\nu(t)}{2} - \mu_X(t) - \mu_Y(t) \]
The General Framework

Basic Model

\[ \log S(t) = h(t) + X(t) + Y(t) \]
\[ T(t) = \theta(t) + Z(t) + W(t) \]

Under the risk-neutral measure the spot price \( S(t) \) can be written as a product of the damped daily forward curve and two log-normal random variables,

\[ S(t) = f(t) e^{-\nu(t)/2} e^{X(t)-\mu_X(t)} e^{Y(t)-\mu_Y(t)}. \]
The Temperature-Leg

- \( T = \theta(t) + Z(t) + W(t) \) with
  \[
  dZ = -\kappa_Z Z dt + \sigma_Z(t) dB_Z, \quad Z(0) = 0 \\
  dW = -\kappa_W W dt + \sigma_W(t) dB_W, \quad W(0) = 0
  \]

- The OU-process \( Z \) describes the short-term dynamic of the temperature, i.e. the mean-reversion to the long term averaged temperature \( \theta(t) \).
- The OU-process \( W \) describes the long term variability of the mean-reversion level \( \theta(t) \).
- We assume that both processes are not correlated.

By standard arguments we find analytically

\[
T(t) = \theta(t) + \int_0^t e^{-\kappa_Z(t-u)} \sigma_Z(u) dB_Z(u) + \int_0^t e^{-\kappa_W(t-u)} \sigma_W(u) dB_W(u)
\]
Assume $\lambda_Z(t), \lambda_W(t)$ are real-valued, measurable and bounded functions and let

$$\Lambda(t) = \left( -\frac{\lambda_Z(t)}{\sigma_Z(t)}, -\frac{\lambda_W(t)}{\sigma_W(t)} \right)$$

and $B = (B_Z, B_W)$. Define

$$Z^\Lambda(t) := \exp \left( \int_0^t \Lambda(u) dB(u) - \frac{1}{2} \int_0^t \|\Lambda(u)\|^2 du \right)$$

as the density process of the probability measure

$$Q^\Lambda(A) = \mathbb{E} \left( I_A Z^\Lambda(T_\infty) \right)$$

Under the risk-neutral measure $Q^\Lambda$ we get additional drift terms and we find

$$T(t) = \theta(t) + \int_0^t \lambda_Z(u) e^{-\kappa Z(t-u)} du + \int_0^t \lambda_W(u) e^{-\kappa W(t-u)} du$$

$$+ \int_0^t e^{-\kappa Z(t-u)} \sigma_Z(u) dB_Z(u) + \int_0^t e^{-\kappa W(t-u)} \sigma_W(u) dB_W(u)$$

The function $\lambda_Z, \lambda_W$ are the market price of risk. The market price of risk defines a risk premium which is added to the long term averaged temperature profile.
The Combined Gas-Temperature Model

In the risk-neutral measure we have finally,

\[ \log S(t) = h(t) + X(t) + Y(t) \]
\[ T(t) = \theta(t) + Z(t) + W(t) \]

with processes

\[ dX = -\kappa_X X \quad dt + \sigma_X(t) dB_X \]
\[ dY = -\frac{\sigma_Y(t)^2}{2} \quad dt + \sigma_Y(t) dB_Y \]
\[ dZ = (\lambda_Z(t) - \kappa_Z Z) \quad dt + \sigma_Z(t) dB_Z \]
\[ dW = (\lambda_W(t) - \kappa_W W) \quad dt + \sigma_W(t) dB_W \]

and instantaneous correlations

\[ dB_X dB_Z = \rho(t) dt, \quad dB_Y dB_W = R dt \]

where \( \rho(t) \) is a time-dependent (for example yearly periodic) function and \( R \) is a constant.
Calibration of the Commodity Leg

- We interpret the stochastic mean reversion level $Y$ as the log price of a synthetic M1 product. This product delivers for 30 days starting instantaneously.

- In this framework we assume that log spot prices mean revert to the M1 level, the OU process $X$ defines the mean reversion.

M1 Process

- Based on standard instantaneous volatility model of forward prices,

$$\Sigma(t, T_1, T_2) = \frac{a}{b(T_2 - T_1)} \left( e^{-b(T_1 - t)} - e^{-b(T_2 - t)} \right) + c$$

we model

$$\sigma_Y(t) = \frac{365a}{30b} \left( e^{-bt} - e^{-b(t+30/365)} \right) + c$$

- We derive variance $v_Y(t)$ and price options using $\sigma_{76}(\tau) := \sqrt{\frac{v_Y(\tau)}{\tau}}$.

- The forward parameters can be calibrated implicitly to (option) markets.
Calibration of the Commodity Leg

OU Process

- We distinguish between summer and winter spot volatility for the OU process \( X \),
  \[
  \sigma_X(t) = \begin{cases} 
  \sigma_{\text{sum}}, & t \in \text{summer} \\
  \sigma_{\text{win}}, & t \in \text{winter} 
  \end{cases}
  \]

- Parameters \( \kappa_X, \sigma_{\text{sum}}, \sigma_{\text{win}} \) are calibrated implicitly by market quotes for strip of daily options.

- A spot option with maturity \( \tau \) can be computed by Black-76 formula using the volatility
  \[
  \sigma_{76}^{\text{spot}}(\tau) = \sqrt{\frac{\nu(\tau)}{\tau}}
  \]
  with the total variance \( \nu(\tau) \) given above.

  
- We can use this pricing formula to calibrated against typically quoted strips of options.
We assume that the seasonal averaged temperature is periodic (one year) and is parameterized by a truncated Fourier series

\[ \theta(t) = c_1 + c_2 \cdot t + c_3 \cos(2\pi t + c_4) \]

with parameters \( c_1, c_2, c_3, c_4 \) which are determined by least squares regression.

The short term mean reversion parameter of the residual (i.e. de-seasonalized) temperature process is estimated using an AR(1) process.

We assume a monthly step-wise volatility function.
Calibration of the market price of risk

The market price of risk can be determined by traded temperature futures like HDD and CDD futures. Under the risk neutral measure we can approximate today’s traded HDD futures price $F_{\text{HDD}}(T_1, T_2)$ for delivery between time $T_1$ and $T_2$ by

$$F_{\text{HDD}}(T_1, T_2) = \mathbb{E}^Q \left( \int_{T_1}^{T_2} (c - T(t))^+ \, dt \bigg| \mathcal{F}_0 \right)$$

where the exercise temperature $c$ is (typically) $18^\circ \text{C} (65^\circ \text{F})$.

Using Bachelier’s option price formulas one can show

$$F_{\text{HDD}}(T_1, T_2) = \int_{T_1}^{T_2} \Sigma_T(t) \left[ d(t) \cdot N(-d(t)) + N'(d(t)) \right] \, dt$$

with $d(t) := \frac{\mu_T(t) - c}{\Sigma_T(t)}$ and normal cumulative distribution function $N(x)$.

The market price of risk can be estimated using temperature forward products.
Calibration of Long-Term Temperature and Correlation

- Long-term temperature parameters are estimated similarly to short term, however data are clustered on months. Residual temperature is clustered and AR(1) process is fitted.

- Short term correlation is typically unstable. Estimate correlation based on stochastic increments over a rolling windows and aggregate to quarterly time buckets.

- For long term correlation estimate stochastic increments and aggregate to a yearly level.
Valuation of Temperature-Gas Swap

- The considered temperature gas swap pays at time $t$
  \[ \Lambda(t) := N(t) \cdot (S(t) - k(t)) \cdot (\tilde{k}(t) - T(t)) \]

- For the valuation we have to determine
  \[ \mathbb{E}(\Lambda(t)|\mathcal{F}_0) \]

- Based on \[ \mathbb{E}(S(t) \cdot (T(t) - \mathbb{E}(T(t)))) = \text{Cov}(S(t), T(t)) \] we find
  \[ \text{Cov}(S(t), T(t)) = f(t)e^{-\nu(t)/2 - \mu_X(t)} \text{Cov} \left( e^{X(t)+Y(t)}, W(t) + Z(t) \right) \]
  \[ = f(t)e^{-\nu(t)/2 - \mu_X(t)} \left( \text{Cov} \left( e^{X(t)+Y(t)}, W \right) + \text{Cov} \left( e^{X(t)+Y(t)}, Z \right) \right) \]
  \[ =: d_1(t) \]
Valuation of Temperature-Gas Swap

We use Stein’s Lemma to find

\[
\text{Cov} \left( e^{X(t)+Y(t)}, W(t) \right) = \mathbb{E} \left( e^{X(t)+Y(t)} \right) \text{Cov} \left( X(t) + Y(t), W(t) \right) = e^{\nu(t)/2 + \mu X(t)} \text{Cov} \left( Y(t), W(t) \right)
\]

\[
\text{Cov} \left( e^{X(t)+Y(t)}, Z(t) \right) = e^{\nu(t)/2 + \mu X(t)} \text{Cov} \left( X(t), Z(t) \right)
\]

Putting all intermediate results together yields

\[
d_1(t) = f(t) \left( \text{Cov} \left( X(t), Z(t) \right) + \text{Cov} \left( Y(t), W(t) \right) \right).
\]

The covariances \( \text{Cov} \left( X(t), Z(t) \right) \) and \( \text{Cov} \left( Y(t), W(t) \right) \) are known analytically. We compute the expected cashflow of a temperature-gas swap and find

\[
\mathbb{E} \left( (S(t) - k)(\tilde{k} - T(t)) \right) = -d_1(t) + (f(t) - k) \left( \tilde{k} - \theta(t) \right)
\]

\text{extrinsic} \quad \text{intrinsic}
Example - A Swap Deal

Deal Description

- Value a TTF gas / temperature swap which delivers in W14. The valuation day shall be **01/04/2014**. The payoff shall be

\[
\sum_{i=1}^{182} (DA_i - k) \cdot (TempStrike_i - T_i)
\]

where \(DA_i\) is the day-ahead price of day \(i\) and \(T_i\) is a temperature index defined at Düsseldorf airport.

- The gas strike is \(k = 26.15\) Euro/MWh (which is ATM of 31/01/2014, e.g. day of contract signing).

- The temperature strike \(TempStrike_i\) differs for each day in delivery period and is derived from a 20 year history of Düsseldorf temperature.
Calibration

- Gas parameter are calibrated implicitly to market (option data).

- Short term temperature parameter are estimated on historical 20 years. For simplicity we add no risk premium here.

- Long term parameter are estimated on same history. We will study sensitivity with respect to these parameters.

- Correlation are estimated on a 3 year history.
Valuation and Sensitivity - Forward Implied Volatility

- The extrinsic valuation is defined by $d_1(t)$. We study the sensitivity of $d_1(t)$ (analytical) and the cashflow distribution (Monte-Carlo) with respect to the gas forward volatility.
- For that we define a base scenario and shift the implied vols parallel by +2% (high) and -2% (low)

![Graph](image)

- $d_1(t)$ - defines expected cashflow
- The cashflow distribution has fatter tails than a normal distribution (red fit)
- The expected cashflow is sensitive to the forward volatility, quantiles are more robust.
Valuation and Sensitivity - Forward Implied Volatility

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\[
\begin{align*}
\text{base} - \text{case cashflow distribution} \\
\text{The cashflow distribution has fatter tails than a normal distribution (red fit)} \\
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\end{align*}
\]
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![Cashflow Distribution](image1)

**base** - case cashflow distribution

- The cashflow distribution has fatter tails than a normal distribution (red fit)

- The expected cashflow is sensitive to the forward volatility, quantiles are more robust.

![Cashflow Distribution](image2)

**low** - case cashflow distribution
Valuation and Sensitivity - Long Term Parameters

- Here we study the sensitivity of $d_1(t)$ and the cashflow distribution with respect to the temperature long term variance.
- For that we define a base scenario and change the OU volatility of the long term temperature process.

$d_1(t)$ - defines expected cashflow

Risk premia can be added to long term temperature parameters.

Analogously, risk premia can be added to correlation parameters (not shown here)
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- **base** - case cashflow distribution
- **low** - case cashflow distribution

Risk premia can be added to long term temperature parameters.

Analogously, risk premia can be added to correlation parameters (not shown here).
Conclusion

- Gas demand is driven by temperature which results in volume risks.

- Risk can be managed by customized products, for example gas-temperature swaps.

- We presented a four factor model to simulate combined temperature and gas dynamics.

- The model allows for analytical valuation expressions and for a market reflective calibration of parameters.