Calibration and Parameter Risk Analysis for Gas Storage Models

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New Abstract

In this paper we provide a comprehensive framework for analyzing the model risk associated with the valuation of Gas Storage products. We derive a consistent and holistic approach to evaluating these deals with respect to the uncertainty associated with both model choice and parameter calibration/estimation. For the latter we consider both calibration to market instruments and estimation from historical data. We follow the recent example of the literature pertaining to exotic option pricing and propose methods for adjusting storage valuations to account for the uncertainty inherent in the parametrization of the pricing model used. We utilize the theory of convex risk measures and previous work in the area of calibration risk functionals in presenting a methodology which unifies market-based calibration risk and historically estimated parameter risk into a single model risk metric. The potential benefit of such a framework will impact equally trading, risk and regulatory stakeholders within a storage business, from model validation through to deriving appropriate bid-offer levels. We provide detailed numerical examples based upon state of the art Lévy based forward curve models and demonstrate how prices can be adjusted to incorporate model risk and also how several different models can be ranked depending upon the risk implicit in their estimation.
Business Education

Financial models create a false sense of security

By Didier Cossin

Business schools are in fact creating a system of flawed models that are used to instruct managers.

And I'd like especially to thank the business school faculty who taught me all I know.

Bankruptcy Court

The flawed maths of financial models

By Pablo Triana

What many suspected has been found true: quantifying in finance may be an oxymoron.

Imagine a car school that specialised in teaching how to build Toyotas. Following the manufacturer's recall of thousands of malfunctioning vehicles, should the school rethink its curriculum or should it trot along unperturbed, delivering the same lectures as before, as if nothing had happened?

A similar quandary is faced today by those universities that offer graduate financial programmes.
Model Risk Management: Regulatory Context

- Basel Committee on Banking Supervision
Fed OCC: Risk Bullething on Model Validation

- Independent Review
- Defined Responsibility
- Model Documentation
- Ongoing Validation
- Audit Oversight

- Model development, implementation and use
- Governance and control mechanisms
- Policies and procedures
- Controls and compliance
- Appropriate incentives and organisational structure
Model Risk Definition

- Derman (1996)
  - Model inapplicability
  - Incorrect model use
  - Incorrect solution to a correct model
  - Incorrect use of a correct model
  - Use of poorly specified model approximations
  - Software and hardware errors
  - Unstable or poor quality data input
Model Risk v Model Uncertainty

- **Model Risk**
  - Exposure to future possible outcomes but with a unique defined probability measure

- **Model Uncertainty**
  - Exposure to future possible outcomes for which there is no one unique defined probability measure

- Knight (1921)
Selected Literature:
- Green and Figlewski (1999)
- Gibson et al. (1999)
- Hull and Suo (2002)
- Nalholm and Poulsen (2006)
- Kerkhof et al. (2009)
- Gupta (2009)
  - with Reisinger
- Morini (2011)
- Deryabin (2012)
- Glasserman and Xu (2013)
- Bannor and Scherer (2013)
- Bannor et al. (2013)
1. Monotonicity: If \( \mu(C_1) \leq \mu(C_2) \), then it should also hold that \( \mu(C_1 + C_2) \leq \mu(C_2) \).

2. Risk is model dependent: It should not reduce the model uncertainty.

3. Replacing the model by its rival should not increase model uncertainty.

4. The model uncertainty is a coherent risk measure.

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**cont.1** For benchmark options, the model uncertainty is no greater than the uncertainty of the market price:

\[
\forall i \in I, \quad \mu(C_i) \leq |V^{(i)bid} - V^{(i)ask}|.
\]

**cont.2** Dynamic hedging with the underlying does not reduce model uncertainty, since the hedge is model dependent:

\[
\forall \phi \in \Phi, \quad \mu \left( X + \int_0^T \phi_t dS_t \right) = \mu(X).
\]

But if the value of a claim can be totally replicated in a model-free way using only the underlying, then the claim has zero model uncertainty:

\[
\text{if } \exists x \in \mathbb{R}, \phi \in \Phi \text{ s.t. } \forall \theta \in \Theta, \quad X = x + \int_0^T \phi_t dS_t \quad \text{\theta-a.s. then } \quad \mu(X) = 0.
\]

**cont.3** Diversification does not increase the model uncertainty of a portfolio:

\[
\forall X_1, X_2 \in X, \forall \lambda \in [0, 1], \quad \mu(\lambda X_1 + (1 - \lambda)X_2) \leq \lambda \mu(X_1) + (1 - \lambda)\mu(X_2).
\]

**cont.4** Static hedging of a claim with traded options is bounded by the sum of the model uncertainty of that claim and the uncertainty in the cost of replication:

\[
\forall X \in X, \forall a \in \mathbb{R}^d, \quad \mu \left( X + \sum_{i=1}^d a_i C_i \right) \leq \mu(X) + \sum_{i=1}^d |a_i||V^{(i)bid} - V^{(i)ask}|.
\]
Worst-Case Coherent Measure (Cont, 2006):

Bounds

\[ \bar{\pi}(X) = \sup_{Q \in \mathcal{Q}} E^Q[X] \quad \pi(X) = \inf_{Q \in \mathcal{Q}} E^Q[X] = -\bar{\pi}(-X) \]

Measure

\[ \mu_Q(X) = \bar{\pi}(X) - \pi(X) \]

Worst-Case Convex Measure (Cont, 2006)

Bounds

\[ \pi^*(X) = \sup_{Q \in \mathcal{Q}} \{ E^Q[X] - \| C^* - E^Q[H] \| \} \]

\[ \pi_*(X) = -\pi^*(-X) = \inf_{Q \in \mathcal{Q}} \{ E^Q[X] + \| C^* - E^Q[H] \| \} \]
Bannor and Scherer (2013)

- Propose calibration risk functional concept
  - Incorporate calibration risk into bid-offer levels

- Theoretical framework consistent with convex risk measure concept

- Derive push-forward distributions for asset values based on calibration error from stochastic models

- Examine parameter risk inherent in a real options model-based approach to valuing power plant infrastructure
- Draw on innovative risk capturing approach of Bannor and Scherer (2013)
- Authors use sophisticated multi-factor setting to model emissions, gas and power prices
- Multidimensional search problem that poses considerable parameter risk
- Given unavailability of joint estimator’s distribution in closed form, parameter risks are separately studied
- Bid-ask spreads from AVaR risk capturing price functional show spike risk is the most important parametric risk!
Motivation for Current Work

- Paucity of academic literature on model risk and model uncertainty

- Growing importance of industry practice of model risk management and model validation activity
  - Regulatory impetus

- Bannor et al (2015) pave the way for further research into model risk issues in energy markets
  - Perfect context given the range of OTC products and structures
  - Extensive use of models for valuation and hedging activity
Motivation for Work

- Gas storage capacity presents a prime candidate for model risk analysis given:
  - Ever increasing importance of gas storage capacity in Europe, and globally
  - Difficulty in deriving competitive prices for this capacity
  - Growing secondary market for capacity ... allowing market players to adjust seasonal flexibility to match portfolio growth
  - Dependency on models for valuation and hedging
  - No industry- or academic-consensus in circulation
Contributions

- Derive holistic approach to evaluating the uncertainty associated with parameter calibration and estimation
  - Calibration to market derivative instruments
  - Estimation to historical data

- Consider an innovative suite of mean-reverting Levy-driven models that offer market consistency
  - Developed in first two papers of three-paper series

- Suggest method for ranking models based on their robustness to calibration errors
  - Relevance to trading, risk and regulatory stakeholders
Henaff et al (2013)

- Examine historically estimated parameter risk associated with storage valuation
- Employ coherent model risk measure of Cont (2006)
  - But deviate from this by replacing the set of benchmark instruments ...
  - ... with a test which determines whether a set of model parameters returns likelihood value “close” to the ML value
- Use two proposed spot price models with price spikes

- We differ from Henaff et al (2013) in two ways:
  - Unified approach to evaluating calibration and estimation risk
  - Use of market consistent model framework that incorporates wider market information and is more in line with practical trading considerations
Bannor and Scherer (2013): Risk capturing functionals

- Risk functional $\Gamma$ ... giving bid-ask spread $[-\Gamma(-X), \Gamma(X)]$.

- $\Gamma$ has certain desirable properties
  - Order Preservation: If one payoff dominates another then it should not incur a greater model risk premium.
    $$X \succsim Y \Rightarrow \Gamma(X) \ll \Gamma(Y)$$
  - Diversification: Combining of payoffs exposed to model risk should not increase ones model risk premium
    $$\Gamma(\lambda X + (1 - \lambda) Y) \leq \lambda \Gamma(X) + (1 - \lambda) \Gamma(Y) \quad \lambda \in [0, 1]$$
  - Model Independence: If the payoff’s value is model independent then there should be no model risk premium incorporated in its price
Bannor and Scherer (2013): Risk capturing functionals

The authors show how the functional $\Gamma$ generated by normalized, law invariant, convex risk measures $\rho$ satisfy the above properties such that

$$\Gamma (X) := \rho (Q \mapsto E_Q [X])$$

- $Q$ is one of a family of potential models $Q$ generally unknown distribution $R$
- In many cases the estimators of the model $Q$ will possess asymptotic normality
- We can exploit this to note that

$$\left( E_{\theta_n} [X] - E_{\theta_0} [X] \right) \sim N \left( 0, (\nabla E_{\theta_0})' \Sigma \nabla E_{\theta_0} \right)$$

where $E_{\theta_n} [X]$ is the expected value of the payoff using the sample parameter estimates. $E_{\theta_0} [X]$ is the expected value under the true model parameters and $\nabla E_{\theta_0}$ denotes the gradient of the value with respect to the model parameters.
Bannor and Scherer (2013): Risk capturing functionals

With this in place ...

The authors introduce the $\theta_n * AVaR$ risk captured functional generated by the AVaR convex risk measure. The offer price of a claim $X$ under this functional is then approximated by

$$
\theta_n * AVaR (X) \approx E_{\theta_0} [X] + \frac{\varphi (\Phi^{-1} (1 - q))}{q \sqrt{N}} \sqrt{(\nabla E_{\theta_0})}' \Sigma \nabla E_{\theta_0}
$$

and so they propose adjusting the mid price by an amount

$$
\frac{\varphi (\Phi^{-1} (1 - q))}{q \sqrt{N}} \sqrt{(\nabla E_{\theta_n} [X])}' \Sigma \nabla E_{\theta_n} [X]
$$

where $q \in (0, 1)$ and

$$
\nabla E_{\theta_n} [X] = \frac{E_{\theta_n + \epsilon} [X] - E_{\theta_n - \epsilon} [X]}{2\epsilon}
$$
Bannor and Scherer (2013): Risk capturing functionals

- Finally ...

  - \( h \) is decreasing. This is to ensure that parameters which yield a higher total error are given less likelihood.

  - \( \int h(\varepsilon(\theta)) \, d\theta = 1 \). This is simply a normalization to ensure the function meets the definition of a probability distribution.

One example of such a transformation function is the normal transformation function

\[
h^N_{\lambda} (t) := c \exp \left( - \left( \frac{t - t^*}{\lambda} \right)^2 \right) \quad t \geq t^*
\]
Energy Model Risk Analysis

- Wish to consider parameter calibration risk and estimation risk jointly. But why?
- Realistic models of natural gas forward curve cannot be calibrated to benchmark instruments alone
  - Due to lack of liquid time-spread options market
- Correlation structure is typically estimated from historical data and then approximated by a suitable model specification
- Storage valuation models are particularly sensitive to the model implied correlation structure
  - Hence exposed to parameter estimation risk!
- Overall level of volatility implied by model constrained to be calibrated to the market
  - To obtain consistency with the products used to hedge volatility risk
  - Hence exposed to calibration risk!
Energy Model Risk Analysis

- We propose the following transformation function form:

\[ h(\varepsilon(\theta)) = h_m(\varepsilon(\theta) | \theta_h) p(\theta_h) \]

- Transformation function decomposed into:
  - Error term density conditional upon the historically estimated parameters
  - Sampling error density associated with the historically estimated parameters

- From Bannor et al (2015), it is known that asymptotically \( (\theta_h - \theta_0) \sim N(0, \Sigma) \)

- Gaussian density suitable specification for sampling error density
Steps follow closely those of Bannor et al (2015)

- We apply the delta method described by the authors in conjunction with the sampling error density
- Construct a joint market and historical parameter risk induced value density
Energy Model Risk Analysis

If we define the covariance matrix of $M$ relative maturity forward curve returns to be $C \in \mathbb{R}^{m \times m}$ with spectral decomposition

$$C = WDW^{-1}$$

and denote the dimensionally reduced approximation of $C$ as $C_e$. Further, let us denote the model approximation of $C_e$ as $C_a$ with approximation error defined as $C_e - C_a = \mu_\theta$ which, as discussed above we take as given. Denote the covariance generating function associated with this model as $g(\theta_h)$ such that $g(\theta_h) = C_a$. We then have

$$C_e^m - C_e \sim N(0, \Sigma)$$

where $\Sigma$ is the inverse sample Fisher Information matrix associated with the sample covariance matrix $C_e^m$ estimated from a sample of size $n$. Therefore

$$C_a^m - C_a \sim N(0, \Sigma)$$

and so

$$(g(\theta_h^m) - g(\theta_h)) \sim N(0, \Sigma)$$
Applying the delta method we then have

\[
(g^{-1} g(\theta^n_h) - g^{-1} g(\theta_h)) \sim N \left( 0, (\nabla g^{-1})' \Sigma \nabla g^{-1} \right)
\]

\[
(\theta^n_h - \theta_h) \sim N \left( 0, (\nabla g^{-1})' \Sigma \nabla g^{-1} \right)
\]

For storage payoff given by \( X \), applying the delta method again gives the sampling error induced storage value density

\[
(E_{\theta_h} [X] - E_{\theta_0} [X]) \sim N \left( 0, (\nabla E_{\theta_0})' (\nabla g^{-1})' \Sigma \nabla g^{-1} \nabla E_{\theta_0} \right)
\]

(5)
Energy Model Risk Analysis

- Last result gives the storage value variance induced by uncertainty over the estimate of the forward curve covariance matrix.

- The relationship can be understood as:
  - First, weighting the matrix $\sum$ by the sensitivity of the model parameters to the sensitivity of the forward curve covariance matrix.
  - Second, weighting the result by the sensitivity of the storage value to the model parameters.
Model Specifications

- **Mean-Reverting Variance Gamma (MRVG)** ... Kiely et al (2015a)

\[
dx(t) = \left( \frac{\partial f(0,t)}{\partial t} \right) - \kappa_j \left( \exp(-\alpha t) \right) + \alpha f(0,t) - \alpha \int_0^t \kappa_j \left( \exp(-\alpha(t-s)) \right) ds - \alpha x(t) \right) dt + dX(t) \tag{6}
\]

where \( f(0,t) \) is the initial log forward price for time \( t \) and \( dX(t) \) is a driftless Variance-Gamma process with \( \kappa_j \) its cumulant.

- **Mean-Reverting Jump Diffusion (MRJD)**
  - Second model ... variant of first ... specified by Deng (2000) and used by Kjaer (2008)
  - Choice of stochastic driver is different
  - Jump-diffusion process with compound Poisson jump process driven by a double exponential distribution
Model Specifications

- MRVG-3x ... Kiely et al (2015b)

\[ dx(t) = \frac{\partial f(0, t)}{\partial t} + dy^{(1)}(t) + dy^{(2)}(t) \]

where

\[ dy^{(1)}(t, T) = (-\kappa_j(1) (b \exp(-\alpha (T - t)) \sigma)) dt + b \exp(-\alpha (T - t)) \sigma dX^{(1)}(t) \]

\[ dy^{(2)}(t, T) = -\frac{1}{2} (\exp(-\epsilon (T - t)) c_1)^2 dt + (\exp(-\epsilon (T - t)) c_1) \sigma dW^{(2)}(t) \]

and \( dW^{(2)}(t) \) is a standard Brownian Motion.

- First factor accounts for majority of forward curve variability
  - Composition of MRVG and MR Diffusion
  - Parameter \( b \) ... proportion of total variance attributed to first factor

- Second factor approximates the typical shape of the sensitivity of the forward curve to the second PC of forward curve returns covariance matrix
Storage Contract and Data

- Simple 20in / 20out storage deal
  - Deal commences immediately on options quote date
  - Lasts for 1 year
  - All values in pence / therm

- Options data
  - 6-month and 1-year monthly options on NBP
  - Strike prices ranging from 0.95-1.05 moneyness
  - 14 option prices in total
  - Quote date 19th December 2012
  - Sourced from Bloomberg
Storage Contract and Data

- Data used to estimate historical covariance matrix
  - 3 years ending 19th December 2012
  - Contains relative maturity returns with a day-ahead quote and month-ahead quotes spanning 11-months

- First two models calibrated using FFT-based swaption pricing method ... Kiely et al (2015a)

- Third model calibrated using moment matching method with FFT-based option pricing ... Kiely et al (2015b)
Mark-Based Calibration Risk

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\sigma$</th>
<th>$\nu$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRVG</td>
<td>0.2162</td>
<td>0.201</td>
<td>0.2560</td>
<td>1.07%</td>
</tr>
</tbody>
</table>

RMSE refers to the relative root mean square error.

Table 1: MRVG model parameters.

- Consider 2808 parameter combinations
- Retain 807 based on 3% limit around minimum RMSE
Table 2: MRVG model storage value push forward distribution.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modus</td>
<td>11.2087</td>
</tr>
<tr>
<td>Expected Value</td>
<td>11.2116</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.38%</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.034</td>
</tr>
</tbody>
</table>

Figure 2:
MRVG Storage Value Push Forward Density
The red line corresponds to the storage value with the smallest calibration error.
Model-Based Calibration Risk

Consider 2548 parameter combinations

Retain 795 based on 3% limit around minimum RMSE

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\sigma$</th>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRVG</td>
<td>0.2099</td>
<td>0.0334</td>
<td>8.7966</td>
<td>0.047</td>
<td>1.10%</td>
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</table>

RMSE refers to the relative root mean square error.

Table 3: MRVG model parameters.
Table 4: MRJD model storage value push forward distribution.

<table>
<thead>
<tr>
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<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Modus</td>
<td>11.2127</td>
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<tr>
<td>Expected Value</td>
<td>11.2039</td>
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<td>Coefficient of Variation</td>
<td>0.36%</td>
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<tr>
<td>Skewness</td>
<td>0.021</td>
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</table>

Figure 3:
MRJD Storage Value Push Forward Density
The red line corresponds to the storage value with the smallest calibration error.
Model-Based Calibration Risk

Conclusion

- MRVG and MRJD models carry comparable levels of calibration risk
- MRVG returns higher expected value than MRJD
- But MRVG also has higher variability than MRJD
Joint Calibration and Parameter Estimation Risk

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\sigma$</th>
<th>$\nu$</th>
<th>$b$</th>
<th>$c_1$</th>
<th>$\epsilon$</th>
<th>RMSE</th>
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<tbody>
<tr>
<td>MRVG-3x</td>
<td>0.1148</td>
<td>0.2518</td>
<td>0.1675</td>
<td>0.7511</td>
<td>0.6254</td>
<td>12.734</td>
<td>1.86%</td>
</tr>
</tbody>
</table>

Table 5: MRVG-3x model parameters.

- Consider 2548 parameter combinations
- Retain 838 based on 3% limit around minimum RMSE
Figure 4:
MRVG-3x Calibration Error Storage Value Density
This density is conditional upon the historically estimated parameters.
The red line corresponds to the storage value with the smallest calibration error.

Table 6: MRVG-3x model conditional storage value push forward distribution.
Joint Calibration and Parameter Estimation Risk

- Comparing to MRVG and MRJD
  - Value is much higher
  - Coefficient of variation is almost double
  - Confidence in calibrated value is lower

- Clustering of storage value at distinct levels
  - Relate to different combinations of $\alpha$ and $\sigma$
  - These combinations appear to fully determine storage value
  - The impact of $\nu$ is minimal
Joint Calibration and Parameter Estimation Risk

- We now proceed to deriving the parameter risk density associated with the historically estimated parameters.
- We can derive a sampling error covariance matrix for each of the volatility parameters.
- Which is turn can be used to derive a parameter risk covariance matrix.
- We must first estimate the Jacobian of our storage value with respect to our model parameters.

The $c_1$ parameter which controls the percentage of spot variance accounted for by the second factor has the greatest impact on the storage value, followed by the $\epsilon$ parameter which is ultimately responsible for the extrinsic value attributable to the second factor. Intuitively, this can be understood as the $\epsilon$ parameter reducing the auto-correlation of the second factor and $c_1$ being responsible for passing this decorrelation through into the forward curve returns. The $b$ parameter has little impact on the storage value which is due to the low levels of extrinsic value attributable to the first factor. The resulting parameter risk induced storage value density is shown in Figure 5.
Figure 5:
MRVG-3x Parameter Risk Storage Value Density
Joint Calibration and Parameter Estimation Risk

\[ h(\varepsilon(\theta)) = h_m(\varepsilon(\theta)) p(\theta_h) \]

<table>
<thead>
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</tr>
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<td>Expected Value</td>
<td>16.8249</td>
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<td>Coefficient of Variation</td>
<td>3.01%</td>
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<tr>
<td>Skewness</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 8: MRVG-3x combined calibration and parameter risk push forward distribution.

- Variability has increased dramatically from inclusion of risk associated with the historically estimated parameters
Deriving Risk-Adjusted Price Levels
Deriving Risk-Adjusted Price Levels

- **MRVG**
  - Bid-offer price levels ... 11.1507-11.2697
  - Spread of 1.06% of mean value

- **MRJD**
  - Bid-offer prices levels ... 11.1473-11.2611
  - Spread of 1.02% of mean value

- **MRVG-3x**
  - Bid-offer price levels ... 16.1866-17.4748
  - Spread of 7.66% of mean value