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A structural model for electricity forward prices

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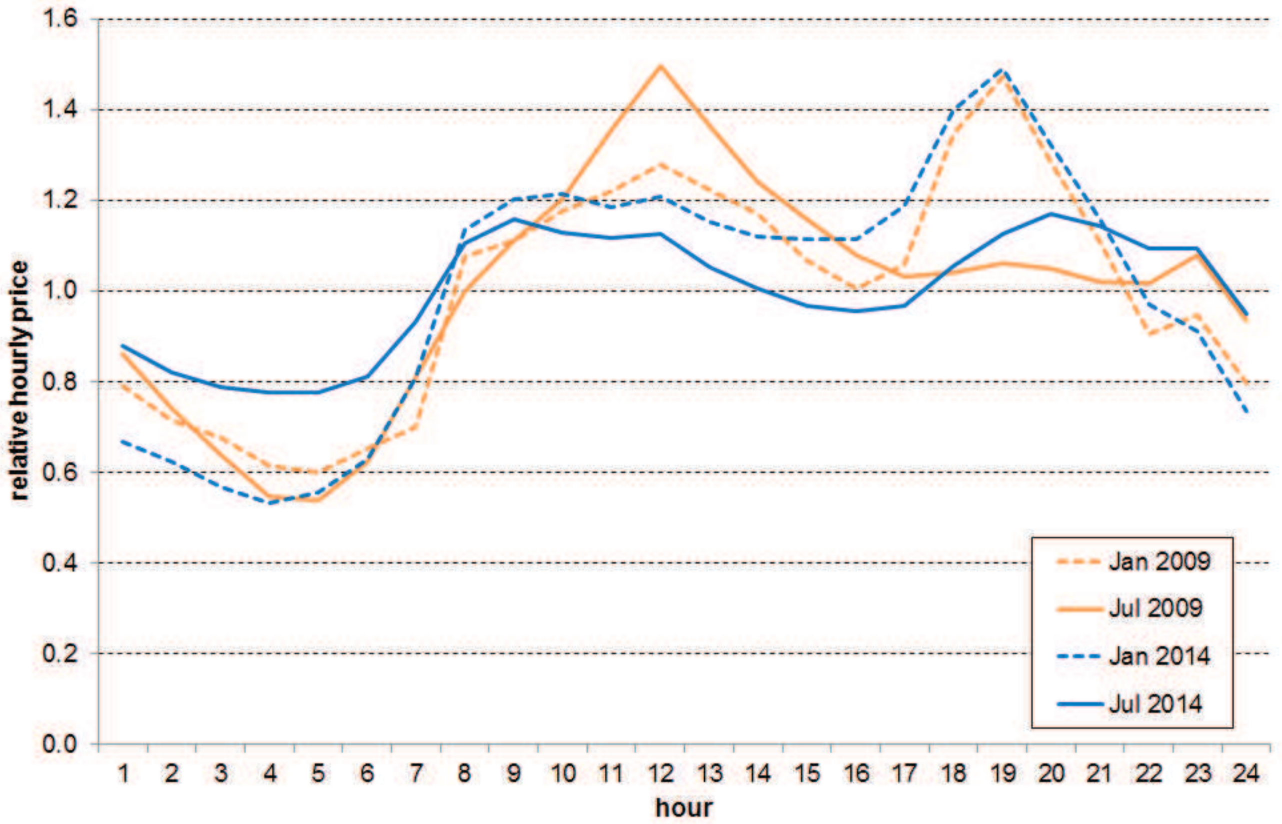
Outlook

- Structural models for forward electricity prices are highly relevant: major structural changes in the market due to the infeed from renewable energy
- Renewable energies infeed reflected in the market expectation – **impact on futures (forward) prices?**



Literature review

- Models for forward prices in commodity/energy:
  - Specify one model for the spot price and from this derive for forwards: *Lucia and Schwartz (2002)*; *Cartea and Figueroa (2005)*; *Benth, Kallsen, and Mayer-Brandis (2007)*;
  - Heath-Jarrow-Morton approach – price forward prices directly, by multifactor models: *Roncoroni, Guiotto (2000)*; *Benth and Koekebakker (2008)*; *Kiesel, Schindlmayr, and Boerger (2009)*;
- Critical view of *Koekebakker and Ollmar (2005)*, *Frestad (2008)*
  - Few common factors cannot explain the substantial amount of variation in forward prices
  - Non-Gaussian noise
- Random-field models for forward prices:
  - *Roncoroni, Guiotto (2000)*;
  - *Andresen, Koekebakker, and Westgaard (2010)*;
- Derivation of seasonality shapes and price forward curves for electricity:
  - *Fleten and Lemming (2003)*;
  - *Bloechlinger (2008).*

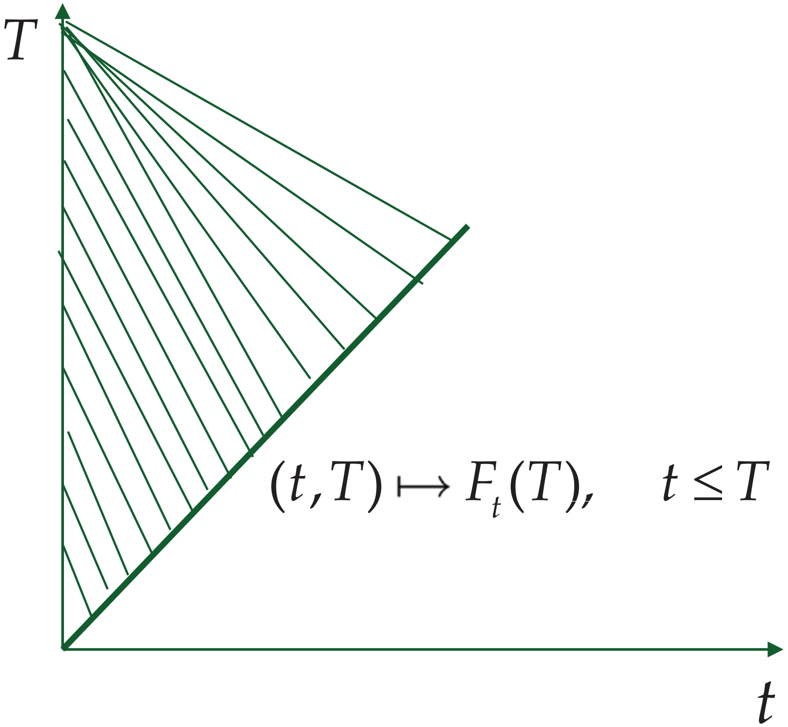
Questions to be answered

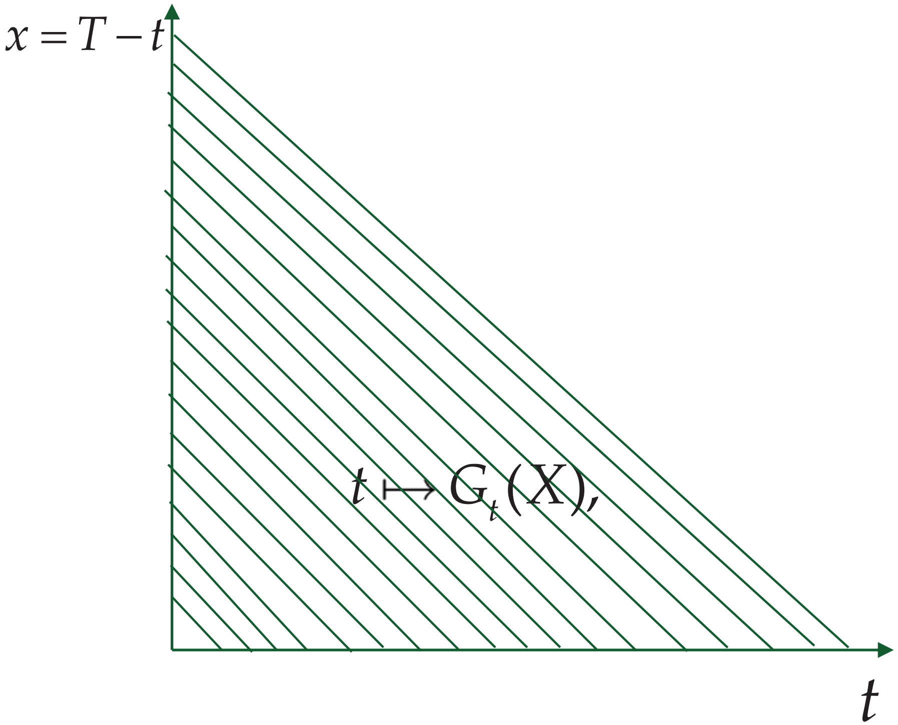
- We will refer to a panel of daily price forward curves derived over time
- We deseasonalize and aim at a structural model for the stochastic component of PFCs
  - Examine and model the dynamics of risk premia, the distribution of noise (non-Gaussian, stochastic volatility), spatial correlations
  - The analysis is spatio-temporal: cross-section analysis with respect to the time dimention and the maturity “space”

Problem statement

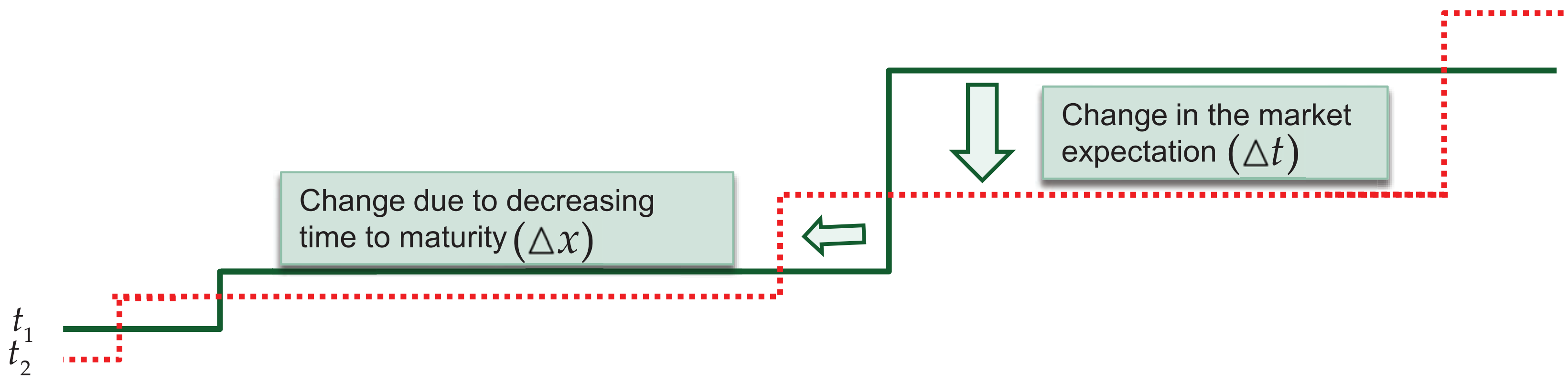
- Previous models model forward prices evolving over time (time-series) along the **time at maturity T**: *Andresen, Koekebakker, and Westgaard (2010)*
- Let $F_t(T)$ denote the forward price at time $t \geq 0$ for delivery of a commodity at time $T \geq t$
- Random field in t:
$$t \mapsto F_t(T), \qquad t \geq 0 \tag{1}$$
- Random field in both t and T:
$$(t, T) \mapsto F_t(T), \qquad t \geq 0, \qquad t \leq T \tag{2}$$
- Get rid of the second condition: **Musiela parametrization** $x = T - t$, $x \geq 0$.
$$F_t(t + x) = F_t(T), \qquad t \geq 0 \tag{3}$$
- Let $G_t(x)$ be the forward price for a contract with time to maturity $x \geq 0$. Note that:
$$G_t(x) = F_t(t + x) \tag{4}$$

Graphical interpretation





Influence of the “time to maturity”



Model formulation: Heath-Jarrow-Morton (HJM)

- The stochastic process $t \mapsto G_t(x), \quad t \geq 0$ is the solution to:

$$dG_t(x) = (\partial_x G_t(x) + \beta(t,x))\ dt + dW_t(x) \tag{5}$$

  - Space of curves are endowed with a Hilbert space structure $\mathcal{H}$
  - ∂_x differential operator with respect to time to maturity
  - β spatio-temporal random field describing the market price of risk
  - W Spatio-temporal random field describing the randomly evolving residuals

- Discrete structure:

$$G_t(x) = f_t(x) + s_t(x)\,, \tag{6}$$

  - $s_t(x)$ deterministic seasonality function $\mathbb{R}_+^2 \ni (t,x) \mapsto s_t(x) \in \mathbb{R}$

Model formulation (cont)

We furthermore assume that the deasonalized forward price curve, denoted by $f_t(x)$, has the dynamics:

$$df_t(x) = (\partial_x f_t(x) + \theta(x) f_t(x))\, dt + dW_t(x)\,, \tag{7}$$

with $\theta \in \mathbb{R}$ being a constant. With this definition, we note that

$$\begin{aligned} dF_t(x) &= df_t(x) + ds_t(x) \\ &= (\partial_x f_t(x) + \theta(x) f_t(x))\, dt + \partial_t s_t(x)\, dt + dW_t(x) \\ &= (\partial_x F_t(x) + (\partial_t s_t(x) - \partial_x s_t(x)) + \theta(x)(F_t(x) - s_t(x)))\, dt + dW_t(x)\,. \end{aligned}$$

In the natural case, $\partial_t s_t(x) = \partial_x s_t(x)$, and therefore we see that $F_t(x)$ satisfy (5) with $\beta(t,x) := \theta(x) f_t(x)$.

The market price of risk is proportional to the deseasonalized forward prices.

Model formulation (cont)

We discretize the dynamics in Eq. (7) by an Euler discretization

$$df_t(x) = (\partial_x f_t(x) + \theta(x) f_t(x))\ dt + dW_t(x)$$

$$\partial_x f_t(x) \approx \frac{f_t(x + \Delta x) - f_t(x)}{\Delta x}$$

$$f_{t+\Delta t}(x) = (f_t(x) + \frac{\Delta t}{\Delta x}(f_t(x + \Delta x) - f_t(x)) + \theta(x) f_t(x) \Delta t + \epsilon_t(x) \tag{8}$$

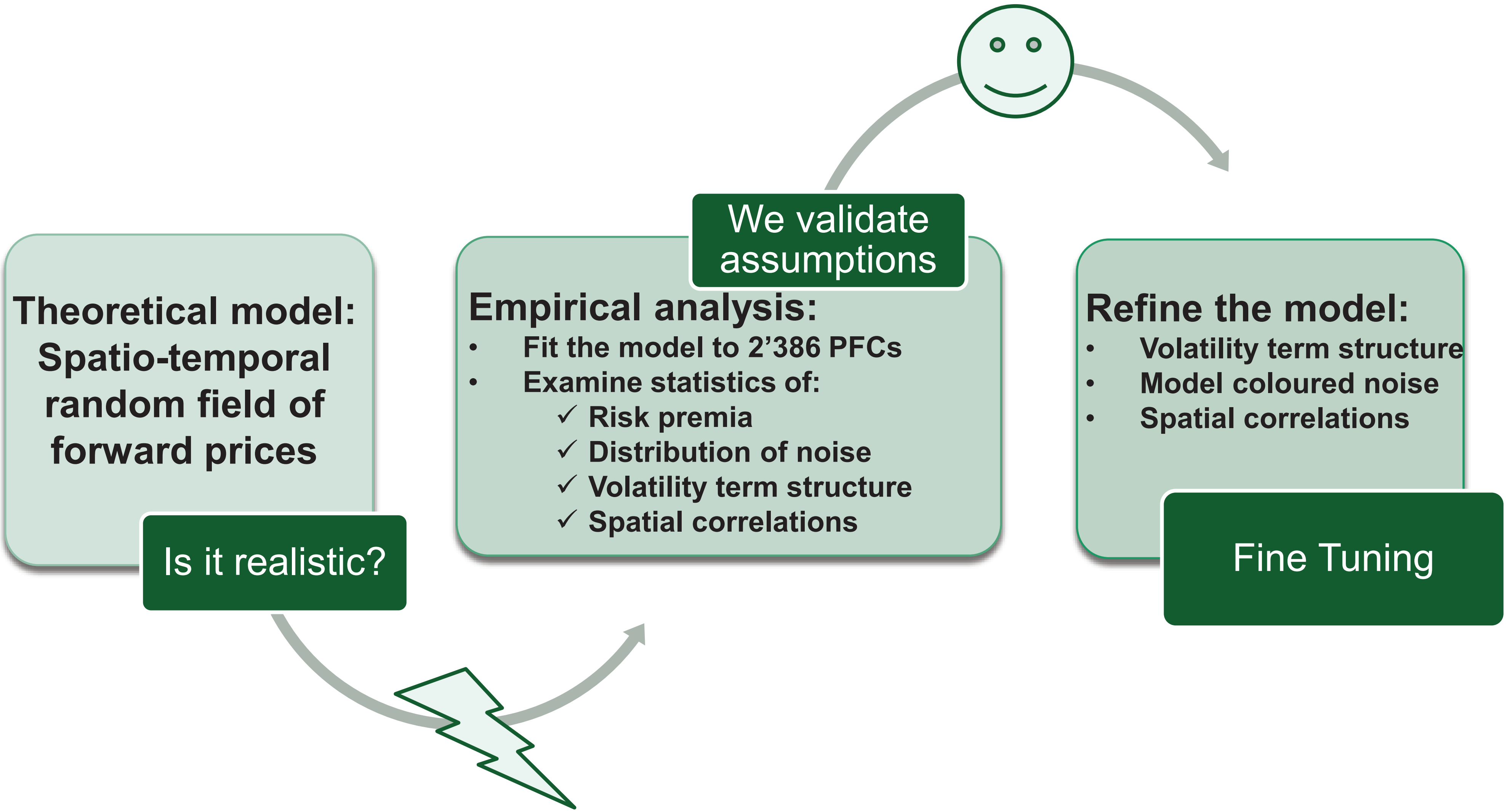
with $x \in \{x_1, \ldots, x_N\}$ and $t = \Delta t, \ldots, (M-1)\Delta t$, where $\epsilon_t(x) := W_{t+\Delta t}(x) - W_t(x)$.

$$Z_t(x) := f_{t+\Delta t}(x) - f_t(x) - \frac{\Delta t}{\Delta x}(f_t(x + \Delta x) - f_t(x)) \tag{9}$$

which implies

$$Z_t(x) = \theta(x) f_t(x) \Delta t + \epsilon_t(x)\,, \tag{10}$$

$$\epsilon_t(x) = \sigma(x) \tilde{\epsilon}_t(x) \tag{11}$$



Theoretical model:
Spatio-temporal random field of forward prices
Is it realistic?
We validate assumptions
Empirical analysis:
Fit the model to 2’386 PFCs
Examine statistics of:
Risk premia
Distribution of noise
Volatility term structure
Spatial correlations
Refine the model:
Volatility term structure
Model coloured noise
Spatial correlations
Fine Tuning

Derivation of price forward curves: seasonality curves

- We firstly **remove the long-term trend** from the hourly electricity prices
- Follow *Blöchlinger (2008)* for the derivation of the seasonality shape for EPEX power prices: very „data specific"; **removes seasonal effects and autocorrelation!**
- In a first step, we identify **the seasonal structure during a year with daily prices**: factor-to-year ($f2y$)
- In the second step, **the patterns during a day are analyzed using hourly prices**: factor-to-day ($f2d$)
- Forecasting models for the factors are derived, such that the resulting shape can be predicted
- The shape is aligned to the level of futures prices

Factor to year

$$f2y_d = \frac{S^{day}(d)}{\sum_{k \epsilon \text{year}(d)} S^{day}(k)\frac{1}{K(d)}} \tag{12}$$

To explain the $f2y$, we use a multiple regression model:

$$f2y_d = \alpha_0 + \sum_{i=1}^{6} b_i D_{di} + \sum_{i=1}^{12} c_i M_{di} + \sum_{i=1}^{3} d_i CDD_{di} + \sum_{i=1}^{3} e_i HDD_{di} + \varepsilon \tag{13}$$

- $f2y_d$: Factor to year, daily-base-price/yearly-base-price
- D_{di}: 6 daily dummy variables (for Mo-Sat)
- M_{di}: 12 monthly dummy variables (for Feb-Dec); August will be subdivided in two parts, due to summer vacation
- CDD_{di}: Cooling degree days for 3 different German cities – $\mathbf{max(}T - 18.3°C, 0\mathbf{)}$
- HDD_{di}: Heating degree days for 3 different German cities – $\mathbf{max(}18.3°C - T, 0\mathbf{)}$

where CDD_i/HDD_i are estimated based on the temperature in Berlin, Hannover and Munich.

Regression model for the temperature

- For temperature, we propose a forecasting model based on fourier series:

$$T_t = a_0 + \sum_{i=1}^{3} b_{1,i} \cos(i \frac{2\pi}{365} YT_t) + \sum_{i=1}^{3} b_{2,i} \sin(i \frac{2\pi}{365} YT_t) + \varepsilon_t \tag{14}$$

  where T_p is the average daily temperature and YT the observation time within one year

- Once the coefficients in the above model are estimated, the temperature can be easily predicted since the only exogenous factor YT is deterministic!

- Forecasts for CDD and HDD are also straightforward

Factor to day

- The $f2d$, in contrast, indicates the weight of the price of a particular hour compared to the daily base price.

$$f2d_t = \frac{S^{hour}(t)}{\sum_{k \epsilon \text{day}(t)} S^{hour}(k) \frac{1}{24}} \tag{15}$$

- with $S^{hour}(t)$ being the hourly spot price at the hour t.
- We know that there are considerable differences both in the daily profiles of workdays, Saturdays and Sundays, but also between daily profiles during winter and summer season
- We classify the days by weekdays and seasons and choose the classification scheme presented in Table 1

Profile classes for each day

Table 1: *The table indicates the assignment of each day to one out of the twenty profile classes. The daily pattern is held constant for the workdays Monday to Friday within a month, and for Saturday and Sunday, respectively, within three months.*

	J	F	M	A	M	J	J	A	S	O	N	D
Week day	1	2	3	4	5	6	7	8	9	10	11	12
Sat	13	13	14	14	14	15	15	15	16	16	16	13
Sun	17	17	18	18	18	19	19	19	20	20	20	17

Profile classes for each day

- The regression model for each class is built quite similarly to the one for the yearly seasonality. For each profile class $c = \{1, \ldots, 20\}$ defined in table 1, a model of the following type is formulated:

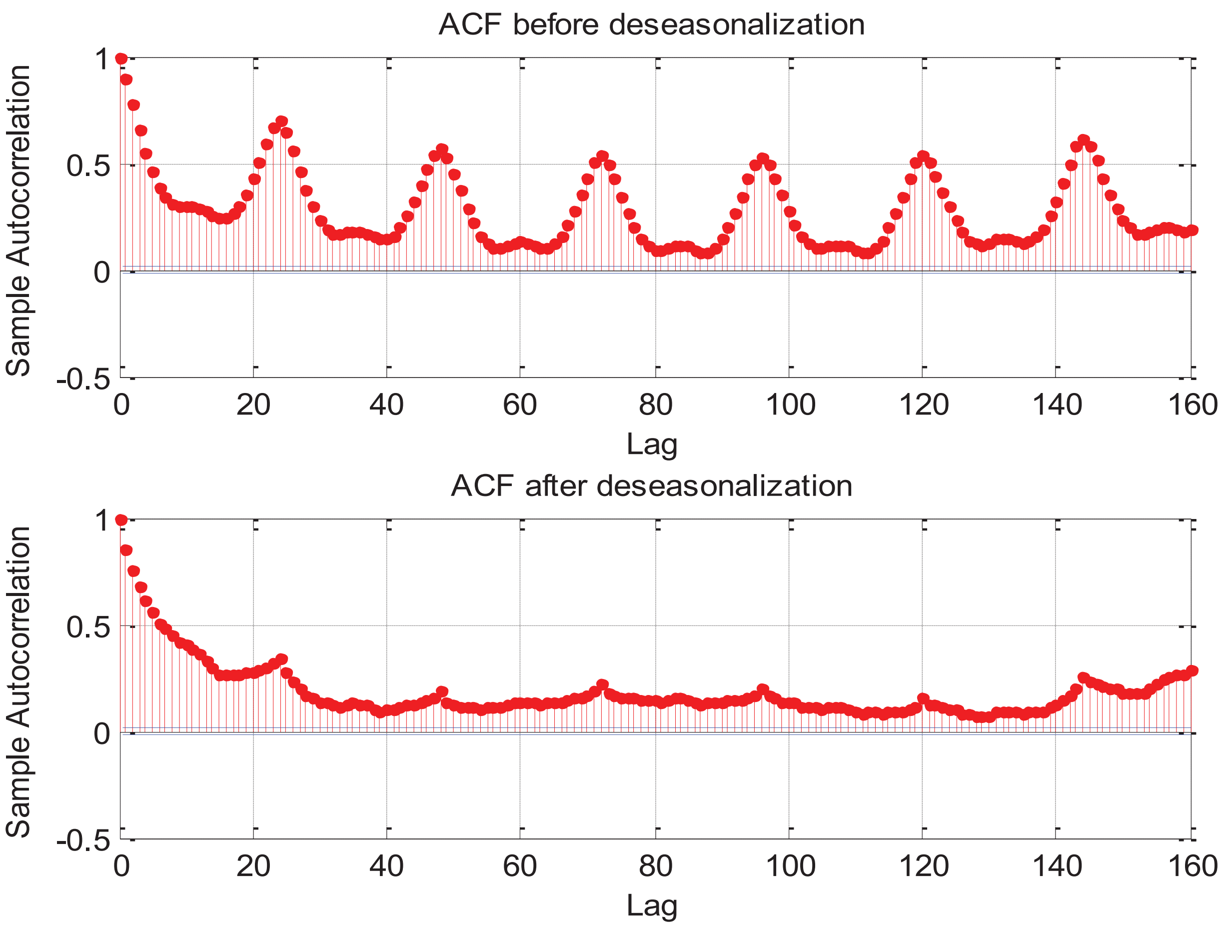
$$f2d_t = a_o^c + \sum_{i=1}^{23} b_i^c H_{t,i} + \varepsilon_t \quad \text{for all } t \epsilon c. \tag{16}$$

  where $H_i = \{0, \ldots, 23\}$ represents dummy variables for the hours of one day

- **The seasonality shape s_t can be calculated by $s_t = f2y_t \cdot f2d_t$.**
- **s_t is the forecast of the relative hourly weights and it is additionally multiplied by the yearly average prices, in order to align the shape at the prices level**
- **This yields the seasonality shape s_t which is finally used to deseasonalize the electricity prices**

Deseasonalization result

The deseasonalized series is assumed to contain only the stochastic component of electricity prices, such as the volatility and randomly occurring jumps and peaks



Derivation of HPFC *Fleten & Lemming (2003)*

- Recall that $F_t(x)$ is the price of the forward contract with maturity x, where time is measured in hours, and let $F_t(T_1, T_2)$ be the settlement price at time t of a forward contract with delivery in the interval $[T_1, T_2]$.
- The forward prices of the derived curve should match the observed settlement price of the traded future product for the corresponding delivery period, that is:

$$\frac{1}{\sum_{\tau=T_1}^{T_2} \exp(-r\tau/a)} \sum_{\tau=T_1}^{T_2} \exp(-r\tau/a) F_t(\tau - t) = F_t(T_1, T_2) \tag{17}$$

  where r is the continuously compounded rate for discounting per annum and a is the number of hours per year.
- A realistic price forward curve should capture information about the hourly seasonality pattern of electricity prices

$$\min \left[\sum_{x=1}^{N} (F_t(x) - s_t(x))^2 \right] \tag{18}$$

Input mix for electricity production Germany

	2009	**2010**	**2011**	**2012**	**2013**	**2014**
Coal	42.6	41.5	42.8	44	45.2	43.2
Nuclear	22.6	22.2	17.6	15.8	15.4	15.8
Natural Gas	13.6	14.1	14	12.1	10.5	9.5
Oil	1.7	1.4	1.2	1.2	1	1
Renewable energies from which	**15.9**	**16.6**	**20.2**	**22.8**	**23.9**	**25.9**
Wind	6.5	6	8	8.1	8.4	8.9
Hydro power	3.2	3.3	2.9	3.5	3.2	3.3
Biomass	4.4	4.7	5.3	6.3	6.7	7.0
Photovoltaic	1.1	1.8	3.2	4.2	4.7	5.7
Waste-to-energy	0.7	0.7	0.8	0.8	0.8	1
Other	3.6	4.2	4.2	4.1	4	4.3

Table 2: Electricity production in Germany by source (%), as shown in *Paraschiv, Bunn and Westgaard (2016).*

Data

- We employed a unique data set of 2'386 daily price forward curves $F_t(x_1), \ldots, F_t(x_N)$ generated each day between 01/01/2009 and 15/07/2015 based on the latest information from the observed futures prices for the German electricity Phelix price index.
- We firstly deaseasonalize the prices:

$$F_t(x) = f_t(x) + s_t(x)\,, \tag{19}$$

- Estimate the parameter of the market price of risk (θ), the volatility term structure ($\sigma(x)$) and analyse the noise $\tilde{\epsilon}_t(x)$

$$Z_t(x) := f_{t+\Delta t}(x) - f_t(x) - \frac{\Delta t}{\Delta x}(f_t(x+\Delta x) - f_t(x)) \tag{20}$$

which implies

$$Z_t(x) = \theta(x) f_t(x) \Delta t + \epsilon_t(x)\,, \tag{21}$$

$$\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x) \tag{22}$$

Increase in renewables: increase in the PFC's volatility

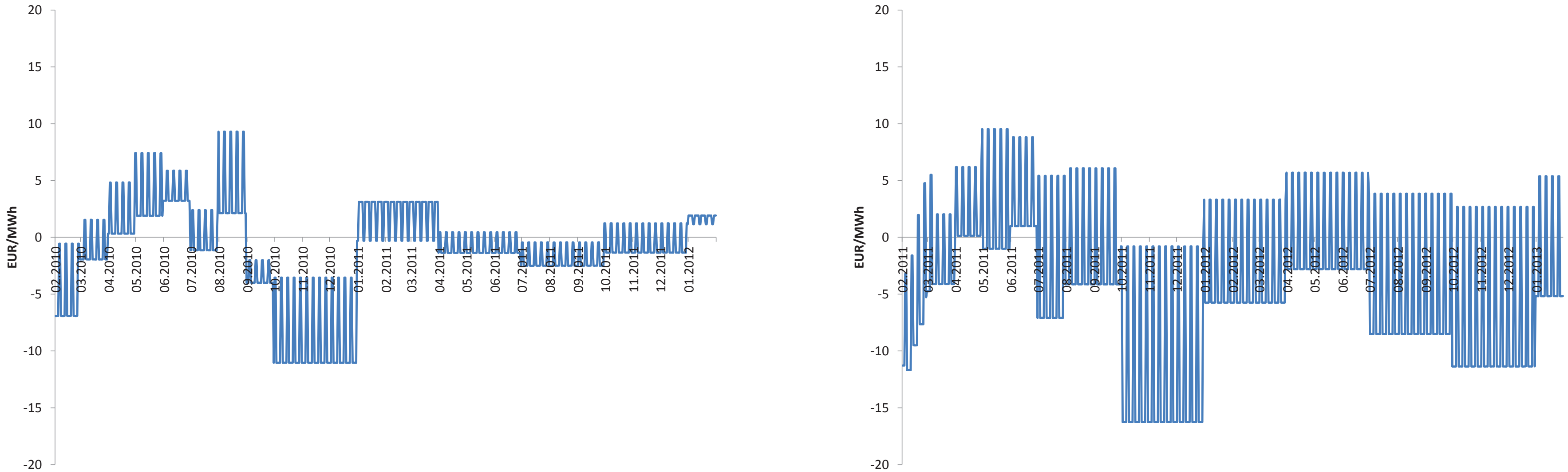


Figure 2: *Stochastic component of PFCs generated at 01/17/2010 (left graph) and 01/17/2011, right graph.*

Increase in renewables: increase in the PFC's volatility

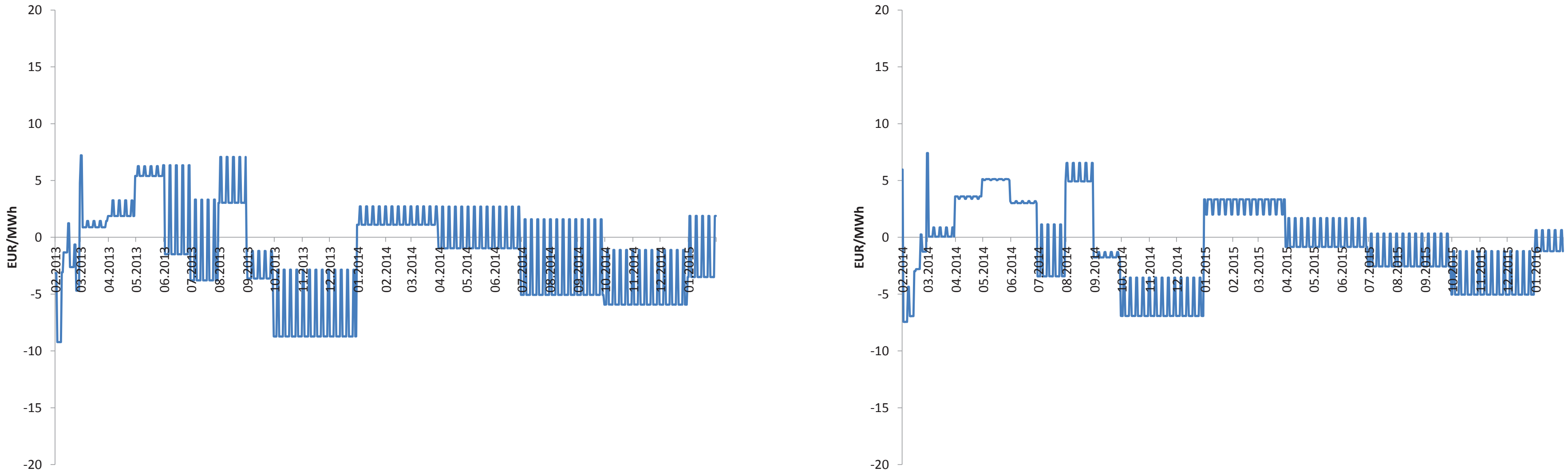
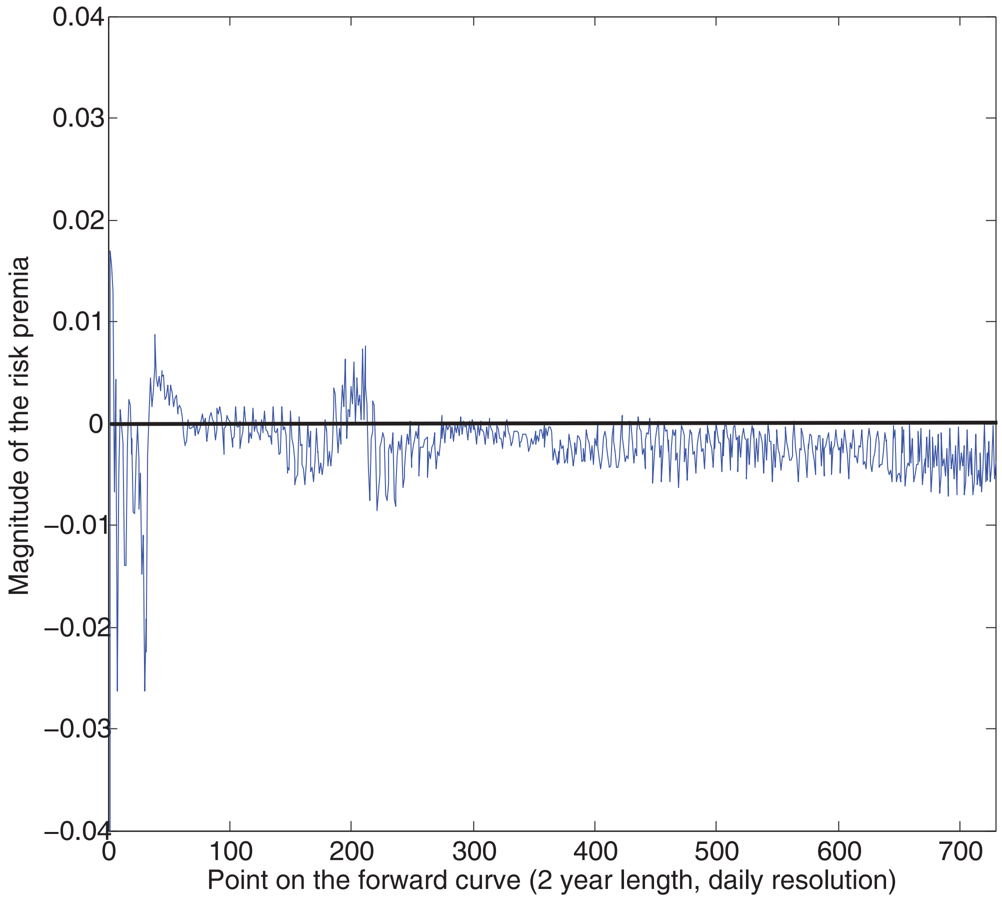


Figure 3: *Stochastic component of PFCs generated at 01/17/2013 (left graph) and 01/17/2014, right graph.*

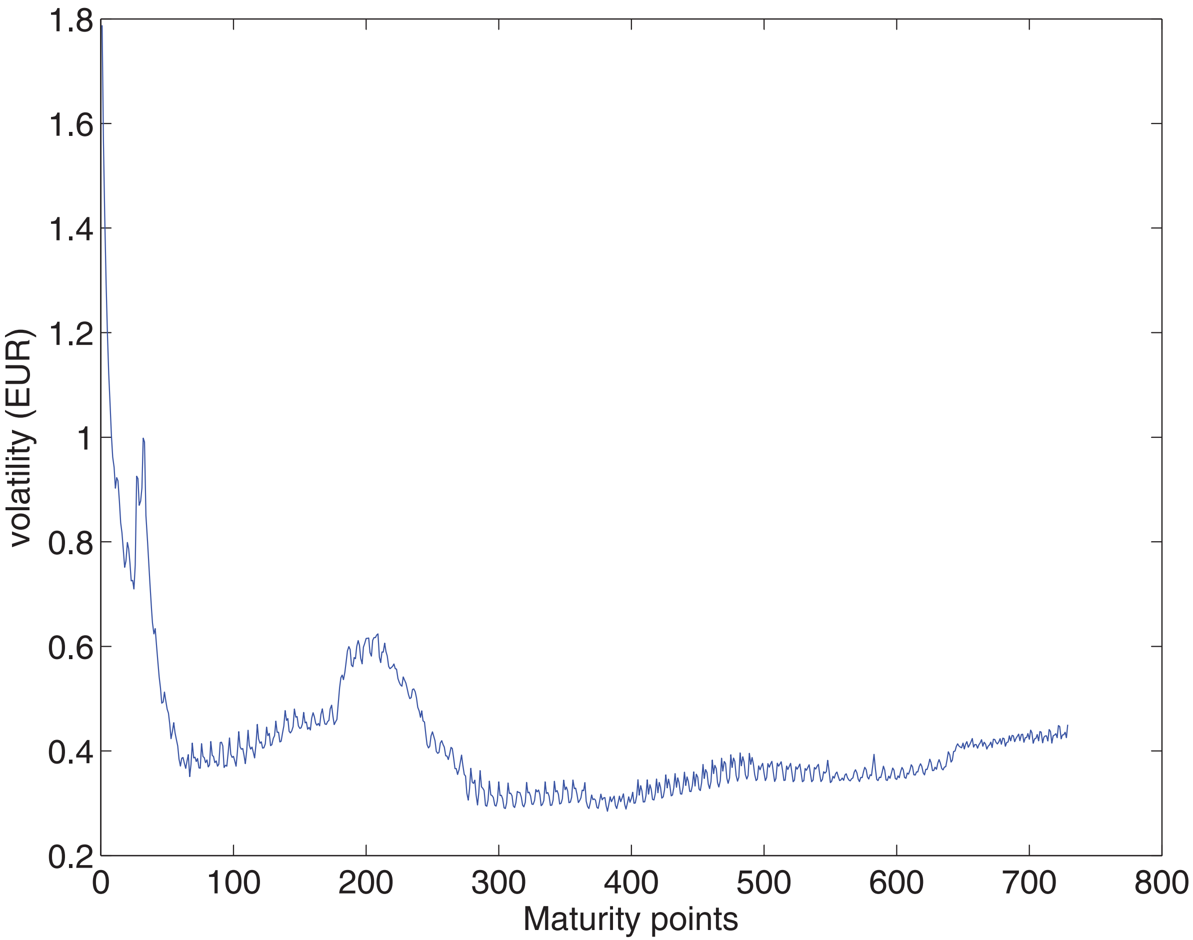
Risk premia

- Short-term: it oscillates around zero and has higher volatility (similar in *Pietz (2009)*, *Paraschiv et al. (2015)*)
- Long-term: is becomes negative and has more constant volatility (*Burger et al. (2007)*): In the long-run power generators accept lower futures prices, as they need to make sure that their investment costs are covered.



Term structure volatility

- We observe Samuelson effect: overall higher volatility for shorter time to maturity
- Volatility bumps (front month; second/third quarters) explained by increased volume of trades
- Jigsaw pattern: weekend effect; volatility smaller in the weekend versus working days



Explaining volatility bumps

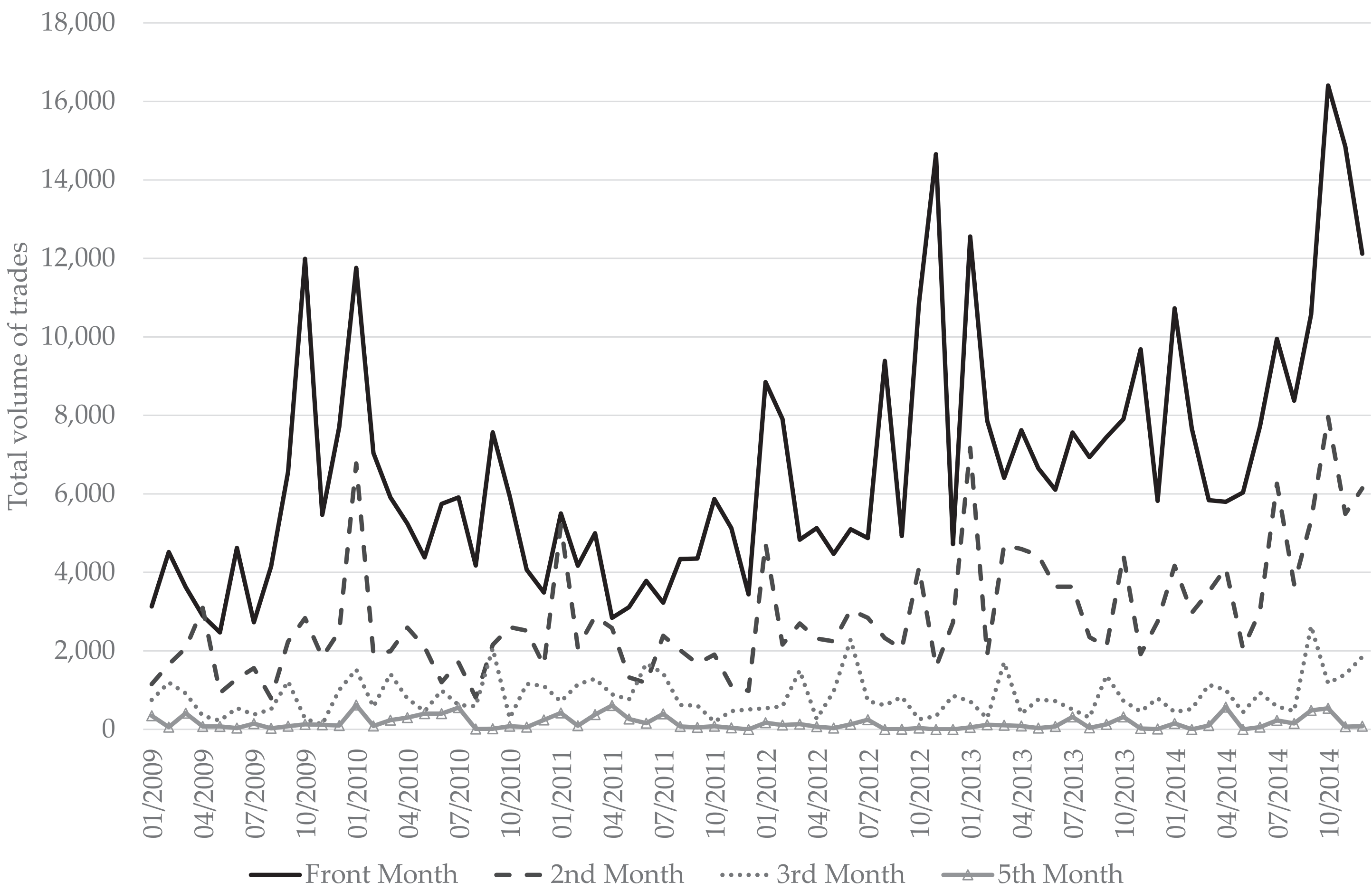


Figure 5: The sum of traded contracts for the monthly futures, evidence from EPEX, own calculations (source of data ems.eex.com).

Explaining volatility bumps

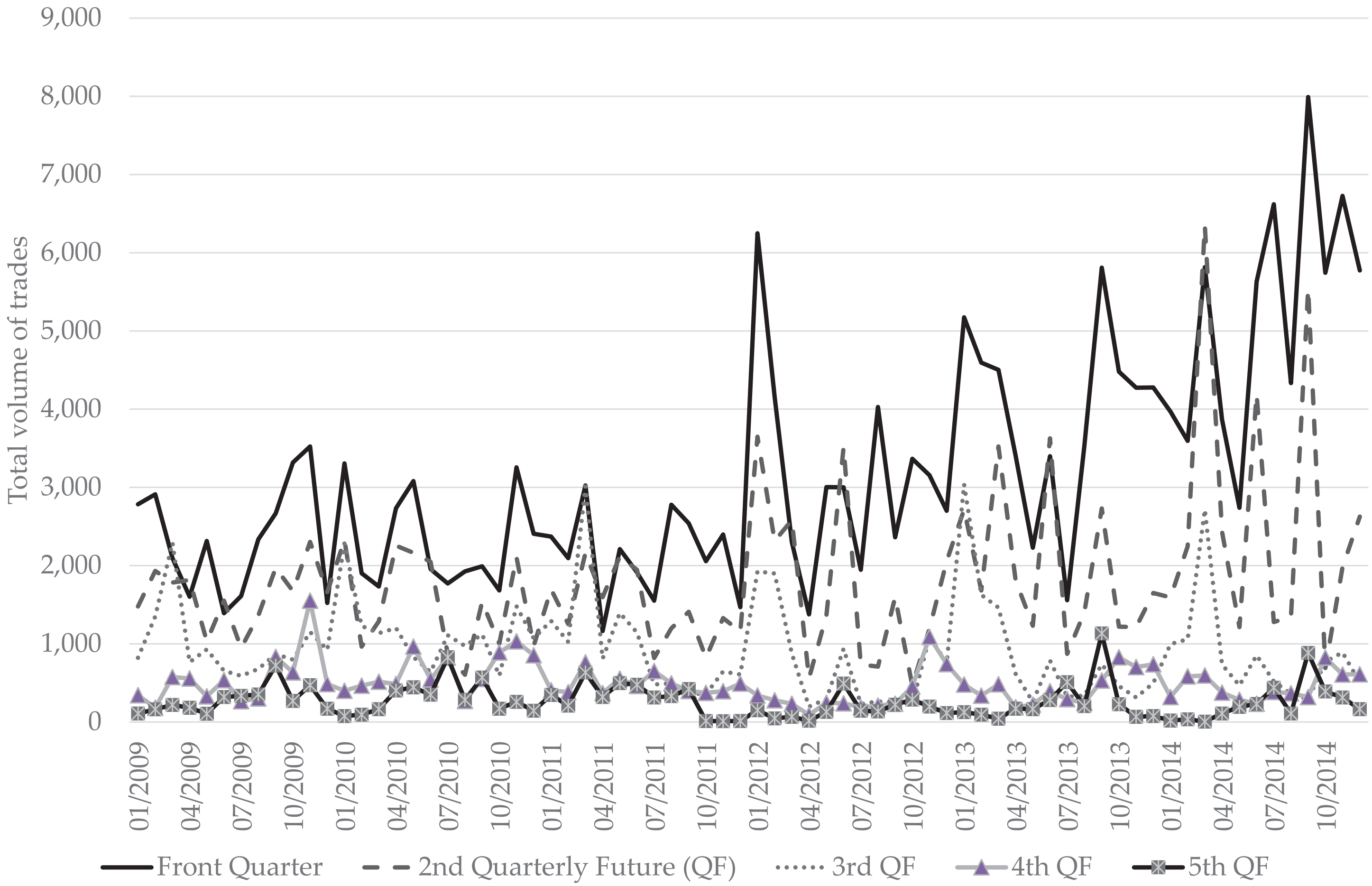


Figure 6: The sum of traded contracts for the quarterly futures, evidence from EPEX, own calculations (source of data ems.eex.com).

Statistical properties of the noise

- We examined the statistical properties of the noise time-series $\tilde{\epsilon}_t(x)$

$$\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x) \tag{23}$$

- We found: Overall we conclude that the model residuals are **coloured noise**, with **heavy tails** (leptokurtic distribution) and with a tendency for **conditional volatility**.

$\tilde{\epsilon}_t(x_k)$	Stationarity	Autocorrelation $\tilde{\epsilon}_t(x_k)$	Autocorrelation $\tilde{\epsilon}_t(x_k)^2$	ARCH/GARCH
	h	h1	h1	h2
Q0	0	1	1	1
Q1	0	0	0	0
Q2	0	1	1	1
Q3	0	0	1	1
Q4	0	1	0	0
Q5	0	1	1	1
Q6	0	1	1	1
Q7	0	1	1	1

Table 3: The time series are selected by quarterly increments (90 days) along the maturity points on one noise curve. Hypotheses tests results, case study 1: $\Delta x = 1day$. In column stationarity, if $h = 0$ we fail to reject the null that series are stationary. For autocorrelation $h1 = 0$ indicates that there is not enough evidence to suggest that noise time series are autocorrelated. In the last column $h2 = 1$ indicates that there are significant ARCH effects in the noise time-series.

Autocorrelation structure of noise time series

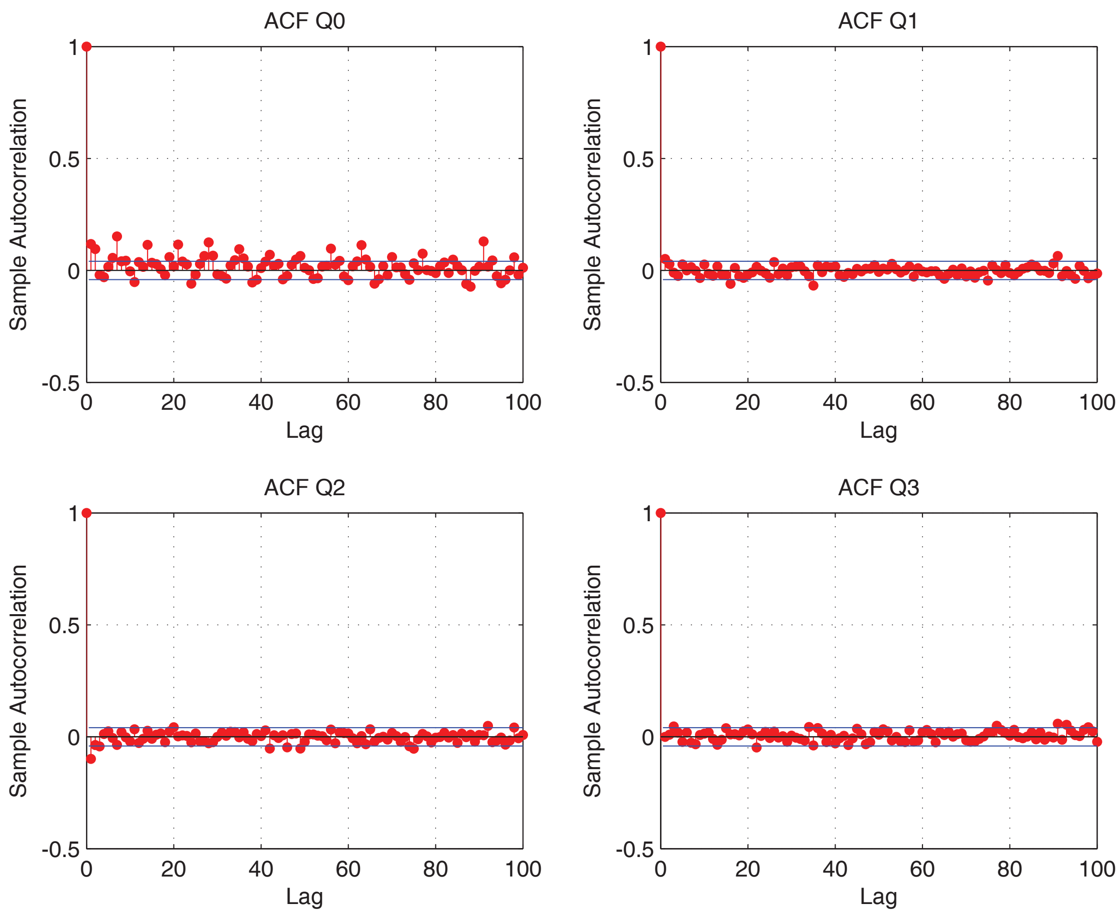


Figure 7: Autocorrelation function in the level of the noise time series $\tilde{\epsilon}_t(x_k)$, by taking $k \in \{1, 90, 180, 270\}$, case study 1: $\Delta x = 1\,day$.

Autocorrelation structure of noise time series (squared)

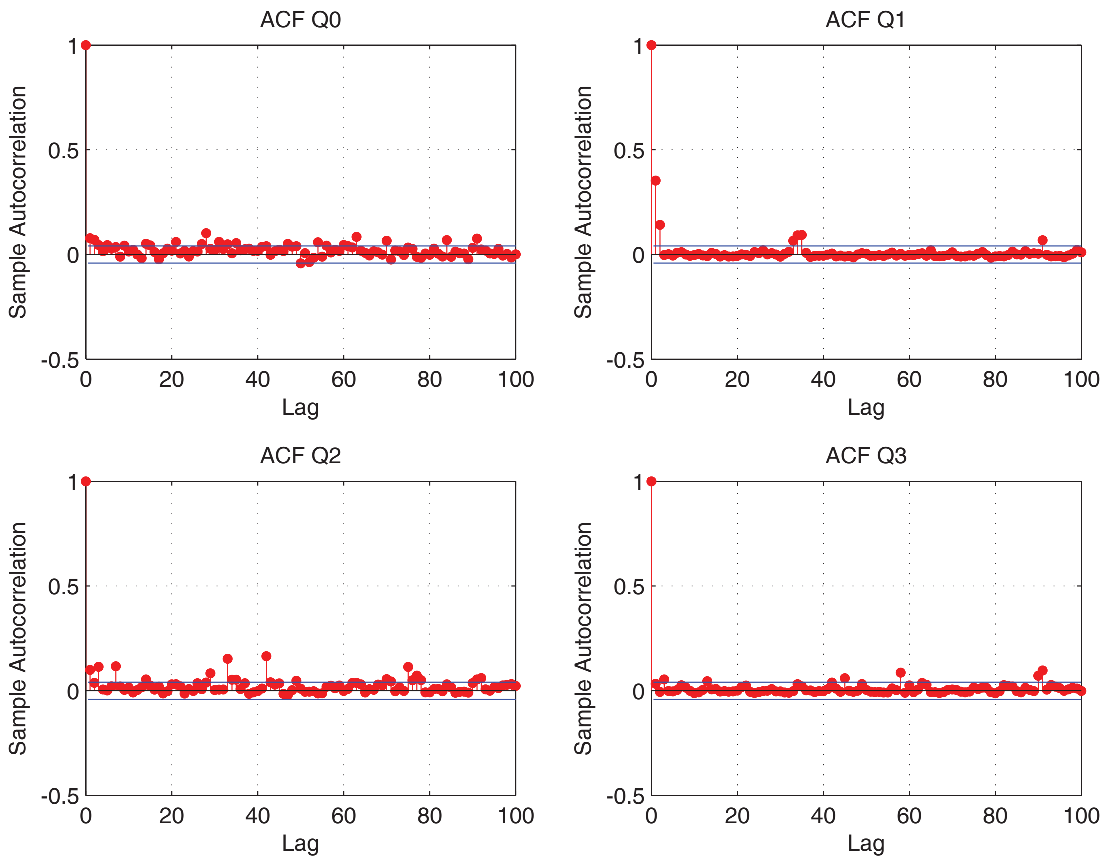
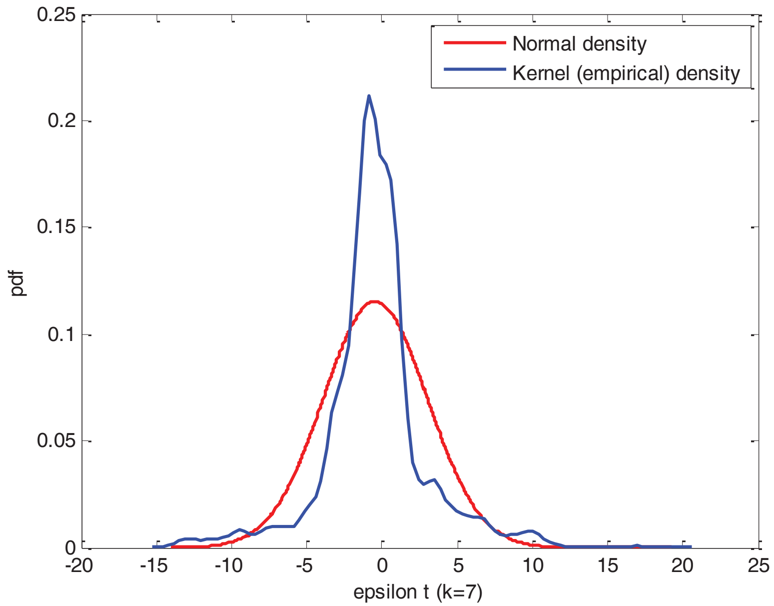
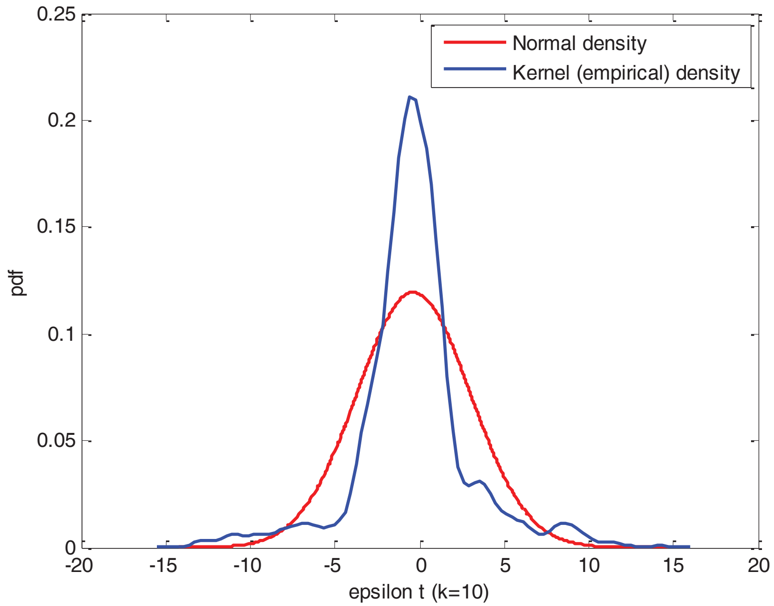


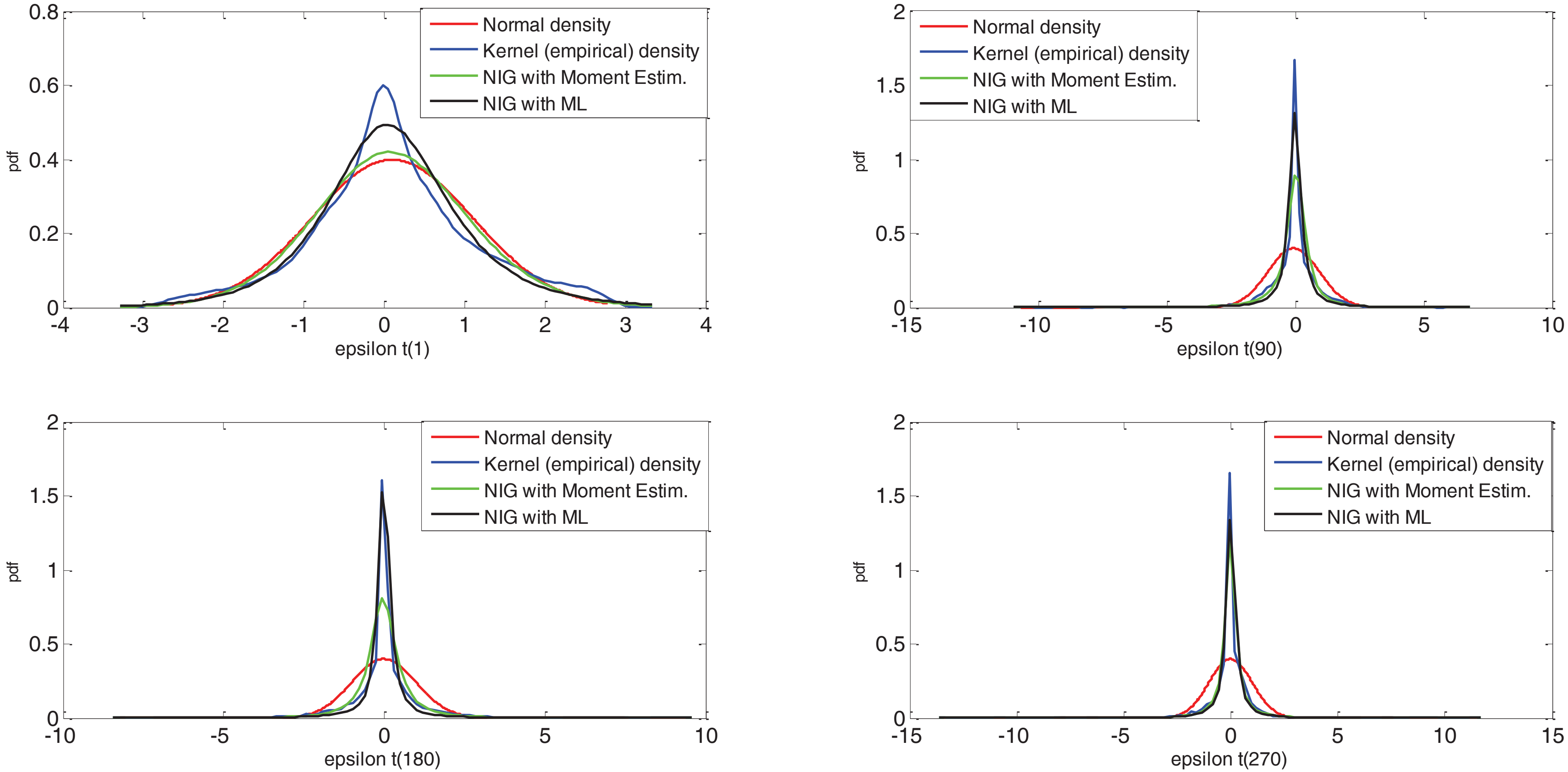
Figure 8: Autocorrelation function in the squared time series of the noise $\tilde{\epsilon}_t(x_k)^2$, by taking $k \in \{1, 90, 180, 270\}$, case study 1: $\Delta x = 1\,day$.

Leptokurtic distribution





Normal Inverse Gaussian (NIG) distribution for coloured noise



Spatial dependence structure

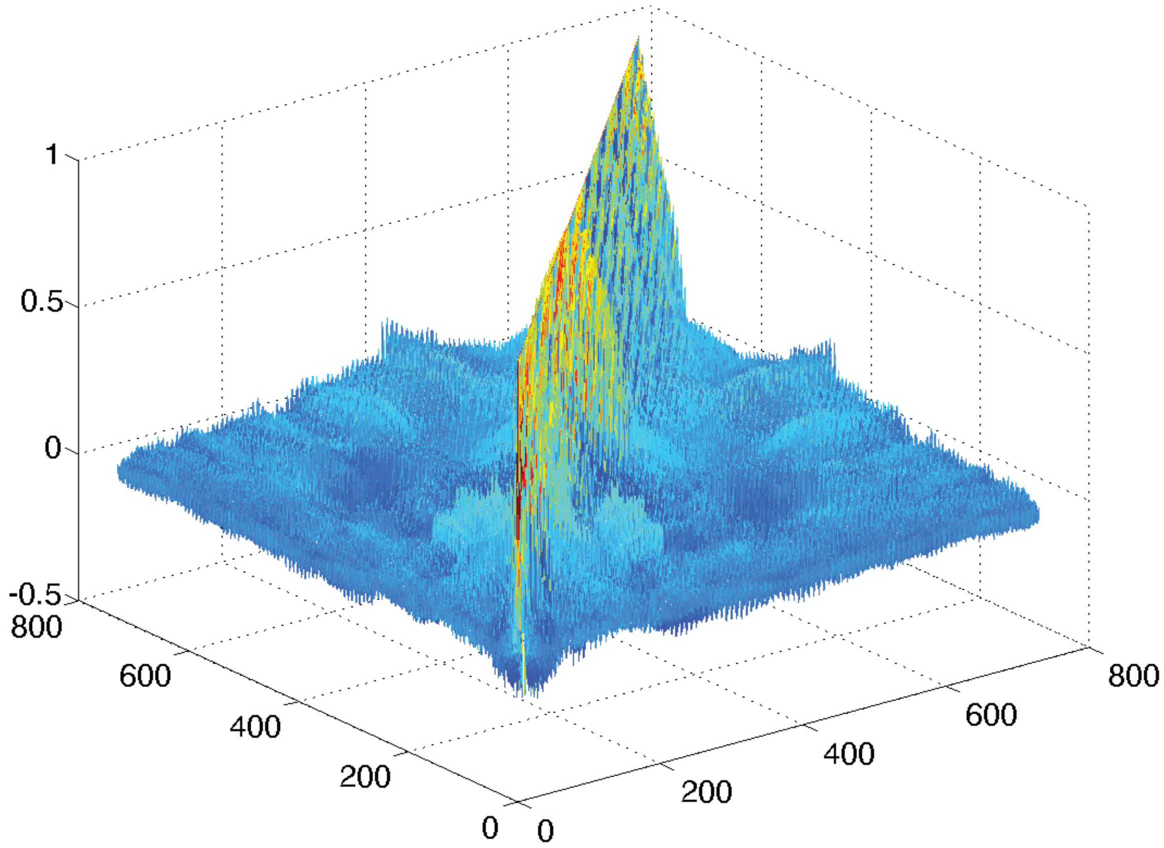


Figure 9: *Correlation matrix with respect to different maturity points along one curve.*

Revisiting the model

- We have analysed empirically the noise residual $dW_t(x)$ expressed as $\epsilon_t(x) = \sigma(x)\widetilde{\epsilon}_t(x)$ in a discrete form
- Recover an infinite dimensional model for $W_t(x)$ based on our findings

$$W_t = \int_0^t \Sigma_s \, dL_s \,, \tag{24}$$

where $s \mapsto \Sigma_s$ is an $L(\mathcal{U}, \mathcal{H})$-valued predictable process and L is a $\mathcal{U}$-valued Lévy process with zero mean and finite variance.

- As a first case, we can choose $\Sigma_s \equiv \Psi$ time-independent:

$$W_{t+\Delta t} - W_t \approx \Psi(L_{t+\Delta t} - L_t) \tag{25}$$

Choose now $\mathcal{U} = L^2(\mathbb{R})$, the space of square integrable functions on the real line equipped with the Lebesgue measure, and assume Ψ is an integral operator on $L^2(\mathbb{R})$

$$\mathbb{R}_+ \ni x \mapsto \Psi(g)(x) = \int_{\mathbb{R}} \widetilde{\sigma}(x,y) g(y) \, dy \tag{26}$$

- we can further make the approximation $\Psi(g)(x) \approx \widetilde{\sigma}(x,x) g(x)$, which gives

$$W_{t+\Delta t}(x) - W_t(x) \approx \widetilde{\sigma}(x,x)(L_{t+\Delta t}(x) - L_t(x)) \,. \tag{27}$$

Revisiting the model (cont)

- Recall the spatial correlation structure of $\widetilde{\epsilon}_t(x)$. This provides the empirical foundation for defining a covariance functional $\mathcal{Q}$ associated with the Lévy process L.
- In general, we know that for any $g, h \in L^2(\mathbb{R})$,

$$\mathbb{E}[(L_t, g)_2 (L_t, h)_2] = (\mathcal{Q}g, h)_2$$

where $(\cdot, \cdot)_2$ denotes the inner product in $L^2(\mathbb{R})$

$$\mathcal{Q}g(x) = \int_{\mathbb{R}} q(x, y) g(y)\, dy\,, \tag{28}$$

- If we assume $g \in L^2(\mathbb{R})$ to be close to δ_x, the Dirac δ-function, and likewise, $h \in L^2(\mathbb{R})$ being close to δ_y, $(x, y) \in \mathbb{R}^2$, we find approximately

$$\mathbb{E}[L_t(x) L_t(y)] = q(x, y)$$

- A simple choice resembling to some degree the fast decaying property is $q(|x-y|) = \exp(-\gamma|x-y|)$ for a constant $\gamma > 0$.
- It follows that $t \mapsto (L_t, g)_2$ is a NIG Lévy process on the real line.

Conclusion and future work

- We developed a spatio-temporal dynamical arbitrage free model for electricity forward prices based on the Heath-Jarrow-Morton (HJM) approach under Musiela parametrization
- We examined a unique data set of price forward curves derived each day in the market between 2009–2015
- We examined the spatio-temporal structure of our data set
  - **Risk premia:** higher volatility short-term, oscillating around zero; constant volatility on the long-term, turning into negative
  - **Term structure volatility:** Samuelson effect, volatility bumps explained by increased volume of trades
  - **Coloured (leptokurtic) noise** with evidence of conditional volatility
  - **Spatial correlations structure:** decaying fast for short-term maturities; constant (white noise) afterwards with a bump around 1 year
- After explaining the Samuelson effect in the volatility term structure, the residuals are modeled by an infinite dimensional NIG Lévy process, which allows for a natural formulation of a covariance functional.

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