Optimal market maker pricing in the German intraday power market

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[TBD]

1. Introduction

Considering exchange-based trading of power supply contracts with delivery in Germany there exist both a futures and a spot market. The main German spot market is operated by EPEX SPOT SE and comprises trading of both power supply contracts with hourly and quarter hourly delivery. For contracts with hourly delivery the spot market comprises a day-ahead auction with submission deadline at 12.00 noon on the day before the delivery day and continuous trading which opens at 3.00 pm on the day before the delivery day and closes 30 minutes before delivery start. For contracts with quarter hourly delivery similar markets are in place. In the following, the markets involving continuous trading of power supply contracts with delivery in Germany are commonly referred to as the German intraday power market. They are mainly used in order to balance forecast errors which evolve after the submission deadlines for the day-ahead auctions, see e.g. Weber (2010).

Participants in the German intraday power market have the choice between three types of orders, i.e. regular orders, iceberg orders and over-the-counter (OTC) orders. Iceberg orders are large-volume orders which are split into several orders with smaller volume and placed into the market sequentially. In case of OTC orders the trader needs to specify the receiving balancing group. OTC orders are not considered further in the following.

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Furthermore, market participants may either place single-contract orders for deliveries in individual hours or block orders for deliveries in two or more contiguous hours. Only single-contract orders are considered in the following. If market participants wish to place single-contract orders they need to make some specifications. Important ones are the delivery hour or quarter hour, whether it is a buy or a sell order, the quantity to be traded, the price limit and potentially also execution restrictions, i.e. immediate-or-cancel, fill-or-kill or all-or-nothing.[1] In case of iceberg orders the peak quantity needs to be specified in addition and execution restrictions may not be specified.

Single-contract orders with no execution restriction are added to a central, open and anonymous order book. Order book entries are sorted firstly by whether they are buy or sell orders, secondly by limit price and thirdly by time of reception. Order books provide market participants with information on the (peak) quantity and the limit price of each active buy and sell order. A buy (sell) order placed into the single-contract market is matched if its limit price is greater (smaller) than or equal to the limit price of at least one sell (buy) order from the order book and if all execution restrictions are adhered to. In that case the order may also be referred to as market order.

Given that the German intraday power market involves continuous trading of power supply contracts by allowing market participants to place (i) buy and sell limit orders which are gathered in a central, open and anonymous order book and which give rise to a bid-ask spread and (ii) buy and sell market orders, significant analaogies with order-driven equity markets prevail. An important trading strategy on equity markets is market making, i.e. placing buy and sell limit orders and being lifted on both sides of the market, thus potentially realizing bid-ask spreads. According to Harris (2003) market makers are exposed to the risk of holding an inventory position whose value may decrease due to an unfavorable price change and to the risk of entering into a contract with an information trader which may trigger an unfavorable price change and hence a decrease in the value of the newly gained inventory position. Adverse selection risk. The question arises of whether it is reasonable to apply the market making strategy also on the German intraday power market. The aim of this work is to go into this question.

This work is organized as follows: In Section 2 the market parameters from a model for market maker pricing in equity markets are adjusted to the German intraday power market. In Section 3 a solution for optimal limit order prices from the perspective of a market maker in the German intraday power market is derived. Section 4 treats the estimation of the parameters of the model for market maker pricing in the German intraday power market and Section 5 gives an overview of backtest results.

2. Components of the German Intraday Power Market Relevant for Modeling Market Maker Pricing

A market maker faces the question of how to price her sell and buy limit orders. Generally speaking, a market maker should price her limit orders such that both the diversifiable

[1] For more details please refer to EPEX SPOT SE (2016).

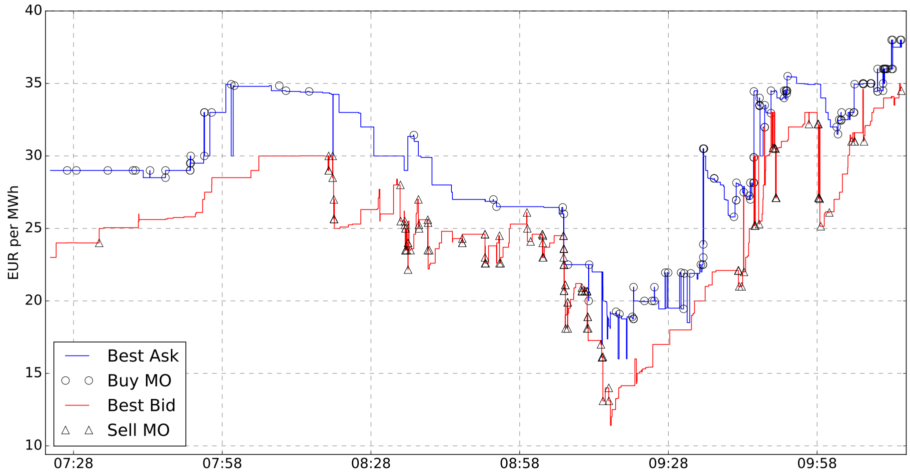

Figure 1: Bid, ask and market orders for product H12 with delivery on April 01, 2015

inventory risk and the adverse selection risk are taken account of. Avellaneda and Stoikov (2008) suggest a model for market maker pricing which takes account of the diversifiable inventory risk. As opposed to that the model suggested by Cartea, Jaimungal, and Ricci (2014) and Ricci (2014) also considers the adverse selection risk. This and the following chapter address how the different elements of their model could be modified so that they resemble better the characteristics of the German intraday power market.

Developing a model for optimal market maker pricing in the German intraday power market requires consideration of a number of relevant market components in the first place. Those components which are stochastic processes or random variables are defined on the completed, filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ where $\mathcal{F}_t$ is a filtration of the σ-algebra $\mathcal{F}$ and $\mathbb{P}$ is the real-world probability measure.

2.1. Mid Price and Half Spread

The mid price of a power supply contract which is traded on the German intraday power market at time t, denoted S_t, is defined as the arithmetic mean of the price of the best buy limit order and the price of the best sell limit order at that time. The mid price changes if a new buy or sell limit order is placed inside the bid-ask spread or if an existing buy or sell limit order is canceled from the first level of the order book. Furthermore, the mid price changes if a buy (sell) market order lifts at least the entire best sell (buy) limit order. The following stochastic differential equation reflects these mid price dynamics:

$$dS_t = \epsilon_\nu^+ dK_t^+ - \epsilon_\nu^- dK_t^- + \epsilon_v^+ dL_t^+ - \epsilon_v^- dL_t^- + \epsilon^+ dM_t^+ - \epsilon^- dM_t^- \tag{1}$$

The processes $K_t^\pm$ are point processes on the real positive line with intensities $\nu_t^\pm$ and reflect the number of upward changes in the mid price due to new buy limit orders being

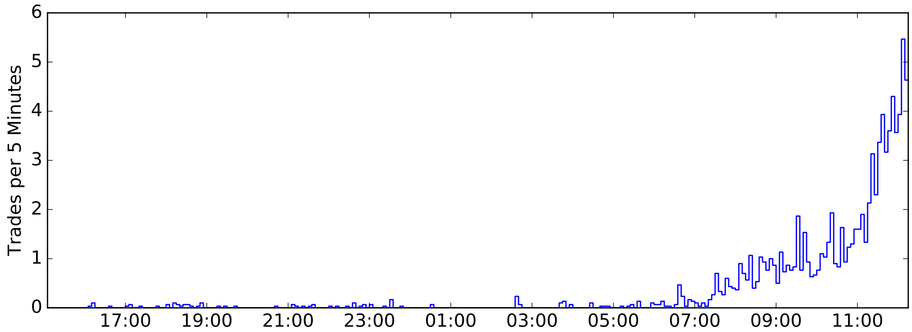

Figure 2: Average buy market order intensity for product H14 with delivery between April 01 and April 30, 2015

placed inside the bid-ask spread and downward changes due to existing buy limit orders being canceled from the first level of the order book. The random variables $\epsilon_\nu^\pm$ reflect by how much the mid price jumps up and down. The processes $L_t^\pm$ with intensities $\upsilon^\pm$ and the random variables $\epsilon_\upsilon^\pm$ refer to sell limit orders and are defined analagously. The processes $M_t^\pm$ are also point processes on the real positive line with intensities $\lambda_t^\pm$ and reflect the number of buy and sell market orders. The random variables $\epsilon^\pm$ reflect by how much the mid price jumps up and down.

The half spread H_t is defined as half the difference between the price of the best buy limit order and the price of the best sell limit order at that time. The dynamics of the half spread depend on the same events as those which drive the mid price. However, the directions of the half spread changes caused by these events are partly different. The following stochastic differential equation reflects these half spread dynamics:

$$dH_t = -\epsilon_\nu^+ dK_t^+ + \epsilon_\nu^- dK_t^- + \epsilon_\upsilon^+ dL_t^+ - \epsilon_\upsilon^- dL_t^- + \epsilon^+ dM_t^+ + \epsilon^- dM_t^- \tag{2}$$

2.2. Intensities

Considering arrivals of buy and sell market orders per time interval on the German intraday power market, there is evidence that their distributions depend on the location of the time intervals. Figure 2 shows the average number of buy market orders per five minutes for product H14 with delivery on weekdays between April 01, 2015 and April 30, 2015. The path suggests that the intensity of buy market order arrivals increases with decreasing time to maturity. Furthermore, the frequency tends to be markedly higher in the last trading hour compared to the second last trading hour and still on a substantial level in the second last trading hour. For sell market orders the same holds true. For that reason the intensity parameters are fitted separately to the data which falls into the last trading hour and the second last trading hour (see Section 4). All data that does not fall into one of these windows is ignored. In the following the last trading hour is referred to as “Late” and the second last as “Mid”.

Figure 3 shows the arrivals of sell market orders per minute product H14 with delivery

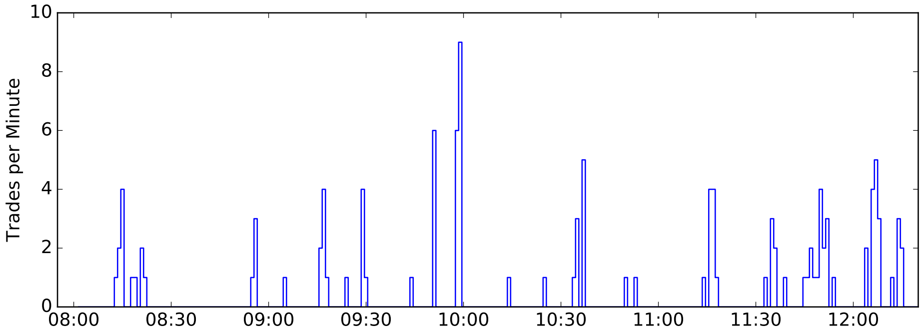

Figure 3: Sell market order intensity for product H14 with delivery on April 08, 2016

on April 08, 2015. The path suggests that occasionally sell market orders arrive in clusters. For buy market orders the same holds true. Given such clustering, buy and sell market order arrivals are hypothesized to follow point processes with intensities which depend on the arrival histories. One such process is the Hawkes point process which according to Daley and Vere-Jones (2003) has the advantage of a linear representation for the conditional intensity. Considering the arrivals of buy and sell market orders, possible forms for the conditional intensities $\lambda_t^{\pm}$ are

$$\lambda_t^+ = \mu^+ + \sum_{t_i^+ < t} \zeta^+ e^{-\rho^+ \left(t - t_i^+\right)} \tag{3}$$

$$\lambda_t^- = \mu^- + \sum_{t_i^- < t} \zeta^- e^{-\rho^- \left(t - t_i^-\right)}. \tag{4}$$

They may be interpreted as follows: The arrival of a buy (sell) market order causes the conditional intensity to jump up by ζ^+ (ζ^-), which means that the probability of another buy (sell) market order arriving increases. The positive impact ζ^+ (ζ^-) of the buy (sell) market order on the conditional intensity decays exponentially with time at rate ρ^+ (ρ^-). They not only have the advantage of being simple to evaluate but also of having first moments which are simple to evaluate. According to Da Fonseca and Zaatour (2014) they are defined as

$$\mathbb{E}\left[\lambda_T^+ \mid \mathcal{F}_t\right] = \frac{\mu^+ \rho^+}{\zeta^+ - \rho^+} \left(e^{\left(\zeta^+ - \rho^+\right)(T-t)} - 1\right) + \lambda_t^+ e^{\left(\zeta^+ - \rho^+\right)(T-t)} \tag{5}$$

$$\mathbb{E}\left[\lambda_T^- \mid \mathcal{F}_t\right] = \frac{\mu^- \rho^-}{\zeta^- - \rho^-} \left(e^{\left(\zeta^- - \rho^-\right)(T-t)} - 1\right) + \lambda_t^- e^{\left(\zeta^- - \rho^-\right)(T-t)}. \tag{6}$$

Having discussed the intensities of buy and sell market order arrivals, the intensities of limit order-driven mid price and half spread changes remain to be considered. While a thorough analysis has been omitted, it is assumed that they also exhibit clustering. Against that background it is hypothesized that the conditional intensities of limit order-driven mid price and half spread changes $\nu_t^{\pm}$ and $\upsilon_t^{\pm}$ and hence also their first moments have the same forms as the conditional intensities of buy and sell market order arrivals.

2.3. Mid Price and Half Spread Impacts

As already indicated the mid price and half spread impacts of buy and sell market orders $\epsilon^{\pm}$ are random variables which represent the absolute mid price and half spread changes after the arrival of a market order. If a market order consumes only part of the best limit order, it does not have any impact on the mid price and half spread. If, however, one or more limit orders are consumed by a market order, the mid price and the half spread change by half the distance between the price of the best limit order before the arrival of the market order and after. With this in mind it becomes clear that the mid price and half spread impacts of market orders depend on the shape of the limit order book (i.e. the volume at each tick above and below the mid price) before market order arrival and the market order volume. Hence, the expected mid price and half spread impacts of buy and sell market orders $\varepsilon^{\pm}$ should depend on processes reflecting the shape of the limit order book and the evolution of market order volumes.

Regarding the limit order books for the products traded on the German intraday power market, it is hypothesized that their shapes may vary significantly over time. For example, in times of large market orders arriving on one side of the market they may thin out quickly on that side and take a while to be filled up again. Considering the evolution of market order volumes on the German intraday power market a positive autocorrelation structure is suspected. One explanation for this is that if a market participant suffers a sudden position imbalance e.g. due to a power plant outage, she may place a number of market orders with substantial volumes on one side of the market over a short period of time in order to compensate the imbalance. However, analyzing these dynamics in detail is left for future research.

The mid price and half spread impacts of limit order-driven mid price and half spread changes $\epsilon_{\nu}^{\pm}$ and $\epsilon_{\upsilon}^{\pm}$ are random variables which represent the absolute mid price and half spread changes after a new limit order has been placed inside the bid-ask spread or after an existing limit order has been canceled from the first level of the order book. Both the mid price and the half spread change by half the distance between the price of the best limit order before the event and after. The mid price and half spread impacts of existing limit orders being canceled from the first level of the order book depend on the shape of the limit order book. Hence, the shape of the limit order book should be represented in their expectations ε_{ν}^{-} and $\varepsilon_{\upsilon}^{+}$. As opposed to this the mid price and half spread impacts of new limit orders being placed inside the bid-ask spread do not depend directly on the shape of the limit order book.

2.4. Fill Probability

Assume that there are market participants placing market orders which lift limit orders contained in a limit order book. The highest (lowest) price of a sell (buy) limit order which is lifted by a buy (sell) market order is denoted Π_t^+ (Π_t^-), whereas the absolute difference between that price and the mid price is denoted $\Delta\Pi_t^+$ ($\Delta\Pi_t^-$) and named mid price impact. The fill probabilities $\Pr\left(\Delta\Pi_t^{\pm} > \delta_t^{\pm}\right)$ reflect the likeliness that a sell or buy limit order placed at distance $\delta_t^{\pm}$ above or below the mid price is lifted. Avellaneda and Stoikov (2008) suggest that the fill probability of a limit order may be derived from the distribution of market order volumes and its relation to the distribution of market order impacts on the mid price. Assuming that the distributions of market order volumes obey power laws and that there is a logarithmic relation between the distributions of market order volumes and the distributions of market order impacts, they come up with parametric equations for the fill probabilities which decay exponentially in $\delta_t^{\pm}$. The way how these fill probabilities work is that the further a limit order is placed away from the mid price the less likely it is to be lifted.

Avellaneda and Stoikov (2008) do not consider explicitly the impact of the shape of the limit order book on the fill probability. Furthermore, they do not consider any variability in the shape of the limit order book or the distributions of market order volumes over time. While confirming the dependence of the fill probabilities on the distributions of market order volumes, Cartea, Jaimungal, and Ricci (2014) state explicitly that the shape of the limit order book is an additional factor influencing the fill probability. However, the parametric equations for the fill probabilities they suggest which are most comparable to the ones proposed by Avellaneda and Stoikov (2008) are exponentially decaying functions of $\delta_t^{\pm}$ with decay rates $\kappa_t^{\pm}$ which reflect the current distributions of market order volumes and the current shape of the limit order book in an aggregated way. The larger κ_t^{+} (κ_t^{-}) is, the smaller is the probability of a sell (buy) limit order with a price greater (smaller) than the mid price of being lifted.

Considering the German intraday power market both variability in the distributions of market order volumes and variability in the shape of the limit order book are suspected (see above). For that reason the parametric equations for the fill probabilities suggested by Cartea, Jaimungal, and Ricci (2014) appear to be a good starting point. It is hypothesized, however, that they should also take account of the half spread. The reason for that is that in fact it is the distance between the price of a sell (buy) limit order and the price of the best sell (buy) limit order which mainly impacts the likeliness of being lifted, not the distance to the mid price. Hence, the fill probabilities of sell and buy limit orders are assumed to have the forms

$$f\left(\delta_t^{+}; H_t, \kappa_t^{+}\right) \coloneqq \Pr\left(\Delta\Pi_t^{-} > \delta_t^{-}\right) = e^{-\kappa_t^{+}\left(\delta_t^{+} - H_t\right)} \tag{7}$$

$$f\left(\delta_t^{-}; H_t, \kappa_t^{-}\right) \coloneqq \Pr\left(\Delta\Pi_t^{-} > \delta_t^{-}\right) = e^{-\kappa_t^{-}\left(\delta_t^{-} - H_t\right)}. \tag{8}$$

In line with Cartea, Jaimungal, and Ricci (2014) these fill probabilities are assumed to be associated with the condition $f\left(\delta_t^{\pm}; H_t, \kappa_t^{\pm}\right) = 1$ for $\delta_t^{\pm} < H_t$. That means that market participants are assumed not to be able to increase the likeliness of their limit orders being lifted by placing them inside the bid-ask spread.

3. A Model for Optimal Market Maker Pricing in the German Intraday Power Market

Having outlined the characteristics of the components of the German intraday power market which are relevant for the market maker pricing model suggested by Cartea, Jaimungal, and Ricci (2014) and Ricci (2014), the model itself may be modified. Again it should be noted that all stochastic processes and random variables are defined on the completed, filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ where $\mathcal{F}_t$ is a filtration of the σ-algebra $\mathcal{F}$ and $\mathbb{P}$ is the real-world probability measure.

3.1. Market Maker's Value Function

The model for market maker pricing comprises on the one hand a market making agent (the Agent) who continuously places sell and buy limit orders and on the other hand other market participants who place limit and market orders. The markup (markdown) which the Agent adds to (subtracts from) the mid price to price her sell (buy) limit orders is labeled δ_t^+ (δ_t^-). If a sell (buy) limit order placed by the Agent is lifted her inventory decreases (increases). The Agent realizes profits if she sells power at prices which are higher than the prices at which she buys power. Hence, she benefits from the price spread between the market's sell and buy side and a favorable mid price and/or half spread development but also takes the risk of an unfavorable mid price and/or half spread development. For that reason the Agent is reluctant to inventory. Furthermore, the Agent splits the trading periods of the products which are traded in the German intraday power market into windows of equal duration such as one hour and operates in each window on its own. If the Agent's inventory is positive (negative) at the end of such a trading window, she liquidates her position by a sell (buy) market order.[2]

Against that background the Agent's performance criterion is hypothesized to consist of three components. The first component is her terminal wealth X_T whose dynamics over a trading window are defined as

$$dX_t = \left(S_t + \delta_t^+\right) dN_t^+ - \left(S_t - \delta_t^-\right) dN_t^- \tag{9}$$

where the processes N_t^+ and N_t^- represent the number of sell and buy limit orders placed by the Agent which have been lifted by buy and sell market orders, respectively. In case of a sell limit order being lifted the Agent's wealth increases by the price of that limit order, whereas in case of a buy limit order being lifted her wealth decreases by the price of that limit order. The intensities of the processes $N_t^\pm$ are labeled $\Lambda_t^\pm$ and consist of the intensities of buy and sell market order arrivals $\lambda_t^\pm$ and the fill probabilities of sell and buy limit orders $f\left(\delta_t^\pm; H_t, \kappa_t^\pm\right)$:

$$\Lambda_t^+ = \lambda_t^+ f\left(\delta_t^+; H_t, \kappa_t^+\right) \tag{10}$$

$$\Lambda_t^- = \lambda_t^- f\left(\delta_t^-; H_t, \kappa_t^-\right). \tag{11}$$

The hypothesized intensities of buy and sell market order arrivals and fill probabilities of sell and buy limit orders have already been discussed above. Given that the processes $N_t^\pm$ count the number of sell and buy limit orders placed by the Agent and subsequently

[2] In particular, she does not pass any position on to the reserve energy market.

being lifted by buy and sell market orders, the inventory level at any time during a trading window is defined as

$$q_t = N_t^- - N_t^+. \tag{12}$$

The second component of the Agent's performance criterion is the liquidation value of her terminal inventory, i.e. her terminal inventory position multiplied by the terminal mid price reduced or increased by the terminal half spread depending on whether the terminal inventory level is positive or negative and reduced by a penalty which is scaled by the terminal inventory position: $q_T\left(S_T - \operatorname{sgn}(q_T) H_T - \alpha q_T\right)$. The terminal mid price is reduced by an inventory-scaled penalty to account for the fact that the greater the terminal inventory position is, the worse is the price at which the terminal inventory position may be liquidated: In case of a negative terminal inventory position the liquidation price tends to be greater greater than the terminal best sell limit order price, whereas in case of a positive terminal terminal inventory position the liquidation price tends to be smaller than the terminal best buy limit order price.

The third component of the Agent's performance criterion is a penalty on inventory held during the trading period: $-\phi \int_t^T q_u^2 du$.

Having determined the components of the Agent's performance criterion the following value function evolves:

$$\Phi\left(t, X_t, S_t, H_t, q_t, \boldsymbol{\nu_t}, \boldsymbol{\upsilon_t}, \boldsymbol{\lambda_t}, \boldsymbol{\kappa_t}\right) = \sup_{\delta^+,\delta^-} \mathbb{E}\left[X_T + q_T\left(S_T - \operatorname{sgn}(q_T) H_T - \alpha q_T\right) - \phi \int_t^T q_u^2\, du \,\middle|\, \mathcal{F}_t \right] \tag{13}$$

$\boldsymbol{\nu_t}$ is a vector comprising the intensities of buy limit orders entering the order book or being canceled which have an impact on the mid price and the half spread, i.e. $\left(\nu_t^+, \nu_t^-\right)'$, $\boldsymbol{\upsilon_t}$, $\boldsymbol{\lambda_t}$ and $\boldsymbol{\kappa_t}$ analogous.

3.2. The Optimal Strategy

With the value function as presented in Equation (13) at hand, employing the dynamic programming principle yields the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}
0 = (\partial_t + \mathcal{L})\,\Phi \\
&+ \nu^+\left[\mathbb{S}^+_{h\nu}\Phi\left(t,x,s+\epsilon^+_\nu\right) - \Phi\right] + \nu^-\left[\mathbb{S}^-_{h\nu}\Phi\left(t,x,s-\epsilon^-_\nu\right) - \Phi\right] \\
&+ \upsilon^+\left[\mathbb{S}^+_{h\upsilon}\Phi\left(t,x,s+\epsilon^+_\upsilon\right) - \Phi\right] + \upsilon^-\left[\mathbb{S}^-_{h\upsilon}\Phi\left(t,x,s-\epsilon^-_\upsilon\right) - \Phi\right] \\
&+ \lambda^+ \sup_{\delta^+}\Big\{ f\left(\delta^+; h, \kappa^+\right)\Big[\mathbb{S}^+_{hq\lambda}\Phi\left(t, x+s+\delta^+, s+\epsilon^+\right) - \Phi\Big] \\
&\qquad + \left(1 - f\left(\delta^+; h, \kappa^+\right)\right)\left[\mathbb{S}^+_{h\lambda}\Phi\left(t,x,s+\epsilon^+\right) - \Phi\right]\Big\} \\
&+ \lambda^- \sup_{\delta^-}\Big\{ f\left(\delta^-; h, \kappa^-\right)\Big[\mathbb{S}^-_{hq\lambda}\Phi\left(t, x-s+\delta^-, s-\epsilon^-\right) - \Phi\Big] \\
&\qquad + \left(1 - f\left(\delta^-; h, \kappa^-\right)\right)\left[\mathbb{S}^-_{h\lambda}\Phi\left(t,x,s-\epsilon^-\right) - \Phi\right]\Big\} \\
&- \phi q^2
\end{aligned} \tag{14}$$

with terminal condition $\Phi(T,\cdot) = x + q(s - \operatorname{sgn}(q)h - \alpha q)$ and $x \coloneqq X_t$, $s \coloneqq S_t$ and $h \coloneqq H_t$. The remaining time indexes are also suppressed in the HJB equation. This way of presentation will be kept up for the rest of this section. Considering the HJB equation $\mathcal{L}$ requires clarification. Given that the HJB equation reflects the local behavior of the value function and that the intensities $\boldsymbol{\nu}$, $\boldsymbol{\upsilon}$ and $\boldsymbol{\lambda}$ as well as the decay rates $\boldsymbol{\kappa}$ vary over time, the derivatives of the value function with respect to these parameters need to be included in the HJB equation. Furthermore, the shift operators $\mathbb{S}^{+}_{h\nu}$ etc. require clarification. They reflect the expectation with regards to the value function in case of a shift in the parameter/s specified in their index, i.e.:

$$\mathbb{S}^{+}_{h\nu}\Phi = \mathbb{E}\left[\Phi\left(t,x,s,h-\epsilon^{+}_{\nu},q,\boldsymbol{\nu}+\left(\zeta^{+}_{\nu},0\right)',\boldsymbol{\upsilon},\boldsymbol{\lambda},\boldsymbol{\kappa}\right)\right] \tag{15}$$

$$\mathbb{S}^{-}_{h\nu}\Phi = \mathbb{E}\left[\Phi\left(t,x,s,h+\epsilon^{-}_{\nu},q,\boldsymbol{\nu}+\left(0,\zeta^{-}_{\nu}\right)',\boldsymbol{\upsilon},\boldsymbol{\lambda},\boldsymbol{\kappa}\right)\right] \tag{16}$$

$$\mathbb{S}^{+}_{h\upsilon}\Phi = \mathbb{E}\left[\Phi\left(t,x,s,h+\epsilon^{+}_{\upsilon},q,\boldsymbol{\nu},\boldsymbol{\upsilon}+\left(\zeta^{+}_{\upsilon},0\right)',\boldsymbol{\lambda},\boldsymbol{\kappa}\right)\right] \tag{17}$$

$$\mathbb{S}^{-}_{h\upsilon}\Phi = \mathbb{E}\left[\Phi\left(t,x,s,h-\epsilon^{-}_{\upsilon},q,\boldsymbol{\nu},\boldsymbol{\upsilon}+\left(0,\zeta^{-}_{\upsilon}\right)',\boldsymbol{\lambda},\boldsymbol{\kappa}\right)\right] \tag{18}$$

$$\mathbb{S}^{+}_{hq\lambda}\Phi = \mathbb{E}\left[\Phi\left(t,x,s,h+\epsilon^{+},q-1,\boldsymbol{\nu},\boldsymbol{\upsilon},\boldsymbol{\lambda}+\left(\zeta^{+},0\right)',\boldsymbol{\kappa}\right)\right] \tag{19}$$

$$\mathbb{S}^{-}_{hq\lambda}\Phi = \mathbb{E}\left[\Phi\left(t,x,s,h+\epsilon^{-},q+1,\boldsymbol{\nu},\boldsymbol{\upsilon},\boldsymbol{\lambda}+\left(0,\zeta^{-}\right)',\boldsymbol{\kappa}\right)\right] \tag{20}$$

$$\mathbb{S}^{+}_{h\lambda}\Phi = \mathbb{E}\left[\Phi\left(t,x,s,h+\epsilon^{+},q,\boldsymbol{\nu},\boldsymbol{\upsilon},\boldsymbol{\lambda}+\left(\zeta^{+},0\right)',\boldsymbol{\kappa}\right)\right] \tag{21}$$

$$\mathbb{S}^{-}_{h\lambda}\Phi = \mathbb{E}\left[\Phi\left(t,x,s,h+\epsilon^{-},q,\boldsymbol{\nu},\boldsymbol{\upsilon},\boldsymbol{\lambda}+\left(0,\zeta^{-}\right)',\boldsymbol{\kappa}\right)\right] \tag{22}$$

Solving the HJB equation yields as by-product solutions for the optimal markups (markdowns) to be added to (subtracted from) the mid price when pricing sell (buy) limit orders. To do so, an ansatz for Φ is required. Referring to Cartea and Jaimungal (2015) the terminal condition of the HJB equation is used to identify the ansatz

$$\Phi = x + q(s - \operatorname{sgn}(q)h - \alpha q) + g(t,h,q,\boldsymbol{\nu},\boldsymbol{\upsilon},\boldsymbol{\lambda},\boldsymbol{\kappa}) \tag{23}$$

with $g(T,\cdot) = 0$. Substituting the ansatz into the HJB equation and taking expectations with regards to $\epsilon^{\pm}_{\nu}$, $\epsilon^{\pm}_{\upsilon}$ and $\epsilon^{\pm}$ yields

$$\begin{aligned}
0 = {} & \mathcal{D}g \\
& + \nu^{+}\left[\varepsilon^{+}_{\nu}q(1+\operatorname{sgn}(q)) + \mathbb{S}^{+}_{h\nu}g - g\right] - \nu^{-}\left[\varepsilon^{-}_{\nu}q(1+\operatorname{sgn}(q)) - \mathbb{S}^{-}_{h\nu}g + g\right] \\
& + \upsilon^{+}\left[\varepsilon^{+}_{\upsilon}q(1-\operatorname{sgn}(q)) + \mathbb{S}^{+}_{h\upsilon}g - g\right] - \upsilon^{-}\left[\varepsilon^{-}_{\upsilon}q(1-\operatorname{sgn}(q)) - \mathbb{S}^{-}_{h\upsilon}g + g\right] \\
& + \lambda^{+}\left[\varepsilon^{+}q(1-\operatorname{sgn}(q)) + \mathbb{S}^{+}_{h\lambda}g - g\right] + \lambda^{-}\left[-\varepsilon^{-}q(1+\operatorname{sgn}(q)) + \mathbb{S}^{-}_{h\lambda}g - g\right] \\
& + \lambda^{+}\sup_{\delta^{+}}\Big\{f\left(\delta^{+};h,\kappa^{+}\right)\Big[\delta^{+} - \varepsilon^{+} - \varepsilon^{+}((q-1)\operatorname{sgn}(q-1) - q\operatorname{sgn}(q)) \\
& \qquad - h((q-1)\operatorname{sgn}(q-1) - q\operatorname{sgn}(q)) - \alpha(1-2q) + \mathbb{S}^{+}_{hq\lambda}g - \mathbb{S}^{+}_{h\lambda}g\Big]\Big\} \\
& + \lambda^{-}\sup_{\delta^{-}}\Big\{f\left(\delta^{-};h,\kappa^{-}\right)\Big[\delta^{-} - \varepsilon^{-} - \varepsilon^{-}((q+1)\operatorname{sgn}(q+1) - q\operatorname{sgn}(q)) \\
& \qquad - h((q+1)\operatorname{sgn}(q+1) - q\operatorname{sgn}(q)) - \alpha(1+2q) + \mathbb{S}^{-}_{hq\lambda}g - \mathbb{S}^{-}_{h\lambda}g\Big]\Big\} \\
& - \phi q^{2}
\end{aligned} \tag{24}$$

where $\mathcal{D} = \partial_t + \mathcal{L}$.

Proposition 1. *Optimal markups are given by*

$$\delta^{+*} = \frac{1}{\kappa^{+}} + \varepsilon^{+}\left(\left(q-1\right)\operatorname{sgn}\left(q-1\right) - q\operatorname{sgn}\left(q\right) + 1\right) \\ + h\left(\left(q-1\right)\operatorname{sgn}\left(q-1\right) - q\operatorname{sgn}\left(q\right)\right) + \alpha\left(1-2q\right) - \mathbb{S}^{+}_{hq\lambda}g + \mathbb{S}^{+}_{h\lambda}g. \quad (25)$$

Optimal mardowns are obtained analogously.

See Appendix A.1 for a proof. Cartea, Jaimungal, and Ricci (2014) refer to $\frac{1}{\kappa^{+}}$ as risk-neutral component. As may be understood from Equation (25) optimal markups δ^{+*} depend on $g\left(\cdot\right)$, more specifically the difference between $g\left(\cdot\right)$ with half spread, inventory and buy market order intensity shifted and $g\left(\cdot\right)$ with only half spread and buy market order intensity shifted. In the attempt to find an analytical solution for $g\left(\cdot\right)$ the solutions for optimal markups and markdowns are plugged back into the HJB:

$$\begin{aligned} 0 = {} & \mathcal{D}g \\ & + \nu^{+}\left[\varepsilon^{+}_{\nu} q\left(1+\operatorname{sgn}\left(q\right)\right) + \mathbb{S}^{+}_{h\nu}g - g\right] - \nu^{-}\left[\varepsilon^{-}_{\nu} q\left(1+\operatorname{sgn}\left(q\right)\right) - \mathbb{S}^{-}_{h\nu}g + g\right] \\ & + \upsilon^{+}\left[\varepsilon^{+}_{\upsilon} q\left(1-\operatorname{sgn}\left(q\right)\right) + \mathbb{S}^{+}_{h\upsilon}g - g\right] - \upsilon^{-}\left[\varepsilon^{-}_{\upsilon} q\left(1-\operatorname{sgn}\left(q\right)\right) - \mathbb{S}^{-}_{h\upsilon}g + g\right] \\ & + \lambda^{+}\left[\varepsilon^{+} q\left(1-\operatorname{sgn}\left(q\right)\right) + \mathbb{S}^{+}_{h\lambda}g - g\right] + \lambda^{-}\left[-\varepsilon^{-} q\left(1+\operatorname{sgn}\left(q\right)\right) + \mathbb{S}^{-}_{h\lambda}g - g\right] \\ & + \lambda^{+}\left\{f\left(\delta^{+*}; h, \kappa^{+}\right)\frac{1}{\kappa^{+}}\right\} + \lambda^{-}\left\{f\left(\delta^{-*}; h, \kappa^{-}\right)\frac{1}{\kappa^{-}}\right\} \\ & - \phi q^{2} \end{aligned} \quad (26)$$

Given that Equation (26) is non-linear due to $f\left(\cdot\right)$ being an exponential function and functions $g\left(\cdot\right)$ contained in optimal markups and markdowns $\delta^{\pm *}$ being shifted pose difficulties, approximations are required in order to be able to find an analytical solution for $g\left(\cdot\right)$.

Proposition 2. *With first-order Taylor expansion for $f\left(\cdot\right)$ around $\frac{1}{\kappa^{\pm}} + h$ and second-order asymptotic expansion for $g\left(\cdot\right)$ in q, optimal markups δ^{+*} only depend on $g_1\left(\cdot\right)$ and $g_2\left(\cdot\right)$. Solutions are*

$$\begin{aligned} g_1 = {} & \varepsilon^{+}_{\nu}\left(1+\operatorname{sgn}\left(q\right)\right)\mathbb{E}\left[\int_t^T \nu^{+}_u du \,\middle|\, \nu^{+}_t = \nu^{+}\right] - \varepsilon^{-}_{\nu}\left(1+\operatorname{sgn}\left(q\right)\right)\mathbb{E}\left[\int_t^T \nu^{-}_u du \,\middle|\, \nu^{-}_t = \nu^{-}\right] \\ & + \varepsilon^{+}_{\upsilon}\left(1-\operatorname{sgn}\left(q\right)\right)\mathbb{E}\left[\int_t^T \upsilon^{+}_u du \,\middle|\, \upsilon^{+}_t = \upsilon^{+}\right] - \varepsilon^{-}_{\upsilon}\left(1-\operatorname{sgn}\left(q\right)\right)\mathbb{E}\left[\int_t^T \upsilon^{-}_u du \,\middle|\, \upsilon^{-}_t = \upsilon^{-}\right] \\ & + \varepsilon^{+}\left(1-\operatorname{sgn}\left(q\right)\right)\mathbb{E}\left[\int_t^T \lambda^{+}_u du \,\middle|\, \lambda^{+}_t = \lambda^{+}\right] - \varepsilon^{-}\left(1+\operatorname{sgn}\left(q\right)\right)\mathbb{E}\left[\int_t^T \lambda^{-}_u du \,\middle|\, \lambda^{-}_t = \lambda^{-}\right] \\ & + 2\exp\left(-1\right)\left(\alpha - g_2\right)\mathbb{E}\left[\int_t^T \lambda^{+}_u du \,\middle|\, \lambda^{+}_t = \lambda^{+}\right] + 2\exp\left(-1\right)\left(-\alpha + g_2\right)\mathbb{E}\left[\int_t^T \lambda^{-}_u du \,\middle|\, \lambda^{-}_t = \lambda^{-}\right] \end{aligned} \quad (27)$$

and

$$g_2 = -\phi\left(T - t\right). \quad (28)$$

See Appendix A.2 for a proof. The expectations contained in Equation (27) may be computed explicitly on the basis of Fubini's Theorem and expected intensities of the form presented in Equations (3) and (4).

We notify that the behavior of the solution for $g(\cdot)$ for $q < -1 \vee q > 1$ in the region $[-1, 1]$ is comparable to the behavior of the second and the third term of Equation (29) in that region (see Section 3.3 for details).

Conjecture 1. *The solution for $g(\cdot)$ for $q < -1 \vee q > 1$ is also the solution for $g(\cdot)$ for $q \in [-1, 1]$.*

Corollary 3. *The asymptotic expansion of optimal markups is*

$$\begin{aligned} \delta^{+*} &= \frac{1}{\kappa^+} + \varepsilon^+ \left((q-1) \operatorname{sgn}(q-1) - q \operatorname{sgn}(q) + 1 \right) \\ &+ h \left((q-1) \operatorname{sgn}(q-1) - q \operatorname{sgn}(q) \right) + \alpha (1 - 2q) - (q-1) \mathbb{S}^+_{q\lambda} g_1 + q \mathbb{S}^+_{\lambda} g_1 - (1-2q) g_2. \end{aligned} \tag{29}$$

The asymptotic expansion of optimal markdowns is obtained analogously.

Given that the Agent is assumed not to be able to increase the likeliness of her limit orders being lifted by placing them inside the bid-ask spread, the optimal markup and markdown have the current half spread as their minimum. However, the asymptotic solutions for optimal markups and markdowns presented above may take values which are smaller than the current half spread. For that reason the optimal markups and markdowns are rewritten as $\max(\delta^{\pm*}, h)$.

3.3. The Behavior of the Strategy

The effects of the components of optimal markups δ_t^{+*} as provided by Equation (29) are analyzed in the following. All parameters are chosen such that they are similar to the parameter estimations presented below.

We refer to Cartea, Jaimungal, and Ricci (2014) with regards to the interpretation of the risk-neutral component. The second component involving ε^+ may be interpreted as follows: In case of a negative or no inventory position, the second term of Equation (29) is twice the expected impact of a buy market order. Thus, the Agent prices into her sell limit order price that she may have to close the position resulting from that order being hit by buying at a higher mid price and a higher half spread. In case of a positive inventory position, however, the second term of Equation (29) is zero. This is explained by the fact that the Agent's willingness to benefit from a mid price increase and her willingness to protect herself from a half spread increase neutralize each other.

The component involving the half spread may be interpreted as follows: In case of a negative or no inventory position the Agent's sell limit order price comprises the current half spread with positive sign. Facing the potential need to equalize her inventory position by buying power from the market's sell side, the Agent prices the current half spread itself into her sell limit order price. In case of a positive inventory position, however, the Agent's sell limit order price comprises the current half spread with negative sign. [No time dependency]

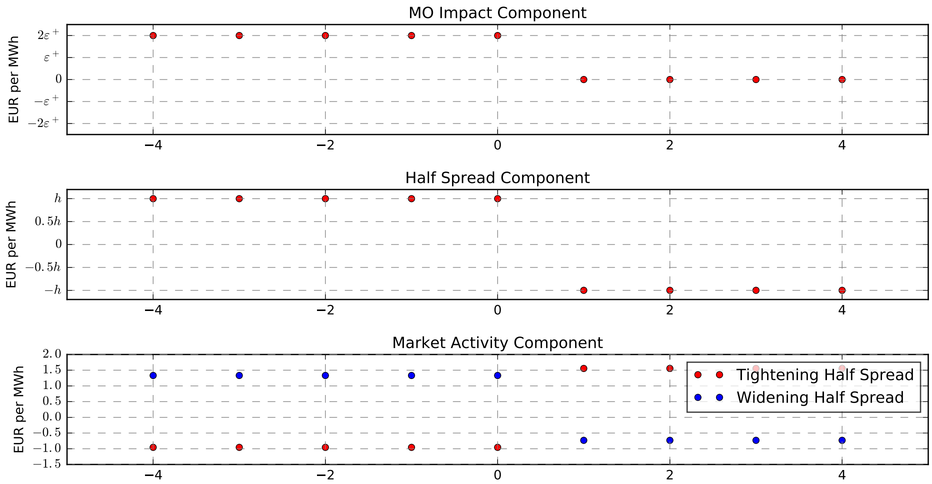

Figure 4: Dependence of different components of optimal markups on inventory position

We continue by considering the terminal inventory penalty term of Equation (29). In case of a negative inventory position, the Agent increases her sell limit order price in order to decrease the likeliness of her sell limit order being lifted, whereas in case of a positive inventory position she decreases her sell limit order price in order to increase the likeliness of her sell limit order being lifted. The amount by which the Agent adjusts her sell limit order price declines linearly in her inventory position. [No time dependency] The last two terms of Equation (27) also depend on the terminal inventory penalty. A possible interpretation is the adjustment of the terminal inventory penalty term of Equation (29) in case of imbalances in the intensities of buy and sell market order arrivals $\lambda_t^{\pm}$.

The running inventory penalty term of Equation (29) declines linearly in time. An interpretation may be that the closer gate closure gets, the smaller is the risk that the mid price and the half spread change unfavorably from the Agent's perspective. Compared to the optimal solution from Cartea and Jaimungal (2015), however, the linear time dependence is not reasonable. The last two terms of Equation (27) also depend on the running inventory penalty. The same interpretation as in case of the terminal inventory penalty may be drawn upon.

We conclude by considering the impact of the first six terms of Equation (27) which we refer to as market activity component. Figure 4 shows that impact for different levels of q. In the case where market activities in combination with expected activity impacts cause the half spread to tighten over time, the sign of the market activity component is negative as long as the Agent has a negative or no inventory position, i.e. it causes the sell limit order price to come closer to the half spread. The intuition behind this is that the Agent takes into account that the negative inventory unit resulting from her sell limit order being hit is expected to be liquidatable at the end of her trading window

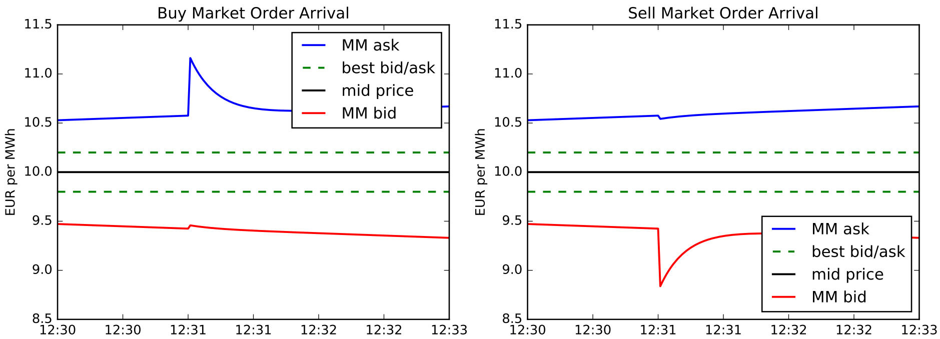

Figure 5: Impact of buy and sell market order arrival on Agent's optimal markups and markdowns

at a lower ask price and a lower half spread. In case of a positive inventory position, however, the market activity component has a positive sign, i.e. it causes the sell limit order price to move away from the best ask. This is due to the fact that the Agent takes into account that the positive inventory position resulting from her sell limit order being hit is expected to be liquidatable at the end of her trading window at a higher bid price and a lower half spread. In the case where market activities in combination with expected activity impacts cause the half spread to widen over time it is the other way around.

Assuming that the Agent does not have an inventory position, Figure 5 shows how the terms involving $g(\cdot)$ react to a buy and a sell market order arrival over time. The arrival of a buy market order causes the optimal markup δ_t^{+*} to increase. Roughly speaking this is due to the fact that the Agent expects an increase in the mid price and therefore protects herself from selling units now which would have to be rebought later at a higher price. The arrival of a sell market order, however, causes the optimal markup δ_t^{+*} to decrease slightly. Roughly speaking this is due to the fact that the Agent expects a decrease in the mid price and therefore wants to sell units now which could then be rebought later at a lower price. [Slightly] Furthermore, it may be observed that the impacts of increases in $\nu_t^{\pm}$ on the sum of the markup components for limit order-driven mid price and half spread changes decay exponentially and return to the base intensities within less than two minutes.

In case of a positive inventory position, the component referring to the arrival of buy market orders comprises 2 as coefficient. This is due to the fact that the Agent wants to protect herself from both the negative impact of a mid price increase and the negative impact of a half spread increase by considering a factor of 2 in the increase of her sell limit order price. By contrast, the component referring to the arrival of sell market orders comprises 0 as coefficient. This is due to the fact that the Agent wants to benefit from the positive impact of a mid price decrease by decreasing her sell limit order price and at the same time wants to protect herself from the negative impact of a half spread increase by increasing her sell limit order price. The interpretations are analogous for negative inventory positions.

4. Parameter Estimation

We estimate the parameters of optimal markups and markdowns with data acquired from EPEX SPOT SE which has emanated from their trading system "M7". The data comprises all limit orders which have been inserted *explicitly* into the market for delivery in GER/AUT between X and Y. Furthermore, it comprises all market orders which have been placed *explicitly* into that market. Limit orders with delivery areas other than GER/AUT which have been filled by market orders with delivery in GER/AUT are comprised but with the insertion timestamp being the same as that of the market order (i.e. not the actual insertion timestamp). All other insertions as well as cancellations of limit orders with delivery areas other than GER/AUT which have also appeared in the market for delivery in GER/AUT due to available interconnection capacity are missing.

Buy and sell market order arrivals are modeled as Hawkes point processes with conditional intensities $\lambda_t^{\pm}$ as defined by Equations (3) and (4). Da Fonseca and Zaatour (2014) provide the log-likelihood function for estimating the parameters of this conditional intensity. We estimate the parameters for each delivery hour and for each trading window (Mid and Late) separately. Since we use multiple trading days for estimation, we use the following log-likelihood function to do so:

$$LL^{\pm} = \sum_{i=1}^{M} \left\{ t_{i,N}^{\pm} - \mu^{\pm} t_{i,N}^{\pm} - \sum_{j=1}^{N} \frac{\zeta^{\pm}}{\rho^{\pm}} \left(1 - \exp\left(-\rho^{\pm}\left(t_{i,N}^{\pm} - t_{i,j}^{\pm}\right)\right)\right) \right.$$
$$\left. + \sum_{j=1}^{N} \ln\left(\mu^{\pm} + \sum_{t_{i,j}^{\pm} < t_{i,k}^{\pm}} \zeta^{\pm} \exp\left(-\rho^{\pm}\left(t_{i,j}^{\pm} - t_{i,j}^{\pm}\right)\right)\right) \right\} \tag{30}$$

where $t_{i,j}^{\pm}$ reflect timestamps of buy and sell market orders with delivery on day i.

To begin with, the suitability of the estimated Hawkes point processes with regards to capturing the intensities of market order arrivals is considered. In this context a test is used which is described in Ogata (1988) and which is based on analyzing residuals. With Hawkes point processes with intensities $\lambda_t^{\pm}$ as defined by Equations (3) and (4) at hand, the residuals $\tau_{i,j}^{\pm}$ may be obtained from the timestamps of buy and sell market orders $t_{i,j}^{\pm}$ one-to-one according to the transformation

$$\tau_{i,j}^{\pm} = \int_{0}^{t_{i,j}^{\pm}} \lambda_u^{\pm} du. \tag{31}$$

The residuals $\tau_{i,j}^{\pm}$ may also be considered as timestamps. The idea of the test is that if the estimated intensities do well in resembling the true intensities, the durations $\tau_{i,j}^{\pm} - \tau_{i,j-1}^{\pm}$ are iid exponentially distributed with unit rate. Probability-probability (PP) and quantile-quantile (QQ) plots are used to assess that. Figure **??** shows unit-rate exponential PP and QQ plots for differences in residuals which are generated on

	Unit	H11		H15		H18	
		Mid	Late	Mid	Late	Mid	Late
μ^+	trades per min						
ζ^+	trades per min						
ρ^+	n/a						
μ^-	trades per min						
ζ^-	trades per min						
ρ^-	n/a						
μ_ν^+	trades per min						
ζ_ν^+	trades per min						
ρ_ν^+	n/a						
μ_ν^-	trades per min						
ζ_ν^-	trades per min						
ρ_ν^-	n/a						
μ_υ^+	trades per min						
ζ_υ^+	trades per min						
ρ_υ^+	n/a						
μ_υ^-	trades per min						
ζ_υ^-	trades per min						
ρ_υ^-	n/a						
ε^+	EUR per MWh						
ε^-	EUR per MWh						
ε_ν^+	EUR per MWh						
ε_ν^-	EUR per MWh						
ε_υ^+	EUR per MWh						
ε_υ^-	EUR per MWh						

Table 1: Parameters of the estimated Hawkes point processes and expected absolute mid price and half spread impacts of market and limit order-driven mid price and half spread changes.

the basis of the parameters of the estimated Hawkes point processes for H11, H15 and H18 buy market orders in the trading window “Late”. Additionally, Figure **??** shows unit-rate exponential PP and QQ plots for differences in residuals which are generated on the basis of the parameters of estimated homogeneous Poisson point processes for H11, H15 and H18 buy market orders in the trading window “Late” for comparison.

[Figure: PP and QQ plots for buy market order arrivals in the trading window “Late”]

[Quality of estimated parameters]

Table 1 presents the estimated parameters of the Hawkes point processes for buy and sell market orders for products [HX], [HY] and [HZ]. [Interpretation of estimated parameters]

Concerning the intensities of buy and sell limit order-driven mid price and half spread changes the same log-likelihood function as for buy and sell market orders is used. [Quality and interpretation]

As pointed out above, the mid price and half spread impacts of buy and sell market orders depend on the shape of the limit order book which appears to change over time, and the market order volumes which appear to exhibit positive autocorrelation. In this work, however, neither of these two factors is attempted to be captured for simplicity reasons. Instead, the expected absolute mid price and half spread impacts of buy and sell market orders $\varepsilon^{\pm}$ are approximated by the means of historic absolute mid price and half spread impacts of buy and sell market orders. The estimates for products [HX], [HY] and [HZ] are presented in Table 1. [Interpretation]

The fill probabilities of buy and sell limit orders are assumed to decay exponentially in the distance of their price to the price of the current best buy and sell limit order at rate $\kappa_t^{\pm}$ (see Section 2.4). Given that for all products and trading windows under consideration the share of buy and sell market orders which do not pass the first level of the limit order book lies around 80%, the decay rates are assumed to be large enough for the risk-neutral component $\frac{1}{\kappa_t^{\pm}}$ to be 0.

5. Backtest

Guéant et al. (2013) describe a backtest to test a model for market maker pricing which is comparable to the one presented above. They point out that their model is intrinsically discrete in space due to the tick size and time due to the priority of older limit orders over younger limit orders with the same price. Against this background they assume the Agent to operate in time windows of length dt. Here dt is chosen to be X seconds.

The backtest itself builds upon trading in a particular stock over a particular time window in the past. Initially, the prices at which the Agent would have placed her sell and buy limit order at the beginning of the first time window of length dt are computed. If a buy (sell) market order with transaction price at or above (below) the price of the Agent's sell (buy) limit order arrived in that time window, it is assumed that her sell (buy) limit order would have been lifted entirely and that she would have immediately placed new sell and buy limit orders. If the Agent's sell or buy limit order would have not been lifted by the end of the first time window of length dt, the prices at which the Agent would have placed her sell and buy limit order at the beginning of the second time window of length dt are computed. If a buy (sell) market order arrived in that time window, transaction and sell (buy) limit order price are compared again to determine whether the Agent's limit order would have been lifted. This procedure is repeated until the end of the time window under consideration. Lastly, the prices of the sell and buy limit orders which would have been lifted and the terminal inventory position and mid price are used to determine the PnL.

Compared to reality the backtest suggested by Guéant et al. (2013) goes along with a number of simplifications. One is that it is not modeled that other market participants may react to the sell and buy limit orders placed by the Agent, in particular if the price of her sell (buy) limit order is lower (higher) than the price of the best sell (buy) limit order. Another is that sell or buy limit orders which have been lifted in reality but would have not been lifted if the Agent had actually placed her sell and buy limit orders are nevertheless removed from the order book. These simplifications are accepted.

Indicator	Measure	Unit	$\alpha = 0$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$
PnL	mean	EUR				
	std	EUR				
	Q_5	EUR				
	Q_{95}	EUR				
	skewness	n/a				
	kurtosis	n/a				
Inventory	mean	MW				
	std	MW				
	Q_5	MW				
	Q_{95}	MW				
	skewness	n/a				
	kurtosis	n/a				
Volume	mean	MW				
	std	MW				
	Q_5	MW				
	Q_{95}	MW				
	skewness	n/a				
	kurtosis	n/a				

Table 2: Backtest results for X to Y and "Late"

In addition to these simplifications Guéant et al. (2013) assume that the terminal inventory position may be liquidated at the terminal mid price. That means that they ignore both the half spread and the shape of the limit order book. In this work, however, the terminal inventory position is assumed to be liquidated at the best sell or buy limit order price. Hence, only the shape of the limit order order book is ignored.

[Figure: Optimal sell and buy limit order prices and buy and sell market orders which would have lifted the Agent's sell and buy limit orders as well as best sell and buy limit order prices for product H14 with delivery on April 15, 2015 in the trading window "Late"]

[Figure: Inventory position for product H14 with delivery on April 15, 2015 in the trading window "Late"]

[Figure: Distribution of PnLs, terminal inventory positions and trading volumes for the trading windows "Late" and "Mid"]

6. Conclusion and Outlook

A. Proofs

A.1. Proposition 1

Proof. Differentiating the functions contained in the arguments of the supremum functions and solving for $\delta^{\pm}$ yields as optimal markups and markdowns $\delta^{\pm *}$. □

A.2. Proposition 2

Proof. g_2 trivial.

g_1 not impacted by sign function. g_0 cancels out. Feynman-Kac. □

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